Section 11.6 – Module 11 Formula Sheet

Probability that (x) survives t years under multiple decrements = ${}_tp_x^{(\tau)}$.

n decrements with force of decrement $\mu_x^{(i)}(t)$, i=1,…,n.

Total force of decrement $\mu_x^{(\tau)}(t) = \mu_x^{(1)}(t) + \ldots + \mu_x^{(n)}(t)$

Probability of surviving all decrements through time t:

${}_tp_x^{(\tau)} = e^{\int_0^t -\mu_x^{(\tau)}(u)du} = e^{\int_0^t -\left[\mu_x^{(1)}(u)+\ldots+\mu_x^{(n)}(u)\right]du}$

Probability of surviving decrement *i* if it is the only decrement acting

${}_tp_x'^{(i)} = e^{-\int_0^t \mu_x^{(i)}(u)dt}$

$${}_tp_x^{(\tau)} = {}_tp_x'^{(1)} \cdot {}_tp_x'^{(2)} \cdots {}_tp_x'^{(n)}$$

Density function for decrement *i*: $f(t,i) = {}_tp_x^{(\tau)}\mu_x^{(i)}(t)$

Probability of termination due to decrement *i* by time t: ${}_tq_x^{(i)} = \int_0^t f(u,i)du$

Total probability of termination due to any decrement:

${}_tq_x^{(\tau)} = {}_tq_x^{(1)} + \ldots + {}_tq_x^{(n)} = 1 - {}_tp_x^{(\tau)}$

Note $q_x'^{(i)} \geq q_x^{(i)}$

${}_\infty q_x^{(i)} = \int_0^\infty f(u,i)du = \lim_{t\to\infty} {}_tq_x^{(i)}$

Relating ${}_tq_x^{(1)}$ and ${}_tq_x'^{(1)}$ assuming UDD in each associated single decrement:

$$q_x^{(1)} = q_x'^{(1)}\left(1 - \frac{q_x'^{(2)}}{2}\right) \quad \text{and} \quad q_x^{(2)} = q_x'^{(2)}\left(1 - \frac{q_x'^{(1)}}{2}\right)$$

$$q_x^{(1)} = q_x'^{(1)}\left[1 - \frac{1}{2}\left(q_x'^{(2)} + q_x'^{(3)}\right) + \frac{1}{3}q_x'^{(2)}q_x'^{(3)}\right], \text{ similar for } q_x^{(2)} \text{ and } q_x^{(3)}$$

Relating ${}_tq_x'^{(1)}$ and ${}_tq_x^{(1)}$ assuming each decrement j and the total decrement are UDD

$${}_tp_x'^{(j)} = \left({}_tp_x^{(\tau)}\right)^{q_x^{(j)}/q_x^{(\tau)}}$$

Multiple decrement life table notation:

$$l_x^{(\tau)} = l_{x-1}^{(\tau)}\, p_{x-1}^{(\tau)} \qquad d_x^{(i)} = q_x^{(i)}\, l_x^{(\tau)}.$$

Section 12.11 – Module 12 Formula Sheet

Equivalence Principle:
APV of gross premiums = APV of insurance benefits + APV of expenses

Total Reserve
= (APV of future benefits) +(APV of future expenses) – (APV of future premiums)

Expense reserve
= APV of future expenses - APV of future level expense premium
Level expense premium = e = Gross premium – Net Premium

Asset Share Calculation

$$ {}_{h+1}AS\ p_{x+h}^{(\tau)} = \left[{}_hAS + G(1-c_h) - e_h \right](1+i) - q_{x+h}^{(1)} b_{h+1} - q_{x+h}^{(2)} \left({}_{h+1}CV \right) $$