Week 4

(60) How much will you have if you deposit \$200 dollars at the end of the year for the next 20 years if the account earns 4% interest per year.

$200 \times \frac{(1.04)^{20}-1}{.04} = \$5955.62$

(23) George is borrowing \$20,000 and will pay 8.5% interest. He will pay off the loan in three annual installments, \$6,000 at the end of the first year, \$7,000 at the end of the second year, and a final payment at the end of the third year. What should the amount of the final payment be?

$20{,}000(1.085)^3 - 6{,}000(1.085)^2 - 7{,}000(1.085) = \$\mathbf{10{,}887.43}$

(28) Amount you will have if you invest \$75 at the end of each month for 10 years if the account pays 7.5% compounded monthly.

$\left(1 + \frac{.075}{12}\right)^{1} = 1.006250000$ .00625

Each month, account grows by: ~~$(1.075)^{\frac{1}{12}}$~~ $= 1.006044919$ ∴ monthly rate is ~~.006044919~~;

$75\frac{(1.00625)^{120}-1}{.00625} = \$13{,}349.78$ ~~\$13,164.34~~

(44) What is the present value of \$25,000 ten years from now at 8% interest?

$PV = 25{,}000 \times (1.08)^{-10} = \$\mathbf{11{,}579.84}$

(46) What is the future value of \$25,000 ten years from now at 8% interest?

$FV = 25{,}000 \times (1.08)^{10} = \$\mathbf{53{,}973.12}$

(50) An account earning annual effective interest i had a balance of \$ 1500 on Jan. 1, 2000 and a balance of \$1773.63 on Jan. 1, 2001. The year's activity in the account consisted of (a) a deposit of \$ 500 on June 1, a withdrawal of \$ 600 on Aug. 1 and a deposit of \$ 300 on Dec. 1. Approximate i using the technique from Example 6.

$1{,}773.63 = 1{,}500(1+i) + 500(1+i)^{\frac{7}{12}} - 600(1+i)^{\frac{5}{12}} + 300(1+i)^{\frac{1}{12}}$

$1{,}773.63 \approx 1{,}500(1+i) + 500\left(1+\frac{7i}{12}\right) - 600\left(1+\frac{5i}{12}\right) + 300\left(1+\frac{i}{12}\right)$

$\therefore 1{,}773.63 - 1{,}500 - 500 + 600 - 300 \approx \left(1{,}500 + 500\left(\frac{7}{12}\right) - 600\left(\frac{5}{12}\right) + \frac{300}{12}\right)i$

$73.63 \approx 1{,}566.67i;\ i \approx \frac{73.63}{1{,}566.67} = .0469978;\ \mathbf{i \approx 4.7\%}$

Week 4

(51) An insurance company earns 7% on their investments. How much must they have on reserve (the present value of claims) on January 1, 2002 to cover the claims for the next 3 years, if they expect claims of $500,000 for 2002, $300,000 for 2003 and $250,000 for 2004. For sake of simplicity, assume that the claims are all paid on Dec. 31 of the stated year.

Let x = total claims (plus forgone interest) that will be paid over 3 years;

$x = 500{,}000(1.07)^2 + 300{,}000(1.07) + 250{,}000;$

$x = 1{,}143{,}450$

$PV = 1{,}143{,}450(1.07)^{-3} = \mathbf{\$933{,}395.81}$

Could also do $x = 500{,}000(1.07)^{-1} + 300{,}000(1.07)^{-2} + 250{,}000(1.07)^{-3}$

(53) An interest rate of 6% compounded three times a year is equivalent to what rate of interest compounded twice a year.

$$\left(1+.\frac{06}{3}\right)^3 = \left(1+\frac{i}{2}\right)^2$$

$$1.061208 = \left(1+\frac{i}{2}\right)^2$$

$$\sqrt{1.061208} = 1+\frac{i}{2}$$

$$2\left(\sqrt{1.061208}-1\right) = i$$

$i = \mathbf{6.02999\%}$