1. a) \( u = \frac{52.54}{50} = 1.0508 \) 
   b) \( d = \frac{43.86}{50} = 0.8772 \) 
   c) \( u \cdot d = e^{(r-s)k \pm d_k} (r-s)(h-\delta_0k) = e^{2(r-s)k} = e^{2(r-s)(\delta_0)} = (1.1708)^2(0.8772) \)  
   \( r = \frac{0.0400}{(r-s)k} \) 
   \[ e^{-0.08(0.25)} = 0.9201 \]
   \( e^{-0.06(0.25)} = 0.9404 \) 
   \( C = e^{-0.08(0.25)} \left[ \begin{array}{c} 0.3(0.96)^3(93.1) + 0.7(0.96)^3(16.8) \end{array} \right] = 10.3025 \) 
   \( \Delta(94.1) e^{-0.3152(0.25)} + B e^{-0.08(0.25)} = 16.8 \) 
   \( \Delta(90.4) e^{-0.3152(0.25)} + B e^{-0.06(0.25)} = 0 \) 
   \( 28.7112 \Delta = 16.8 \) \( \Rightarrow \) \( \Delta = \frac{16.8}{28.7112} \Rightarrow \Delta = 0.5851 \Rightarrow \Delta S = 51.4920 \) 
   \( \Rightarrow B = -43.9129 \) (i.e., we borrow 43,912.9)
   d) Early exercise value = 88 - 80 = 8 \( \sqrt{\text{value of call if held}} = \Delta S + B = 8.5791 \) 
   \( \Rightarrow \) Early exercise would occur

2. a) \( F_{0,t} = 100 - 5 e^{-0.08(0.5)} = 95.1961 \) \( \Rightarrow \sigma_p = \frac{\text{Std of } F_{0,t}}{F_{0,t}} = \frac{2}{95.1961} = 0.02101 \) 
   b) \( u = e^{(0.08)(0.25) + 0.2101} = 1.1332 \) 
   c) \( d = e^{(0.08)(0.25) - 0.2101} = 0.9185 \) 
   d) \( F_{0,t} = 95.1961 \) 
   e) \( F_u = 95.1961(1.1332)^4 = 107.8767 \) 
   f) \( S_u = F_u + PV(d) = 107.8767 + 5 e^{-0.08(0.25)} = 112.7777 \)

3. Convexity requirement:
   \( \frac{20 - x}{50 - 50} \geq \frac{x - 5}{x - 5} \Rightarrow \frac{20 - x}{20} \geq \frac{x - 5}{10} \) 
   \( \Rightarrow 200 - 10x \geq 20x - 100 \Rightarrow 300 \geq 30x \Rightarrow x \geq 10 \)
   Thus, an arbitrage opportunity exists if \( x > 10 \)

4. Convexity violated:
   \( \frac{35 - 10}{50 - 30} \leq \frac{40 - 35}{60 - 50} \Rightarrow 1.25 \leq .5 \) Convexity is violated

5. Thus, the arbitrage is to buy 1 30-strike put, buy 2 60-strike puts, and sell 3 50-strike puts.
6. No-arbitrage requirement: \( x - 25 \leq 50 - 20 \Rightarrow x \leq 55 \)

Thus, there is an arbitrage opportunity if \( x > 55 \). In such a case, the arbitrage would be to buy the 20-strike put and sell the 50-strike put.

7. \( \rho_1 = 0.04 \), \( \rho_2 = 0.02 \), \( x_0 = 101 \frac{Y}{b} \)
   
   - \( \rho \)-denominated call: right to exchange \( \$1 \) for \( 100 \) \( Y \) at \( t = \frac{1}{2} \) (price is \( \frac{d}{\beta} \))
   
   - \( \rho \)-denominated put: right to exchange \( \$1 \) for \( 125 \) \( Y \) at \( t = \frac{1}{2} \) (price is \( \frac{c}{\beta} \))

\[ \Rightarrow E = (1.25)(101) x = 126.25 \times \frac{Y}{b} \]

8. \( \sigma = 0.12 \), \( u = e^{(0.02 - 0.04)(\frac{1}{2}) \times \frac{\sigma^2}{2}} + 0.12(\frac{\sigma^2}{2}) = 1.0170 \)

9. \( C - P = (S - PV(div)) - PV(K) \Rightarrow 6.50 - P = 74.20 - 1.1 e^{-0.06(\frac{1}{2})} - 1.1 e^{-0.07(\frac{1}{2})} - 70 e^{-0.08(\frac{1}{2})} \)

\[ \Rightarrow P = 2.3823 \text{, the theoretical price of the put} \]

Thus, the arbitrage is to sell the put for 2.50 and synthetically create the put, which costs 2.3823 for a risk-free profit of 0.1177.

10. \( 0.114 - 0.098 = x_0 e^{-0.06(\frac{1}{2})} - 0.94 e^{-0.07(\frac{1}{2})} \Rightarrow x_0 = \frac{0.9518}{\epsilon} \)