

MA 366 Spring 2009 Assignments

Exercises will be added as the week progresses. All assignments will be complete by noon Saturday for an assignment due on Monday and by noon Wednesday for an assignment due on Friday.

Week 1:

For Friday: Read 1.2–1.3.

Exercises p. 25: 7, 9, 16

p. 360, The answer to Exercise 8 is given in the back of the text. Substitute the given functions x_1 and x_2 into the the system to show that they do solve the system. Show that they also satisfy the desired initial conditions stated in the exercise.

Read p. 31-39

Exercises p. 39: 1(use dfield), 4(use dfield), 13,15, 16

Week 2:

For Wednesday 1/21:

Read p. 21, first paragraph, p. 31-39, p. 42-47

Exercises : p. 25, 13, 14

p. 360, The answer to Exercise 10 is given in the back of the text. Substitute the given functions x_1 and x_2 into the the system to show that they do solve the system. Show that they also satisfy the desired initial conditions stated in the exercise.

p. 39: 14,19, 24 (use dfield)

p. 24: The differential equation in exercise 4 is not linear. Change one single number to make the equation linear. Do not just change one of the coefficients to zero!

p. 47, 1, 7, 10 (use dfield), 23. (Hint: Set $y' = 0$ and solve for x . You don't need to compute y' to do this! How can you tell that it is a minimum and not a maximum or a point of inflection?)

For Friday 1/23 (This is the complete assignment.):

In all of these problems, do not do the computer part!

Read Example 1, p. 52. Then do Exercise 4, p. 60.

p. 48, 22 (This is similar to 23, but now you want $y' = \infty$.), 24 (Prove that your value really is a maximum.), 30 (a)-(e)

p. 62, 16.

For Friday 1/30 This is the complete assignment:

In all of these problems, do not do the computer part!

p. 50, 31(a), (b)

p. 60, 5, 20 (You may assume that $F = mg$, as in Example 1, p. 2.)

p. 100 1, 2, 3, 21

For Monday, 2/1 (This is the complete assignment.)

1. For the following differential equation (a) find the general solution (b) find a value of y_0 for which the initial value problem $y(0) = y_0$ has two solutions. (c) Explain, using the Existence and Uniqueness theorem, why there is only one possible choice of y_0 in (b) (d) Solve the initial value problem $y(0) = 1$. There is only one solution! (You need to use the initial condition to determine the sign of the square root.)

Hint Completing the square helps in solving for y in terms of x .

$$(3 + 2y)y' = 3x^2 - 1$$

2. For the following differential equation (a) find the general solution (c) find a value of t_0 for which the initial value problem $y(t_0) = 1$ has no solution. (c) Explain, using the Existence and Uniqueness theorem, why there is only one possible choice of y_0 in (c)

$$ty' + 2y = t^2e^{-t}$$

In all of these problems, do not do the computer part!

p. 50, 32(a), (b)

p. 60, 22(a), (b) (You may assume that $F = mg$, as in Example 1, p. 2.)

p. 77, 27(b) *Hint* You need to express $\frac{dy}{dx}$ in terms of v and $\frac{dv}{dx}$.

For Friday, 2/6/09. This is the complete assignment

Read p. 139-143, p.157-163

- (a) Find the solution to the following initial value problem. You should find that the solution is unique and exists for all x . Why, in terms of the existence and uniqueness theorem, is this somewhat surprising?

$$3(y - 2)^2 y' = 4x^3, \quad y(0) = 2.$$

In all of these problems, do not do the computer part

p. 100, 7, 11

p. 144, 9 (Do not graph),

I did not cover the necessary material for the following problems. Hence they will put onto Monday's assignment.

p. 163, 1, 3, 7, 18 (Do not graph)

For Monday, 2/9/09 This is the complete assignment.

In all of these problems, do not do the computer part

p. 144, 5,6,7, 12 (Do not graph)

p. 163, 1, 3, 7, 18 (Do not graph)

For Monday, 2/16/09 This is the complete assignment.

In all of these problems, do not do the computer part

p. 144, 17,21,

p. 163, 11, 19 (Do not graph), 34(a), (b)

p. 171, 2, 5

p. 183, 1, 2, 5

For Friday, 2/20/09

In all of these problems, do not do the computer part This is the complete assignment.

p. 144, 23

p. 165, 35

p. 171, 11 (Don't graph)

p. 183, 29

p. 189, 5, 17.

In the problems on p. 189 DO NOT use formulas (26) and (27). Instead, explicitly write the equations

$$\begin{aligned}u_1' y_1 + u_2' y_2 &= 0 \\u_1' y_1' + u_2' y_2' &= g(t)\end{aligned}$$

as I did in class where y_1 and y_2 and g are EXPLICIT functions. (The y_i are the fundamental set of solutions and the g is the function on the right side of the differential equation.) Then express this system in terms of matrices and use Cramer's rule to find u_1' and u_2' . Finally, integrate to find u_1 , u_2 , and the general solution to the equation, which is

$$y = u_1 y_1 + u_2 y_2.$$

For Monday, 2/23/09

In all of these problems, do not do the computer part This is the complete assignment.

p. 88, 3

The "phase line" is the same as the "phase plane". "Graph $f(y)$ " means graph y' against y . In the example done in class, this turned out to be a parabola.

p. 144, 24

p. 165, 36

p. 171, 12 (Don't graph)

p. 183, 30

p. 189, 10.

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$$y = u_1 y_1 + u_2 y_2.$$

For Friday, 2/27/09

This is the complete assignment.

p. 189, 11.

In the problems on p. 189 DO NOT use formulas (26) and (27). Instead, explicitly write the equations

$$\begin{aligned}u_1' y_1 + u_2' y_2 &= 0 \\ u_1' y_1' + u_2' y_2' &= g(t)\end{aligned}$$

as I did in class where y_1 and y_2 and g are EXPLICIT functions. (The y_i are the fundamental set of solutions and the g is the function on the right side of the differential equation.) Then express this system in terms of matrices and use Cramer's rule to find u_1' and u_2' . Finally, integrate to find u_1 , u_2 , and the general solution to the equation, which is

$$y = u_1 y_1 + u_2 y_2.$$

p. 202,

1,

5 *Note:* The information in the first sentence of the statement of the problem is needed to find the spring constant k . See Example 1, p. 194.

9: Do only the part up to “Plot u versus t .” (You may use a computer.) Also plot on the same graph $y = \left(\sqrt{2^2 + (5/\sqrt{6})^2}\right) e^{-10t}$ and $y = -\left(\sqrt{2^2 + (5/\sqrt{6})^2}\right) e^{-10t}$. *Note:* The information in the first sentence of the statement of the problem is needed to find the spring constant k .

Additional problem:

- (a) In Exercise 5 on p. 202, for which coefficients of damping would the motion be over damped?

For Monday, 3/2/09

This is the complete assignment.

p. 202 6,

10: Do only the part up to “Plot u versus t .” (You may use a computer.)

p. 215, 1 (See Friday’s class notes), 5, 6, 7(a)

For Friday, 3/6/09 This is the complete assignment

p. 202 2,

13: The quasi period is $\mu/(2\pi)$ where μ is as in formula (25), p. 197.

p. 215: 9 (Do not graph), 15, 17(a)

For Monday, 3/9/09 This is the complete assignment

p. 216: 17, (b), (c), (d) *Note:* This exercise demonstrates two important ideas: (1) The amplitude never equals infinity. Perfect resonance does not happen when there is resistance. (2) The maximum amplitude DOES NOT occur at the resonate frequency of the mass-spring combination. In this case, without resistance, resonance occurs when $\omega = \sqrt{2}$. The maximum amplitude occurs at $\omega = 1.4031$ which is just slightly less than $\sqrt{2}$. Friction shifts the resonant frequency.

p. 259: 2 (See Example 2, p. 255), 15(a), (b)

For Friday, 3/27/09 This is the complete assignment

p. 259: 7(a), (b), 17(a), (b), 19(a),

3. Derive the fact that

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left(\frac{x}{2}\right)^{2n}$$

is a solution to Bessel's equation $x^2y'' + xy' + x^2y = 0$ by seeking a solution of the form $f(x) = \sum_0^{\infty} a_n x^n$. You should discover that a_1 must be zero.

4. Determine the largest r such that $f(x)$ has a series expansion

$$f(x) = \sum_0^{\infty} a_n x^n$$

which is valid for all x , $|x| < r$ for the following functions $f(x)$. (See Examples 1, 2 on p. 263-264.) *You are not asked to find the expansion.*

(a) $f(x) = \frac{x^3}{x^2-x+12}$. *Ans: $r = 3$.*

(b) $f(x) = \frac{x^3}{x^2+4x+8}$. *Ans: $r = 2\sqrt{2}$.*

For Monday, 3/30/09 This is the complete assignment

Read the discussion of **Reduction of order** on p. 170-171. Then do p. 173: 23

p. 259: 3, Solve by replacing x by $t - 1$ as was done in problem 19.

p. 265: 9, Problem 6. Use the following theorem which follows from Theorem 5.3.1 on p. 262 of the text:

Theorem 1. *Suppose that y satisfies*

$$P(x)y'' + Q(x)y' + R(x)y = 0$$

where P , Q , and R are polynomials with $P(0) \neq 0$. Then $y(x)$ has a series expansion

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

which converges at least for all $|x| < r$ where r is the length of the smallest zero of $P(x)$. (Complex zeros count!)

For Friday, 4/3/09 This is the complete assignment

Read the discussion of **Reduction of order** on p. 170-171. Then do

p. 173: 28

p. 265: 6

p. 282, 1 (To see that it has regular singularities, first multiply by x .)

5. For the differential equation in Exercise 8 on p. 282, give the approximating Euler equation. (See equation (6) on p. 278.) Use it to find the indicial equation and its roots. Use Theorem 5.6.1 on p. 289 to describe the expected form of the solution. **Do not find the coefficients!** This is meant to be a *short* exercise.

For Friday, 4/10/09

p. 282, 5

6. For the differential equation in Exercise 9 on p. 282, give the approximating Euler equation. (See equation (6) on p. 278.) Use it to find the indicial equation and its roots. Use Theorem 5.6.1 on p. 289 to describe the expected form of the solution. **Do not find the coefficients!** This is meant to be a *short* exercise.
7. Repeat the preceding exercise for the equation in Exercise 17, p. 290. **Do NOT find the terms of the solutions.**

p. 398, 1 (a) Just find the solution—do not describe the behavior

For Friday, 4/17/09 This is the complete assignment

p. 359, 7(a). By "solve for x_2 ", they mean write $x_2 = x_1' + 2x_1$. Show also that you get the same answer using the eigenvector technique.

p. 290 7, 8,

Read Example 3 p. 397. (See Example 5 on p. 381.)

p. 398, 2(a), (b), 3(a), (b), When you plot the trajectories be sure to plot the four trajectories that lie along the eigenvectors as well as a few others. The "keyboard input" option in pplane is useful for this.

11, 15, Do not describe the behavior of the solution.

For Monday, 4/20/09 This is the complete assignment

p. 290 10

p. 398, 17 Do not describe the behavior of the solution.

8. Compute e^A for the following matrices A using formula (22) on p. 416.

(a)

$$A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(c)

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

(d)

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

9. Let A be the 2×2 matrix in Exercise 1 on p. 428.

(a) Use the method demonstrated in class to find e^{tA} .

(b) Use the answer from 10a to find the general solution to the given system. **Other methods will not be accepted.** *Note:* Your answer might differ somewhat from that in the text due to a different choice of basis for the solution.

For Friday, 4/24/09 This is the complete assignment

p. 409, 1(a)

10. Let A be the 2×2 matrix in Exercise 2 on p. 428.

(a) Use the method demonstrated in class to find e^{tA} .

- (b) Use the answer from 10a to find the general solution to the given system. **Other methods will not be accepted.** *Note:* Your answer might differ somewhat from that in the text due to a different choice of basis for the solution.

11. Let A be the 3×3 matrix in Exercise 5 on p. 428.

- (a) Use the method demonstrated in class to find to find a basis $\{X_1, X_2\}$ for the space of generalized eigenvectors corresponding to the eigenvalue $r = 2$. **Other methods will not be accepted.**
- (b) Use the method demonstrated in class to find $e^{tA}X_1$ and $e^{tA}X_2$.
- (c) Use the answer from 12b to find the general solution to the given system, given that an eigenvector corresponding to $r = -1$ is

$$Y_o = \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix}.$$

Other methods will not be accepted. *Note:* Your answer might differ somewhat from that in the text due to a different choice of basis for the solution.

For Wednesday, 4/29/09 This is the complete assignment

12. Let

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

- (a) Use the method demonstrated in class to find to find a basis $\{X_1, X_2\}$ for the space of generalized eigenvectors corresponding to the eigenvalue $r = 3$. **Other methods will not be accepted.**
- (b) Use the method demonstrated in class to find $e^{tA}X_1$ and $e^{tA}X_2$.
- (c) Use the answer from 12b to find the general solution to the given system, given that an eigenvector corresponding to $r = 2$ is

$$Y_o = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}.$$

Other methods will not be accepted.

13. A certain 7×7 matrix A has characteristic polynomial $-(r-1)^4(r+5)^3$. Let X be a generalized eigenvector for A corresponding to $r = 1$. Give a formula for $e^{tA}X$ that does not require summing an infinite series. Your formula should use as few matrix products as possible relative to the given information.
14. Repeat Exercise 13 for a generalized eigenvector Y corresponding to $r = -5$.
15. A certain 4×4 matrix A has characteristic polynomial $(r-3)^4$. Give a formula for $e^{tA}X$ that does not require summing an infinite series. Your formula should use as few matrix products as possible relative to the given information.
16. A certain real 3×3 matrix A has eigenvalues -2 and $2+i$ with corresponding eigenvectors

$$X_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 1 \\ 1+i \\ 2 \end{bmatrix}.$$

Give a fundamental set of **real** solutions for the equation $X'(t) = AX(t)$.

p. 311, 7, 15 p. 320, 11