MA/STAT 472 Fall 2009 Assignments

Assignment 1:

Note: The answers to the exercises from the notes are posted on the web page.

For Thursday: Read notes, p. 127-137.

Exercises:

p. 148, 7-1, 7-2(a)(c)(e), 7-3, 7-4(a), 7-5(a),(b), 7-13 (a)-(e)

In the following questions, assume that mortality follows the Excel Mortality Table posted on the web page. If you use a spread sheet to do find the answer, include a print out of it in you homework.

1. A 30 year old man is married to a 20 year old woman. What is the probability that the woman dies between age 40 and 41 and that the man is still alive at the end of this period.
2. A 30 year old man is married to a 20 year old woman. What is the probability that there is a natural number n such that the woman dies between age n and $n+1$ and that the man is still alive at the end of this period. Hence, the man outlives the woman.

Assignment 2: Due Tuesday, 9/1

Read notes, p. 155-159

Exercises:

1. Notes: p. 148, 7-2(b), (d),7-4(b), 7-5(c), (d)7-6, 7-8 (Assume UDD (Uniform Distribution of deaths over each year–i.e. $l_{x+t} = (1-t)l_x + tl_{x+1}$, $0 \leq t \leq 1$), 7-9(c) *Remember this formula! We will use it later.*(Assume UDD)

2. We seek a 5% return on our investments. What is the minimum we should charge for a $100,000 whole life annuity issued to a female age 20, payable at the beginning of each year she remains alive. (Use the Excel Mortality table posted on the web.)

3. Assume that mortality is described in Exercise 7-1 on p. 148. You sell an annuity due to a new born insect that pays $10 for each year it is alive. What is the single payment premium for this annuity, assuming a 4% interest rate?

4. Let assumptions be as in the preceding exercise, except that the annuity is a two year deferred annuity. What is the premium now?

Assignment 3: Due Thursday, 9/3
Read notes, p. 160

1. Notes: p. 145, 7-7 (a),(c) *Hint:* See Exercise 7-5 (b) on p. 148,
   7-10

2. Notes: p. 181, 8-1 (a), (b), (d) *Hint on (b):* See Exercise 7-5 (b) on p. 148
   8-2, 8-3, 8-4 (a), 8-13.

3. Use the ILT posted on the web page to find, at 6% interest:

   (a) The single payment benefit premium for a $150, whole life annuity due sold to a 25 year old person.

   (b) The single payment benefit premium for a $150, 7 year pure endowment sold to a 25 year old person.

   (c) The single payment benefit premium for a $150, 7 year deferred annuity due sold to a 25 year old person.

   (d) The single payment benefit premium for a $150, 7 payment temporary annuity due sold to a 25 year old person.

Assignment 4: Due Thursday, 9/10

Note: I am giving you until Thursday on this assignment because (a) Monday is a holiday and (b) this assignment is somewhat harder than the ones we have been doing so far. I **strongly** recommend that you try to do it for Tuesday so that you know what questions to ask and will get more out of any hints I am asked to give.

Read notes, p. 189-192

Note: The formulas proved at the end of the last class were

$$\begin{aligned} 1 &= A_x + d\ddot{a}_x \\ 1 &= A_{x:\overline{n}|} + d\ddot{a}_{x:\overline{n}|} \\ A_{x:\overline{n}|} &= A^{\,1}_{x:\overline{n}|} + {}_nE_x \\ d &= 1 - \frac{1}{1+i} = \frac{i}{1+i}. \end{aligned}$$

Note: I accidentally misstated the definition of d in class. The above is the correct formula.

Exercises:

Note: In all of these exercises "premium" means "single payment benefit premium." Also "EOY" means "payable at the end of the year of death". All annuities are annuities-due.

Remark. The answers denoted "My Ans:" were compute personally by the instructor who is infamously bad a getting exact answers to ANYTHING. Take them with much salt!

1. Notes, p. 210. *All of these problems assume EOY.*: 9-1(a), 9-2(a), 9-3(a), 9-4(a).

2. For a whole life annuity due of 1 on (x) $q_x = .01$, $q_{x+1} = .05$, $i = .05$, $\ddot{a}_{x+1} = 6.951$. Calculate the change in the APV of this annuity if p_{x+1} is increased by .03. (Answer: .177.)

3. Use the ILT to find the premium at 6% for a 7 year, \$1,000 EOY term policy issued at age 25. (My ans: \$7.706749680)

4. Do Problem 3 in the sample MLC Exam that is posted on the web page. (Ans: (a)).

5. An individual's mortality expectation initially is given by the ILT. At age 30, she becomes "select" due to being accepted for insurance. Assume that the select period is 3 years and that during this period

$$q_{[x]+t} = (1 - (3 - t) * .01)q_{x+t}$$

where q_{x+t} is the value from the ILT.

(a) Find $l_{[30]}, l_{[30]+1}, l_{[30]+2}$ given that l_{33} is as stated in the ILT. (My ans: 9,500,530, 9,486,430, 9,471,462)

(b) Compute the APV at 6% interest of a \$1,000 life annuity sold to this individual. (My ans: \$15,857.42)

(c) Compute the APV at 6% interest of a \$1,000 EOY whole life policy sold to this individual. (My ans: \$102.4072543)

6. Purdue Life tells us that the premium for a \$1,000 whole life annuity issued at age 40 is \$15,000 while the premium for a \$10,000 EOY whole life insurance policy issued at the same age is \$1,600. What interest rate are they assuming? (My ans: 5.93%)

7. Let the assumptions be as in Exercise 6. Suppose additionally it is given that the premium for a \$1,000 whole life annuity is issued at age 50 is \$13,000 and that the premium for a 10 year \$1,000 temporary annuity issued at age 40 is \$7,700. What is the premium for a 10 year EOY term policy issued at age 40. (My ans: \$7.26)

8. Let $s(x) = \frac{(100-x)^{\frac{1}{3}}}{100^{\frac{1}{3}}}$, $\quad 1 \le x \le 100$. Find $F_X(x)$, $f_X(x)$, $\mu(x)$, ${}_5p_{20}$ and ${}_7q_{10}$.

9. Suppose that $\mu(x) = x^2$. Find $s(x)$, $F_X(x)$, ${}_5p_{20}$ and ${}_7q_{10}$.

Assignment 5: Due Tuesday, 9/15

1. Suppose that $\mu(x) = A + Bc^x$. Compute $s(x)$. The distribution is called "Makeham's law." See the discussion on p. 78 of Bowers. The comments at the bottom of the page are particularly interesting.

2. Do Bowers, p. 84: 3.4(a), (b) *Hint* Graph $s(x)$ using a graphing calculator. p. 126, 4.1, 4.6. *Note:* If the benefit at time t is $b(t)$, then the APV is

$$\overline{A}^{\,1}_{x:\overline{n}|} = \int_0^n {}_tp_x \mu(x+t) b(t) \nu^t \, dt$$

3. Exercises : Notes: p.213, ,9-26, 9-27 9-28. *Note:* For an n-year deferred whole life PTD policy the APV is, of course, ${}_nE_x\overline{A}_{x+n}$.

Assignment 6: Due Thursday, 9/17

1. Derive the following formula under the assumption of constant force of mortality μ and force of interest δ.

$$\overline{A}^{\,1}_{x:\overline{n}|} = \frac{\mu}{\mu+\delta}\left(1 - e^{-(\mu+\delta)n}\right)$$

2. An insurance payable at the moment of death of (x) has the following benefit schedule:
$b_t = \begin{cases} (1.05)^t & 0 \le t < 10 \\ (1.05)^{10} & t \ge 10 \end{cases}$
You are given (i) $\delta = 0.05$, (ii) $\mu_x(t) = 0.02$ for t ≥ 0.
Find the net single premium for this policy.

3. Do Bowers, 3-36, on p. 89.

4. Assume that $l_x = 100 - x$, $0 \le x \le 100$ and $i = .05$. Compute (a) $A^{\,1}_{30:\overline{20}|}$, (b) A_{30}, (c) $\ddot{a}_{30:\overline{20}|}$, and (d) $\ddot{a}_{30}$ using the identities from class. *Do not compute summations!*

5. Using ILT data approximate $\overline{A}_{30}$ and $\overline{A}_{30:\overline{10}|}$.

Assignment 7: Due Tuesday, 9/22

1. Using the Excel mortality table posted on the web page, compute your own curtate expectation of life (a) using the first line of equation (3.5.6) on p. 69 of Bowers and (b) using equation (3.5.7).

2. Assume $l_x = \omega - x$ for $0 \le x \le \omega$. Prove that, as you would guess,

$$\overset{\circ}{e}_x = \frac{1}{2}(w - x).$$

3. On the web page, under "Important Formulas," there is a formula for $\overline{a}_{x:\overline{n}|}$ in the exponential case. Use this formula to derive a formula for $\overset{\circ}{e}_x$ in the exponential case. Are you surprised that it doesn't depend on x?

4. Assume that mortality is exponential with force of mortality μ. Derive the identity

$$a_{x:\overline{n}|} = e^{-(\mu+\delta)}\frac{1 - e^{-n(\mu+\delta)}}{1 - e^{-(\mu+\delta)}}.$$

Hence
$$e_{x:\overline{n}|} = e^{-\mu}\frac{1-e^{-n\mu}}{1-e^{-\mu}}.$$
Remark. Recall that in interest theory
$$\begin{aligned} a_{\overline{n}|i} &= \frac{1-\nu^n}{i} \\ &= \nu\frac{1-\nu^n}{1-\nu} \quad (\text{since } i = \frac{1-\nu}{\nu}) \\ &= e^{-\delta}\frac{1-e^{-n\delta}}{1-e^{-\delta}}. \end{aligned}$$

Hence, in the exponential case
$$a_{x:\overline{n}|} = a_{\overline{n}|i}$$
where the force of interest is $\delta + \mu$.

5. Do Problem 2 on "Sample MLC Exam 3" which is posted on the web page. (Ans: (e))

6. At age 25 an individual has a year long medical problem that increases his force of mortality from $\mu_{25}(t)$ to $\mu_{[25]}(t) = \mu_{25}(t) + 0.1(1-t)$, $0 \leq t \leq 1$. For $t \geq 1$ his force of mortality equals $\mu_{25}(t)$–i.e. the selection period is one year.
   (a) Prove that ${}_tp_{[25]} = e^{-.1(t-t^2/2)}\ {}_tp_{25}$, $0 \leq t \leq 1$.
   (b) If $e_{25} = 10$, find $e_{[25]}$. *Hint:* $a_{x:\overline{n}|} + {}_nE_x a_{x+n} = a_x$. *Ans:* 9.51

Assignment 8: Due Thursday, 9/24

1. Assume that $\delta = .06$ and
$$\mu(x) = \begin{cases} .04 & 0 < x < 40 \\ .05 & 40 \leq x \end{cases}$$
   Calculate $\overset{\circ}{e}_{25:\overline{25}|}$. (Ans: 15.59852015)

2. Do Problem 30 on "Sample MLC Exam 4" which is posted on the web page. (Ans: (b))

3. Do Problem 3 on "Sample MLC Exam 2" which is posted on the web page. (Ans: (c))

4. Do Problem 11 on "Sample MLC Exam 2" which is posted on the web page. (Ans: (a))

Assignment 9: Due Tuesday, 10/06

1. Use the ILT to find (assuming that $i = 0.06$):

   (a) $Var(Z)$ where $Z = \nu^{K(40)+1}$. The appropriate formula is:

   $$Var(Z) = {}^2A_{40} - (A_{40})^2$$

   where ${}^2A_x$ denotes A_x computed at twice the force of interest. *Ans:* 0.02260

   (b) $Var(Z)$ where $Z = \nu^{T(40)}$. Assume UDD. The appropriate formula is:

   $$Var(Z) = {}^2\overline{A}_{40} - (\overline{A}_{40})^2.$$

   Hint: To approximate ${}^2\overline{A}_{40}$ you will need to use the "$\frac{i}{\delta}$ rule" where i and δ correspond to twice the force of interest of the ILT. *Ans:* 0.02398.

2. We have issued \$500 whole life policies, payable at the end of the year of death, to each of 100 independent lives at age 40. Assume that $i = .06$ and that the survival function for each of the lives is described by the ILT. Use the normal approximation to approximate the size of the fund necessary to have on hand in order to be 97.5% certain of being able to pay any claim. *Ans:* 9539.4539.

3. Re do Exercise 2 assuming the insurance is payable at the time of death. Assume UDD. *Ans:* 9823.3201.

4. Re do Exercise 1 assuming a De Moivre distribution with $\omega = 100$ and $\delta = .05$. *Hint:* Use the formulas for the De Moivre distribution posted on the course web page. *Ans:* (a)0.06266932579, (b) 0.0659308063.

5. Re do Exercise 1, part (b), assuming an exponential distribution with $\mu = .02$ and $\delta = .05$. *Hint:* Use the formulas for the exponential distribution posted on the course web page. *Ans:* 0.08503401365

6. Compute the mean and variance of the benefit random variable for a continuous whole life annuity issued at age 25 that pays \$1000. (So we are talking $\overline{a}_{25}$.) Assume $\delta = .03$ and $\mu = .04$.

Assignment 9: Due Thursday, 10/08

1. Show that if $l_x = \omega - x$, then $\mu(x) = \frac{1}{\omega - x}$.

2. Do problem 3 on "Sample MLC Exam 4" which is posted on the web page. (Ans: (c)) *Note:* In our notation, the assumption is $\mu(x) = \frac{1}{104-x}$. Note also the the "rule of 2" works for a pure endowment.

3. Do problem 27 on "Sample MLC Exam 1" which is posted on the web page. (Ans: (b)) *Hint:* See the variance formulas in the sheet of Important Formulas posted on the web.

4. Calculate the 85^{th} percentile of the distribution of Z where Z is the present value of the benefit for a \$1000 whole life PTD policy issued to (30) assuming a De Moivre distribution with $\omega = 100$ and $\delta = .05$.

5. Do problem 14 on "Sample MLC Exam 4" which is posted on the web page. (Ans: (a)) Note: For a continuous whole life annuity $Y = \frac{1-\nu^{T(x)}}{\delta}$.

6. Do problem 9 on "Sample MLC Exam 12" which is posted on the web page. (Ans: (b))*Hint:* Let the death benefit be D. The PV random variable of the new product is $W = 25000Y + DZ$ where Y is the PV random variable for $\ddot{a}_x$ and Z is the PV random variable for A_x. Find a formula expressing $Var(W)$ in terms of $Var(\nu^{K(x)+1})$ and D.

Assignment 10: Due Thursday, 10/22.

1. Assume ILT data. I buy a \$500 whole life PTD policy at age 20 which I pay for with annual payments of \$P made at the beginning of the year. (The premium). Find P, assuming UDD. The appropriate formula is

$$\ddot{a}_{20} P = 500\overline{A}_{20}.$$

   Thus "the APV of the payments must equal the APV of the insurance." *Ans:* 2.035310390.

2. Repeat Exercise 1 for a 10 year PTD term policy issued at age 20. Note that you only pay premiums for 10 years. *Ans:* 0.5799195715

3. Repeat Exercise 2 assuming a De Moivre distribution with $\omega = 120$ and $\delta = .05$.*Ans:* 5.084994420

4. One student asked why the answers to Exercises 2 and 3 are so different. To see why, try computing ${}_{10}q_{20}$ for both ILT and De Moivre mortality. How should increasing the probability of death effect the price of the insurance? Explain. How should it effect the APV of an annuity? How about the annual premium?

Assignment 11: Due Tuesday, 10/27.

The MLC Tables are posted on the course web page. All policies have a benefit of $1.

1. For each of the following policies, find the benefit premium at $i = 6\%$ under the UDD assumption. You may use the values of $\alpha(m)$ and $\beta(m)$ given on the MLC Tables. (See my list of Important Formulas on the web.*Note:* I want the per period payment, not an annual payment.
   (a) A whole life policy, PYD, paid with monthly premiums, issued to a 20 year old. Ans: 0.0003389449550.
   (b) A whole life policy, PTD, paid with monthly premiums, issued to a 20 year old. Ans: 0.0003490145595
   (c) A whole life policy, PTD, paid with quarterly premiums, issued to a 20 year old. Ans: 0.001041610696
   (d) A 30 year term policy, PTD, paid quarterly, issued to a 20 year old. Ans: 0.0004695394142

2. Re-do Exercises 1b and 1c using the approximations $\alpha(m) \approx 1$ and $\beta(m) \approx \frac{m-1}{2m}$ given in class. Ans: 1b 0.0003489023236, 1c 0.001041302090

3. Assume that for $i = .05$, $A_{30} = .2$. Compute the premiums for a whole life policy, PTD, paid with quarterly premiums, issued to a 30 year old. (Assume UDD). Ans: 0.01240601954

Assignment 12: Due Tuesday, 10/27.

1. Do Problem 11 on "Sample MLC Exam 11" which is posted on the web page. (Ans: (e))
2. Assume that mortality is De Moivre and $\delta = .06$. Compute $Var(L)$ where L is the loss at issue random variable for a whole life, completely discrete policy with benefit \$1 and with the benefit premium issued to an individual age x where $\omega - x = 50$. Ans: 0.129286
3. Assume that $\delta = .06$ and $\mu = .03$. Compute the variance of the loss at issue random variable for a completely continuous whole life policy with \$500 benefit and benefit premium issued to (20). Ans: 50,000.
4. Do Problem 10 on "Sample MLC Exam 7" which is posted on the web page. (Ans: (c))

Assignment 13: Due Thursday, 11/5.

1. Turn in the corrected version of Assignment 12.
2. Do Problem 18 on "Sample MLC Exam 3" which is posted on the web page. (Ans: (b))

   Outline of solution:

   (a) Use the unrevised mortality data to find A_{42} noting that ${}_kp_{42}q_{42+k} = {}_{k|}q_{42}$.

   (b) Use an identity to find $\ddot{a}_{42}$ and hence $P(A_{42})$.

   (c) Now compute the reserve at age 52 using the revised mortality data.
3. Do Problem 12 on "Sample MLC Exam 1" which is posted on the web page. (Ans: (a))
4. Do Problem 16 on "Sample MLC Exam 1" which is posted on the web page. (Ans: (a))
5. Do Problem 11 on "Sample MLC Exam 10" which is posted on the web page. (Ans: (e))*Hint:* One of the identities on the formula sheet makes quick work of this.
6. Do Problem 12 on "Sample MLC Exam 10" which is posted on the web page. (Ans: (e))

Assignment 14: Due Tuesday, 11/10.

Remark. There are many different notations for the reserve. The most specific is similar to the notation for premiums. For example, ${}_{10}V(B\overline{A}^{\,1}_{20:\overline{40}|})$ is the reserve at age 30 on a 40 year PTD term policy with benefit B, issued to a 20 year old paid with annual premiums, where the premium is the benefit premium $P(B\overline{A}^{\,1}_{20:\overline{40}|})$. ${}_{10}\overline{V}(\overline{A}^{\,1}_{20:\overline{40}|})$ refers to a PTD policy paid continuously while ${}^{12}_{10}V(A^{1}_{20:\overline{40}|})$ refers to a PYD policy paid monthly. ${}_{10}V(\overline{A}_{20:\overline{40}|})$ would, of course, be endowment insurance. Symbols such as ${}_kV$ and ${}_k\overline{V}$are more general and depend on the context for their meaning, although the first would be a totally discrete policy and the second a totally continuous policy. Both the benefits and the premiums may be non-constant and the premiums are not necessarily the benefit premiums. One also sees symbols such as ${}_kV^{1}_{x:\overline{n}|} = {}_kV(A^{1}_{x:\overline{n}|})$ and ${}_k\overline{V}_x = {}_k\overline{V}(\overline{A}_x)$.

Just remember "${}_kV_x$ is the APV at age $x+k$ of what you still expect to pay out minus the APV of what you still expect to receive."

1. The remaining life time random variable is uniform over $[0,100]$ and $\delta = .04$.
   (a) Find ${}_{10}\overline{V}(\overline{A}_{30})$. Ans: 0.06536890545
   (b) Find ${}_5V$ for a \$500, 20 year pure endowment sold to (30) paid with 10 annual payments. Assume the benefit premium. Ans: 120.0280450
   (c) Find ${}_{10}\overline{V}^{1}_{30:\overline{40}|}$. Ans: 0.0379734020
   (d) Find ${}_{10}\overline{V}_{30:\overline{40}|}$. Ans: 0.04310706021.
   Note The identity that you use to compute the reserve for a completely continuous whole life policy also works for endowment insurance. This is because it is based on the identity $1 = \overline{A}_x + \delta\overline{a}_x$ which also holds for endowment insurance.
   (e) Find $Var({}_{10}L)$ where L is the prospective loss function for a completely continuous whole life policy issued to (30). Ans: .1428207469
2. Assume ILT data.
   (a) Find ${}_{10}V(A^{1}_{30:\overline{40}|})$. Ans: 0.03673300635
   (b) Find ${}_{40}V(A^{1}_{30:\overline{40}|})$. Ans: 0

3. Given ${}_{10}V_{50} = .16$, ${}_{10}V_{60} = .14$, and $\ddot{a}_{50} = 12.5$, find $\ddot{a}_{60}$. Ans: 10.500

4. For a fully discrete \$1,000 whole life policy issued to (x), the annual premium determined by the equivalence principle is 4. Given that $q_x = .004$, $q_{x+1} = .008$, and $i = .04$, find ${}_2V$. *Hint:* Since the premium is determined by the equivalence principle, ${}_0V = 0$. Ans: -3.702552144

   Remark. Having a negative reserve is highly unusual, but not at all impossible. All this says is that the stated values, which I made up, are not realistic.

For Tuesday, Nov. 17

1. Assume that male mortality is described by $l_x^m = 100 - x$ and female by $l_x^f = 105 - x$. I am currently 50 (I wish) and my wife is 40. Assume that $\delta = .05$ and that our time until death random variables are independent.

   (a) Find ${}_{10}p_{50:40}$ and ${}_{10}p_{\overline{50:40}}$.

   (b) Find $\mu_{50:40}(t)$ and $\mu_{\overline{50:40}}(t)$.

   (c) Find $\overline{A}_{50:40}$ and $\overline{A}_{\overline{50:40}}(t)$.

   (d) What would be the premium on a \$1,000 joint life policy, PTD, issued to us now if we pay with a continuous annuity? How about a \$1,000 last survivor policy?

   (e) Approximate the annual premium for the joint life policy described in 1d.

   (f) How many years do we expect to pass before one or the other of us is dead? How about before we both are dead?

   (g) Find $Var(\text{“}\overline{A}_{50:40}\text{”})$.

   (h) Find ${}_{10}\overline{V}(\overline{A}_{50:40})$.

2. Let the assumptions be as in Exercises 1a-1c except that the female has constant force of mortality $\mu = .03$. Find $\overline{A}_{50:40}$.

3. Two future lives (x) and (y) are independent. Find ${}_{2|}q_{xy}$, given the following data.

k	q_{x+k}	q_{y+k}
0	0.06	0.07
1	0.08	0.08
2	0.10	0.15

4. Two lives (x) and (y) are not independent. Assume that their constant forces of mortality are respectively $\mu_1 = .02$ and $\mu_2 = .03$. Find $\overline{a}_{\overline{xy}}$ given that $\delta = .03$ and

$$ {}_tp_{xy} = \frac{1}{2}\, {}_tp_y + \frac{1}{2}\, {}_tp_x\, {}_tp_y. $$

For Tuesday, Dec. 1

1. Do Problem 18 on "Sample MLC Exam 12" which is posted on the web page. (Ans: (a))

2. Do Problem 16 on "Sample MLC Exam 13" which is posted on the web page. (Ans: (d))

3. Do Problem 15 on "Sample MLC Exam 13" which is posted on the web page. (Ans: (c))

4. Do Problem 28 on "Sample MLC Exam 8" which is posted on the web page. (Ans: (e)) *Hint:* What kind of a policy is this?

5. Do Problem 19 on "Sample MLC Exam 12" which is posted on the web page. (Ans: (e)) *Hint:* You can build such an annuity/insurance product by combining whole life and joint life instruments with differing benefits.

6. Do Problem 14 on "Sample MLC Exam 1" which is posted on the web page. (Ans: (c)) *Hint:* One of the identities makes quick work of this problem.

7. Do Problem 13 on "Sample MLC Exam 10" which is posted on the web page. (Ans: (e))

For Thursday, Dec. 3

1. Do Problem 14 on "Sample MLC Exam 10" which is posted on the web page. (Ans: (c))

2. Do Problem 15 on "Sample MLC Exam 10" which is posted on the web page. (Ans: (d))*Hint:* Use the recursion formula for the variance from the formula sheet.

3. Let the data be as in question 7 of our last exam. (See the solution posted on the web page.)

   (a) Find ${}_3V$. Use the recursion relation for the reserve to find ${}_kV$ for $k = 0, 1, 2, 3$.

   Ans: $-150.14, -49.10, 23.07, 42.99$

   (b) Use the recursion relation for the variance of the loss function to find $Var({}_kL)$ for $k = 0, 1, 2, 3$.
   Ans: $770544.33, 717, 981.75, 610, 765.50, 395, 143.68$.

   Remark. This provides an alternative way of solving the exam question. The slight differences between the posted solution to the exam and my answers are attributable to roundoff error.

4. I am age 60. I will certainly retire by age 70. The probability of my retiring is uniformly distributed over $[60, 70]$. Assume that for me $l_x = 100 - x$.

   (a) I have an insurance policy that pays \$1,000 if I cease work due either to retirement or death. Assume that $\delta = .03$. Write an integral that expresses the APV of of this policy. You need not evaluate it.

   (b) I have an annuity that pays \$1,000 continuously up until the time that I cease work due either to retirement or death. Assume that $\delta = .03$. Write an integral that expresses the APV of of this annuity. You need not evaluate it.

   (c) Write an integral that expresses my expected time until I cease employment. You need not evaluate it.

(d) Suppose that my continued employment is subject to an additional decrement with force of mortality $\mu_3 = .01$. This might, for example, represent accidental death. What is the total force of mortality that I am subject to? Write an integral that expresses the APV of the insurance policy in 4a under the added decrement. You need not evaluate it.

For Thursday, Dec. 3

1. In Exercise 4d from Tuesday's assignment, write integrals that express (a) the probability that I cease work due to non-accidental death and (b) the probability that I cease work due to accidental death.

2. Do Problem 22 on "Sample MLC Exam 10". Ans (b)

3. Do Problem 23 on "Sample MLC Exam 10". Ans: (c)

4. Do Problem 21 on "Sample MLC Exam 11". Ans: (c)

5. Do Problem 22 on "Sample MLC Exam 11". Ans: (c)

   Remark. This requires a few formulas that we have not discussed in class:

$$\mu_x^{(j)}(t) = \frac{-\dfrac{d}{dt}\ {}_tp_x^{(j)}}{{}_tp_x^{(\tau)}}$$
$${}_tp_x^{(\tau)} = 1 - tq_x^{(\tau)} \qquad (UDD)$$
$${}_tp_x^{(j)} = 1 - tq_x^{(j)} \qquad (UDD)$$

   I will discuss the first in class on Tuesday. The second is simple. Under UDD

$$\begin{aligned} l_x^{(\tau)}(t) &= (1-t)l_x^{(\tau)} + tl_{x+1}^{(\tau)} \\ &= 1 - t\left(l_x^{(\tau)} - l_{x+1}^{(\tau)}\right) \\ \frac{l_x^{(\tau)}(t)}{l_x^{(\tau)}} &= 1 - t\frac{l_x^{(\tau)} - l_{x+1}^{(\tau)}}{l_x^{(\tau)}} \\ {}_tp_x^{(\tau)} &= 1 - tq_x^{(\tau)} \end{aligned}$$

   The derivation of the third is similar.

However, it should be noted that the UDD assumption applies to the ${}_tp_x^{(j)}$, not the ${}_tp_x'^{(j)}$. If the ${}_tp_x'^{(j)}$ all UDD and independent, then the total mortality *cannot* be UDD. However, if the ${}_tp_x^{(j)}$ are UDD and independent, then the total mortality is automatically UDD.

6. Use the recursion formula to do Problem 29 on “Sample MLC Exam 3”. Ans: (c)

7. Do Problem 14 on “Sample MLC Exam 7”. Ans: (c) *Hint:* If this takes more than 30 seconds, you are doing it wrong.

8. What is the probability that 5 of the 6 MLC questions I chose would all have the answer (c)?