A Field Guide to Some formulas

Crucial formulas:

$$
\begin{aligned}
{}_tE_x &= {}_tp_x\nu^t \\
A^{1}_{x:\overline{n}|} &= \sum_{t=0}^{n-1} {}_tp_x q_{x+t}\nu^{t+1} \\
A^{1\,(m)}_{x:\overline{n}|} &= \sum_{t=0}^{mn-1} {}_{t/m}p_x\ {}_{1/m}q_{x+t/m}\nu^{t/m+1} \\
A^{(m)}_x &= A^{1\,(m)}_{x:\overline{\infty}|} \\
A_{x:\overline{n}|} &= A^{1}_{x:\overline{n}|} + {}_nE_x \\
\ddot{a}_{x:\overline{n}|} &= \sum_{t=0}^{n-1} {}_tp_x\nu^t \\
\ddot{a}^{(m)}_{x:\overline{n}|} &= \frac{1}{m}\sum_{t=0}^{nm-1} {}_{t/m}p_x\nu^{t/m} \\
\ddot{a}_x &= \ddot{a}_{x:\overline{\infty}|} \\
a_{x:\overline{n}|} &= \sum_{t=1}^{n} {}_tp_x\nu^t \\
&= \ddot{a}_{x:\overline{n}|} - 1 + {}_nE_x \\
e_{x:\overline{n}|} &= \sum_{t=1}^{n} {}_tp_x \\
&= a_{x:\overline{n}|} \text{ with } \nu = 1\,(i = 0) \\
P(\overline{A}_x) &= \frac{\overline{A}_x}{\ddot{a}_x} \\
P^{(m)}(\overline{A}_x) &= \frac{\overline{A}_x}{\ddot{a}^{(m)}_x}
\end{aligned}
$$

Note: This needs to be divided by m to give the $m$thly payment.

$$
\begin{aligned}
{}_tL(x) &= B\nu^{T(x)-t} - \pi\frac{1-a^{T(x)-t}}{\delta}, \quad T(x) \geq t \\
E({}_tL(x) \mid L(x) \geq t) &= B\overline{A}_{x+t} - \pi\overline{a}_{x+t}
\end{aligned}
$$

Continuous:

$$\begin{aligned}
\overline{A}^{1}_{x:\overline{n}|} &= \int_0^n {}_tp_x \mu(x+t)\nu^t \, dt \\
&= -\int_0^n \frac{l'(x+t)}{l(x)}\nu^t \, dt \\
\overline{A}_x &= \overline{A}^{1}_{x:\overline{\infty}|} \\
\overline{A}_{x:\overline{n}|} &= \overline{A}^{1}_{x:\overline{n}|} + {}_nE_x \\
\overline{a}_{x:\overline{n}|} &= \int_0^n {}_tp_x \nu^t \, dt \\
\overline{a}_x &= \overline{a}_{x:\overline{\infty}|} \\
\mathring{e}_{x:\overline{n}|} &= a_{x:\overline{n}|} \text{ with } \nu = 1 \, (i = 0)
\end{aligned}$$

De Moivre, also called “uniform”:

$$\begin{aligned}
l_x &= \omega - x \\
\overline{A}^{1}_{x:\overline{n}|} &= \frac{1 - e^{-n\delta}}{\delta(\omega - x)} \\
\overline{A}_x &= \overline{A}^{1}_{x:\overline{\omega - x}|} \\
A^{1}_{x:\overline{n}|} &= \frac{\delta}{i}\overline{A}^{1}_{x:\overline{n}|}
\end{aligned}$$

Exponential, also called “constant force of mortality”

$$\begin{aligned}
s(x) &= e^{-\mu x} \\
{}_xp_t &= e^{-\mu t} \\
\overline{A}^{1}_{x:\overline{n}|} &= \frac{\mu}{\mu + \delta}\left(1 - e^{-(\mu+\delta)n}\right) \\
\overline{A}_x &= \frac{\mu}{\mu + \delta} \qquad (1) \\
\overline{a}_{x:\overline{n}|} &= \mu^{-1}\overline{A}^{1}_{x:\overline{n}|} \\
&= \frac{1}{\mu + \delta}\left(1 - e^{-(\mu+\delta)n}\right)
\end{aligned}$$

Table 1: ILT Formulas

Formula	*Valid*
$a_{x:\overline{n}\vert} = a_x - {}_nE_x a_{x+n}$	C,D,(m)
$\ddot{a}_{x:\overline{n}\vert} = \ddot{a}_x - {}_nE_x \ddot{a}_{x+n}$	C,D,(m)
$A^1_{x:\overline{n}\vert} = A_x - {}_nE_x A_{x+n}$	T, C, D,(m)
$\overline{A}_x + \delta\overline{a}_x = 1$	WL, E, C, D,(m)
$\overline{A}_x = \frac{i}{\delta} A_x$	UDD, De Moivre, WL, T,(m), not $\ddot{a}_x$
$Var(\text{“}\overline{A}_x\text{”}) = {}^2\overline{A}_x - (\overline{A}_x)^2$	C, D,WL, T, E, (m)
$Var(\text{“}\overline{a}_x\text{”}) = \frac{1}{\delta^2}\left({}^2\overline{A}_x - (\overline{A}_x)^2\right)$	C, D,WL, T,(m)
$Var(\text{“}B\overline{A}_x - \pi\overline{a}_x\text{”}) = \left(B + \frac{\pi}{\delta}\right)^2 \left({}^2\overline{A}_x - (\overline{A}_x)^2\right)$	C, D,WL
$= B^2 \frac{{}^2\overline{A}_x - (\overline{A}_x)^2}{(1-\overline{A}_x)^2}$	benefit premium, C, D,WL
$E({}_tL(x) \mid L(x) \geq t) = {}_tV$	
${}_tV = BA_{x+t} - \pi\ddot{a}_{x+t}$	WL
$= B\left(1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}\right)$	benefit premium, C, D,WL,E
$= B\frac{A_{x+t} - A_x}{1 - A_x}$	benefit premium, C, D,WL,E
${}_tV = (b_{t+1}q_{x+t}v - \pi_t) + p_{x+t}v({}_{t+1}V)$	Discrete, T, WL
$Var({}_tL(x) \mid L(x) \geq t) = \left(B + \frac{\pi}{\delta}\right)^2 \left({}^2\overline{A}_{x+t} - (\overline{A}_{x+t})^2\right)$	C, D,WL
$= B^2 \frac{{}^2\overline{A}_{x+t} - (\overline{A}_{x+t})^2}{(1-\overline{A}_x)^2}$	benefit premium, C, D,WL
$Var({}_kL) = p_{x+k}q_{x+k}[v(b_{k+1} - {}_{k+1}V)]^2$	
$+ v^2 p_{x+k} Var({}_{k+1}L)$	General
$\ddot{a}^{(m)}_x = \alpha(m)\ddot{a}_x - \beta(m)$	UDD, not temp., not $A^{(m)}_x$
$A^{(m)}_x = \frac{i}{i^{(m)}} A_x$	UDD, WL, T, De Moivre, not $\ddot{a}^{(m)}_x$
$T(\overline{xy}) + T(xy) = T(x) + T(y)$	general
${}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_{xy}$	independent lives
$A_{\overline{xy}} + A_{xy} = A_x + A_y$	independent lives, WL, T, D,C
$\ddot{a}_{\overline{xy}} + \ddot{a}_{xy} = \ddot{a}_x + \ddot{a}_y$	independent lives, WL, T, D,C