Test 1
MATH/STAT 170 October 18, 2007

1. Huntington Bank offers an account that pays 5%, compounded daily. Lafayette Bank wants to offer an account with the same annual effective rate, but compounded quarterly? What interest rate should they advertise. State your answer as a nominal quarterly rate--i.e. i% per year, compounded quarterly.

$$\left(1+\frac{.05}{365}\right)^{365} = \left(1+\frac{i}{4}\right)^{4}$$

$$\left(1+\frac{.05}{365}\right)^{\frac{365}{4}} = 1+\frac{i}{4}$$

$$4\left[\left(1+\frac{.05}{365}\right)^{\frac{365}{4}}-1\right] = i$$

$$i = 0.050310339$$

5.031%, compounded quarterly

2. You borrow $25,000 to buy a really cool car. What will your monthly payments be if you take out a 5 year loan at 8% interest, compounded monthly?

$$25{,}000\left(1+\frac{.08}{12}\right)^{60} - \frac{\left(1+\frac{.08}{12}\right)^{60}-1}{\frac{.08}{12}}P = 0$$

$$25{,}000\left(1+\frac{.08}{12}\right)^{60} = \frac{\left(1+\frac{.08}{12}\right)^{60}-1}{\frac{.08}{12}}P$$

$$25{,}000\left(1+\frac{.08}{12}\right)^{60}\left(\frac{.08}{12}\right) = P\left[\left(1+\frac{.08}{12}\right)^{60}-1\right]$$

$$\frac{25{,}000\left(1+\frac{.08}{12}\right)^{60}\left(\frac{.08}{12}\right)}{\left(1+\frac{.08}{12}\right)^{60}-1} = P$$

$$P = 506.90985\overline{73}$$

$506.91

3. What is the least amount I can have in my retirement fund to allow me to withdraw $5,000 at the end of each month for the next 30 years, assuming that my fund earns 8% interest, compounded monthly?

$$P\left(1+\frac{.08}{12}\right)^{30(12)} - \frac{\left(1+\frac{.08}{12}\right)^{30(12)}-1}{\frac{.08}{12}}(5{,}000)=0$$

$$P\left(1+\frac{.08}{12}\right)^{360} = \frac{\left(1+\frac{.08}{12}\right)^{360}-1}{\frac{.08}{12}}(5{,}000)$$

$$P = \frac{\left(1+\frac{.08}{12}\right)^{360}-1}{\frac{.08}{12}\left(1+\frac{.08}{12}\right)^{360}}(5{,}000)$$

$$P = 681{,}417.4707$$

$\$681,417.47$ ✓

4. The estimated cost for an instate student to attend Purdue for 2007-2008 is $17,704. Assuming that the cost remains constant, and it is paid at the beginning of the year, what is the present value of a four year education for an incoming freshman at 5% interest?

$$PV = 17{,}704 + 17{,}704(1.05)^{-1} + 17{,}704(1.05)^{-2} + 17{,}704(1.05)$$

$$PV = 17{,}704 + 16{,}860.95 + 16{,}058.05 + 15{,}293.38$$

$$PV = 65{,}916.38$$

$\$65,916.38$ ✓

5. The following chart shows the deposits and withdrawals in an account that earns i% compound interest per year. The balance on Jan. 1 was \$2000 and the Dec. 31 balance was \$2300. Approximate i.

March 1	June. 1
-400	+600

$$2000(1+i)^1 - 400(1+i)^{\frac{10}{12}} + 600(1+i)^{\frac{7}{12}} = 2{,}300$$

$$2000(1+i) - 400\left(1+\tfrac{5}{6}i\right) + 600\left(1+\tfrac{7}{12}i\right) \approx 2{,}300$$

$$2000 + 2000i - 400 - \tfrac{1000}{3}i + 600 + 350i \approx 2{,}300$$

$$\tfrac{6000}{3}i - \tfrac{1000}{3}i + \tfrac{1050}{3}i \approx 100$$

$$\tfrac{6{,}050}{3}i \approx 100$$

$$i \approx \tfrac{3}{6{,}050}(100)$$

$$i \approx 0.049586777$$

4.96 %

6. What price should you pay for a $1000 face value, 10 year bond which has $75 annual coupons, assuming that you want a 3% yield (compounded annually)? (Thus you receive 10 payments of $75 at the end of the year for 10 years, plus a final payment of $1,000.)

$$FV = \frac{(1.03)^{10} - 1}{.03}(75) + 1{,}000$$

$$FV = 859.79 + 1{,}000$$

$$FV = 1{,}859.79$$

$$PV = FV(1.03)^{-10}$$

$$PV = 1{,}383.859128$$

$\$1{,}383.86$

7. You sell the bond in Exercise 6 after 2 years to an individual wanting a 4% yield. What is the selling price? *Hint:* The price is the present value of all of the future payments at the stated interest rate.

$$FV = \frac{(1.04)^{8} - 1}{.04}(75) + 1{,}000$$

$$FV = 691.07 + 1{,}000$$

$$FV = 1{,}691.07$$

$$PV = FV(1.04)^{-8}$$

$$PV = 1{,}235.646671$$

$\$1{,}235.65$