

MA351 Exam 3 Study Sheet

The exam covers only Chapter 3, Sections 3.1, 3.2, 3.3, 3.5, and Chapter 6, only Section 6.1. It does not cover 3.4 and not any of the subsections such as 3.3.1, 3.3.2, etc. There will be some proofs similar to the problems noted below. Also, you should be able to prove Theorem 3 on p. 195, both parts. **Your proof should use the argument from the text.** Other arguments will not be accepted.

Note Unfortunately, there is a mistake in the proof of Theorem 3 on p. 195 in the text. Here is the correct proof:

Theorem. *Let A be an $n \times n$ matrix. If either of the following statements hold, then A is invertible and $B = A^{-1}$.*

(a) *There is a matrix B such that*

$$BA = I \tag{1}$$

(b) *There is a matrix B such that*

$$AB = I \tag{2}$$

Proof. We begin with part (a). To prove the invertibility of A , recall that $\text{rank}(BA) \leq \text{rank} A$. Hence, from equation (1),

$$n = \text{rank} I \leq \text{rank} A$$

Since A is $n \times n$, this shows that $\text{rank} A = n$, proving the invertibility.

Then $B = A^{-1}$ follows from multiplying both sides of equation (1) on the right by A^{-1} :

$$(BA)A^{-1} = IA^{-1}$$

$$B(AA^{-1}) = A^{-1}$$

$$BI = A^{-1}$$

$$B = A^{-1}$$

Part (b) is similar and left as an exercise for the reader. □

The Instructions for the exam are as follows:

No notes, calculators, or other electronic devices such as cell phones are allowed during the exam. Violation of this policy will

result in an automatic 0 on the test. There should be nothing on your desk other than the test and something to write with. Please put all work and answers on either the test sheets or, if extra space is needed, on the attached scratch paper.

Important! Do not put answers on the backs of the pages of the test. Your work will be scanned and e-mailed to me. **The backs of the sheets will NOT be scanned.** Hence, I will not grade work on the backs of the pages.

Justify all answers. A correct answer without supporting justification is worth NO credit!

Chapter 3

1. p. 163, 1, 6, 7, 20, 24, 25, 27, 28, 29. In 24 and 25 you should be able to prove that the given transformations are linear, except of course in 25(c).
2. p. 183, 14, 17, 18, 23, 27, 38
3. p. 198, 3, 7, 14, 15, 16, 17, 20, 23, 27
4. p. 234, 4, 5, 6, 17, 18, 21

Chapter 6

1. p. 321, 7, 8, 9, 10, 17