James was (as usual) correct

In class, James suggested that one of the two proposed solutions to question 3, "Use the ILT to compute $A^1_{\overline{40:50}:\overline{10}|}$ for $i = .06$" was incorrect. He was correct.

The correct solution (which I used) uses the equalities

$$\begin{aligned} A^1_{\overline{40:50}:\overline{10}|} &= A^1_{40:\overline{10}|} + A^1_{50:\overline{10}|} - A^1_{40:50:\overline{10}|} \\ A^1_{40:50:\overline{10}|} &= A^1_{40:50} - {}_{10}E_{40:50} A^1_{50:60} \end{aligned}$$

The incorrect solution (which I proposed) begins

$$\begin{aligned} A^1_{\overline{40:50}:\overline{10}|} &= A^1_{\overline{40:50}} - {}_{10}E_{\overline{40:50}} A^1_{\overline{40:50}+10} \\ &= A^1_{\overline{40:50}} - {}_{10}E_{\overline{40:50}} A^1_{\overline{50:60}} \end{aligned} \tag{1}$$

The first equality in (1) is correct; the second isn't. The problem is the symbol "$\overline{40:50}+10$", which refers to the survival status of me and my wife when I am age 60 and she age 50. There are four possibilities, each with a different probability: only I am alive, only she is alive, we both are alive, or neither is alive. This is a mixed state. The symbol "$\overline{50:60}$" assumes that we both are alive.

It is perhaps clearer to to consider the following: $A^1_{\overline{40:50}}$ is the APV of a whole life, last survivor policy, issued to my wife and me at the respective ages 40 and 50. We can achieve the effect of such a policy, by: (1) purchasing a 10 year, last survivor policy, APV $A^1_{\overline{40:50}:\overline{10}|}$, (2) ten years later, if we are both still alive, we purchase a whole life, last survivor policy, APV, $A^1_{\overline{50:60}}$, if only I am still alive, I purchase an ordinary whole life policy, APV A^1_{60}, if only she is still alive, she purchases an ordinary whole life policy, APV A^1_{50}. Taking account of the probabilities, and discounting back to the present, we see

$$\begin{aligned} &A^1_{\overline{40:50}} = A^1_{\overline{40:50}:\overline{10}|} \\ &+ \nu^{10} \left({}_{10}p_{50}(1 - {}_{10}p_{40}) A^1_{60} + {}_{10}p_{40}(1 - {}_{10}p_{50}) \right) A^1_{50} + {}_{10}p_{40}\, {}_{10}p_{50} A^1_{\overline{50:60}}) \end{aligned}$$

which, using $A^1_{\overline{50:60}} = A^1_{50} + A^1_{60} - A^1_{50:60}$, simplifies to

$$\begin{aligned} A^1_{\overline{40:50}} &= A^1_{\overline{40:50:\overline{10|}}} + \nu^{10}({}_{10}p_{50})A^1_{60} + \nu^{10}({}_{10}p_{40})A^1_{50} - \nu^{10}\left({}_{10}p_{40}\,{}_{10}p_{50}\right)A^1_{50:60} \\ &= A^1_{\overline{40:50:\overline{10|}}} + {}_{10}E_{50}A^1_{60} + {}_{10}E_{40}A^1_{50} - {}_{10}E_{40:50}A^1_{50:60}, \end{aligned}$$

which is not equivalent with (1).

Such difficulties do not occur with the joint life status:

$$A^1_{40:50} = A^1_{40:50:\overline{10|}} + {}_{10}E_{40:50}A^1_{50:60}$$

(Think in terms of splitting the policy as done above.)