

MA366 Final Study Sheet

Show all work. A correct answer without supporting work is worth NO credit! (Some calculators can solve differential equations.) You will not be asked to graph solutions on the test. *I will not ask questions about Wronskians or phase plane plots.*

- (1) p. 259, 6, 8, , p. 259, 10, p. 265, 7, p. 173 24, 26, p. 282, 3, 4, p. 290, 12.
- (2) For the differential equation in Exercise 2 on p. 282, give the approximating Euler equation. Use it to find the indicial equation and its roots. Use Theorem 5.6.1 on p. 289 to describe the expected form of the solution. **Do not find the coefficients! If I refer to Theorem 5.6.1 on p. 289 of the text on the final, you should know what I am talking about. I will not provide the theorem for you!**
- (3) Repeat Exercise 2 for the differential equations in Exercises 3 and 10 on p. 282.
- (4) Find all **real** solutions to the following problems. *No complex numbers allowed!* If no initial conditions are given, give the general solution. *On the exam I will provide the eigenvectors. If I don't provide the eigenvalue, you can determine it from the equation $AX = rX$. On the generalized eigenvector questions you should use the method demonstrated in class.*
p. 398, 4,6, 14, 16, 18
p. 409, 2, 6, 8, 10,
- (5) Find e^{tA} for the 2×2 matrices in Exercises 4 and 10 on p. 428. Then use your answer to find the general solution to the system.
- (6) Find e^{tA} for the following matrix. Then use your answer to find the general solution to the system $X' = AX$.

$$A = \begin{bmatrix} 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- (7) Let A be the 3×3 matrix in Exercise 12 on p. 429.

- (a) Use the method demonstrated in class to find to find a basis $\{X_1, X_2\}$ for the space of generalized eigenvectors corresponding to the eigenvalue $r = 1$. **Other methods will not be accepted.**
- (b) Use the method demonstrated in class to find $e^{tA}X_1$ and $e^{tA}X_2$.
- (c) Use the answer from 7b to find the general solution to the given system, given that an eigenvector corresponding to $r = 2$ is

$$Y_o = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Other methods will not be accepted.

- (8) A certain 6×6 matrix A has characteristic polynomial $-(r - 2)^4(r - 3)^2$. Let X be a generalized eigenvector for A corresponding to $r = 2$. Give a formula for $e^{tA}X$ that does not require summing an infinite series. Your formula should use as few matrix products as possible relative to the given information.
- (9) Repeat Exercise 8 for an generalized eigenvector Y corresponding to $r = 3$.