James was (as usual) correct

In class, James suggested that one of the two proposed solutions to question 3, “Use the ILT to compute $A_{40:50:10}$ for $i = .06$” was incorrect. He was correct.

The correct solution (which I used) uses the equalities

$$A_{40:50:10} = A_{40:10} + A_{10:50} - A_{40:50:10}$$

$$A_{40:50:10} = A_{40:50} - 10E_{40:50}A_{50:60}$$

The incorrect solution (which I proposed) begins

$$A_{40:50:10} = A_{40:50} - 10E_{40:50}A_{40:50+10}$$

The first equality in (1) is correct; the second isn’t. The problem is the symbol “$40:50 + 10$”, which refers to the survival status of me and my wife when I am age 60 and she age 50. There are four possibilities, each with a different probability: only I am alive, only she is alive, we both are alive, or neither is alive. This is a mixed state. The symbol “$50:60$” assumes that we both are alive.

It is perhaps clearer to to consider the following: $A_{40:50}$ is the APV of a whole life, last survivor policy, issued to my wife and me at the respective ages 40 and 50. We can achieve the effect of such a policy, by: (1) purchasing a 10 year, last survivor policy, APV $A_{40:50:10}$, (2) ten years later, if we are both still alive, we purchase a whole life, last survivor policy, APV, $A_{50:60}$, if only I am still alive, I purchase an ordinary whole life policy, APV $A_{60}$, if only she is still alive, she purchases an ordinary whole life policy, APV $A_{50}$. Taking account of the probabilities, and discounting back to the present, we see

$$A_{40:50} = A_{40:50:10}$$

$$+ \nu^{10} \left( 10p_{50}(1 - 10p_{40})A_{60} + 10p_{40}(1 - 10p_{50}) A_{50} + 10p_{40} \ 10p_{50} A_{50:60} \right)$$
which, using $A_{50:60}^1 = A_{50}^1 + A_{60}^1 - A_{50:60}^1$, simplifies to

$$A_{40:50}^1 = A_{40:50:60}^1 + \nu^{10}(10p_{50})A_{60}^1 + \nu^{10}(10p_{40})A_{50}^1 - \nu^{10}(10p_{40}10p_{50})A_{50:60}^1$$

$$= A_{40:50:60}^1 + 10E_{50}A_{60}^1 + 10E_{40}A_{50}^1 - 10E_{40:50}A_{50:60}^1,$$

which is not equivalent with (1).

Such difficulties do not occur with the joint life status:

$$A_{40:50}^1 = A_{40:50:60}^1 + 10E_{40:50}A_{50:60}^1$$

(Think in terms of splitting the policy as done above.)