Test 1
MATH/STAT 170  October 18, 2007

1. Huntington Bank offers an account that pays 5%, compounded daily. Lafayette Bank wants to offer an account with the same annual effective rate, but compounded quarterly? What interest rate should they advertise. State your answer as a nominal quarterly rate—i.e. 1% per year, compounded quarterly.

\[
(1 + \frac{0.05}{365})^{365} = (1 + \frac{i}{4})^4
\]
\[
(1 + \frac{0.05}{365})^{\frac{365}{4}} = 1 + \frac{i}{4}
\]
\[
4 \left[ (1 + \frac{0.05}{365})^{\frac{365}{4}} - 1 \right] = i
\]
\[
i = 0.0503103939
\]

5.031% compounded quarterly

2. You borrow $25,000 to buy a really cool car. What will your monthly payments be if you take out a 5 year loan at 8% interest, compounded monthly?

\[
\frac{25,000 \left( 1 + \frac{0.08}{12} \right)^{60} - (1 + \frac{0.08}{12})^{60} - 1}{\frac{0.08}{12}} = 0
\]
\[
25,000 \left( 1 + \frac{0.08}{12} \right)^{60} = (1 + \frac{0.08}{12})^{60} - 1 \cdot P
\]
\[
25,000 \left( 1 + \frac{0.08}{12} \right)^{60} \frac{0.08}{12} = P \left[ (1 + \frac{0.08}{12})^{60} - 1 \right]
\]
\[
\frac{25,000 \left( 1 + \frac{0.08}{12} \right)^{60} \frac{0.08}{12}}{(1 + \frac{0.08}{12})^{60} - 1} = P
\]

\[
P = 506.9098573
\]

$ 506.91
3. What is the least amount I can have in my retirement fund to allow me to withdraw $5,000 at the end of each month for the next 30 years, assuming that my fund earns 8% interest, compounded monthly?

\[
P \left(1 + \frac{0.08}{12}\right)^{360} \left(1 + \frac{0.08}{12}\right)^{30(12)} - 1 = \frac{5,000}{\frac{0.08}{12}} (5,000) = 0
\]

\[
P \left(1 + \frac{0.08}{12}\right)^{360} = \frac{5,000}{\frac{0.08}{12}} (5,000)
\]

\[
P = \frac{\left(1 + \frac{0.08}{12}\right)^{360} - 1}{\frac{0.08}{12} \left(1 + \frac{0.08}{12}\right)^{360}} (5,000)
\]

\[
P = 681,417.4707
\]

\[
\text{\$681,417.47}
\]

4. The estimated cost for an instate student to attend Purdue for 2007-2008 is $17,704. Assuming that the cost remains constant, and it is paid at the beginning of the year, what is the present value of a four year education for an incoming freshman at 5% interest?

\[
PV = 17,704 + 17,704(1.05)^{-1} + 17,704(1.05)^{-2} + 17,704(1.05)^{-3}
\]

\[
PV = 17,704 + 17,704(0.95) + 17,704(0.90) + 17,704(0.85)
\]

\[
PV = 17,704 + 16,860.95 + 16,058.05 + 15,278.97
\]

\[
PV = 65,916.38
\]

\[
\text{\$65,916.38}
\]
5. The following chart shows the deposits and withdrawals in an account that earns \( i\% \) compound interest per year. The balance on Jan. 1 was $2000 and the Dec. 31 balance was $2300. Approximate \( i \).

<table>
<thead>
<tr>
<th>March 1</th>
<th>June 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-400</td>
<td>+600</td>
</tr>
</tbody>
</table>

\[
2000 \left(1 + i\right)^{10/12} - 400 \left(1 + i\right)^{10/12} + 600 \left(1 + i\right)^{7/12} = 2300
\]

\[
2000 \left(1 + i\right) - 400 \left(1 + \frac{5}{8}i\right) + 600 \left(1 + \frac{7}{12}i\right) \approx 2300
\]

\[
2000 + 2000i - 400 + \frac{1000}{3}i + 600 + 350i \approx 2300
\]

\[
\frac{6000}{3}i - \frac{1000}{3}i + \frac{1050}{3}i \approx 100
\]

\[
\frac{6050}{3}i \approx 100
\]

\[
i \approx \frac{3}{6050} (100)
\]

\[
i \approx 0.049586777
\]

\[
4.96\%
\]
6. What price should you pay for a $1000 face value, 10 year bond which has $75 annual coupons, assuming that you want a 3% yield (compounded annually)? (Thus you receive 10 payments of $75 at the end of the year for 10 years, plus a final payment of $1,000.)

\[
FV = \frac{(1.03)^{10} - 1}{.03} \times 75 + 1,000
\]

\[
FV = 859.79 + 1,000
\]

\[
FV = 1,859.79
\]

\[
PV = FV \times (1.03)^{-10}
\]

\[
PV = 1,383.859128
\]

7. You sell the bond in Exercise 6 after 2 years to an individual wanting a 4% yield. What is the selling price? *Hint:* The price is the present value of all of the future payments at the stated interest rate.

\[
FV = \frac{(1.04)^{8} - 1}{.04} \times 75 + 1,000
\]

\[
FV = 691.07 + 1,000
\]

\[
FV = 1,691.07
\]

\[
PV = FV \times (1.04)^{-8}
\]

\[
PV = 1,235.646671
\]

\$1,235.65