STAT 472 Test 2 Study Sheet Fall 2007

Coverage The test covers Chapters 6, 7, and 8. We did not discuss utility theory from Chapter 6. I do not plan to ask questions on the "percentile premiums," pure endowments, endowment insurance, or deferred insurance despite the fact that all of these topics ARE on on the MLC syllabus. Note, however, that endowment insurance occurs in formulas (5.2.15), p.138 and (5.3.15), p. 144, which you certainly should know.

Formulas Only the MLC tables (on web page) will be provided on the test. **You will NOT be allowed a formula sheet as you were on the last test.**

Instructions Assume uniform distribution of deaths over each year (UDD) whenever necessary to answer the question. You must indicate each time UDD is used! Be careful: Saying that you need UDD when you don't will be penalized.

PTD means "payable at the time of death" and "PYD" means "payable at the end of the year of death." "ILT" means illustrative life table. "Premium" means the benefit premium–i.e. the premium determined by the equivalence principle.

Important: In all cases, unevaluated expressions, such as $(1 - e^{-(.03)6})/(1 + (1.03)^{-2})$ and $\frac{1}{65}\int_0^5 e^{-.05\,t}\,dt$ are just as acceptable as numeric answers, as long as the answer can in principle be reduced to a number. It is even O.K. to give the answer as "$P = A_x/\ddot{a}_x$ where A_x is as stated in problem 1a and $\ddot{a}_x = (1 + (1.03)^{-2})$", or whatever. This can save a lot of time!

Notation $T(x)$ is the time until death at age (x), $K(x) = [T(x)]$ is the years until death at age (x).

(1) We issue a \$1,000, twenty year term policy at age 25, PTD. Assuming $i = .06$, use the ILT to answer the following questions:
   (a) What is the single payment premium for the policy?
   (b) What is the annual premium?
   (c) What is the quarterly premium?
   (d) What is the continuous premium?

(e) Compute the reserve 5 years after issue, assuming annual premiums.
(f) Compute the reserve 5 years after issue, assuming quarterly premiums.
(g) Give the loss function L for the policy assuming annual premiums.
(h) Give the loss function L for the policy assuming continuous premiums.
(i) Give the loss function L for the policy assuming annual premiums and PYD.

(2) Assume that $\delta = .03$ and $l_x = 115 - x$.
(a) Answer all of the questions from Exercise 1 for a \$1,000, twenty year term policy issued at age 25, PTD.

Remark. UDD holds here. Hence, once we know $\overline{A}^{1}_{x:\overline{n}|}$, which is easily computed by

$$\overline{A}^{1}_{x:\overline{n}|} = \int_0^n \frac{e^{-t\delta}}{100 - x}\, dt,$$

we can then find all of the other actuarial functions by using the i/δ conversion along with formulas (5.2.15), p.138 and (5.3.15), p. 144. In UDD, the formulas involving $\alpha(m)$ and $\beta(m)$ are also exact.

(b) Assume that we issue a whole life, \$1000 policy, PTD, at age 30. What is the single payment premium for this policy?
(c) Suppose that the policy is paid with 3 premiums of P, $2P$ and $3P$ at the beginning of the first, 5th, and 15th years respectively. What is P?
(d) Compute ${}_5V$ and ${}_{10}V$ for the policy in 2c.
(e) Assume that the policy in 2b is paid continuously.
(i) Give the loss function L for the policy.
(ii) Find $P(L(T(30)) \geq 0)$.
(iii) Find $Var(L)$.
(iv) Find the reserve 5 years after issue.
(v) Give a formula for the loss function ${}_5L$.
(vi) Compute $E[{}_5L|T(30) > 5]$.
(vii) Find $Var[{}_5L|T(30) > 5]$.

(3) Given ${}_5V(A_{40}) = .3$ and ${}_{10}V(A_{40}) = .4$, find ${}_5V(A_{45})$.

(4) Given $q_x = .004$, $q_{x+1} = .003$, and $i = .03$, find ${}_2V_x$ for a \$1,000 policy issued at age x, payable at the end of the year of death, given that the first and second year premiums are 10 and 12 respectively.

(5) Given ${}_{10}\overline{V}_{40} = .075$, $\overline{A}_{50} = .375$ and $\delta = .05$, find $\overline{P}(\overline{A}_{40})$.

Answers:

(1) See "Practice Question Solutions" on the web page.

(g) $L(T) = 1000\nu^T - P(\nu^{K+1} - 1)/d$ where P is the answer to 1b, $\nu = (1.05)^{-1}$, and $d = .05/(1.05)$.

(h) $L(T) = 1000\nu^T - P(\nu^T - 1)/\delta$ where P is the answer to 1d and $\nu = (1.05)^{-1}$.

(i) $L(T) = 1000\nu^{K+1} - P(\nu^{K+1} - 1)/d$ where $\nu = (1.05)^{-1}$, $d = .05/(1.05)$ and $P = A_{25}/\ddot{a}_{25}$ where $\ddot{a}_{25}$ is as computed in 1b, and (from the ILT) $A_{25} = .08165$.

(2) See "Practice Question Solutions" on the web page.

2(e)v

$$ {}_5L(t) = \begin{cases} 1000\nu^{t-5} - P(\nu^{t-5} - 1)/\delta & t \geq 5 \\ 0 & t < 5 \end{cases} $$

where $P = 1000\overline{A}_{30}/\overline{a}_{30}$ with $\overline{A}_{30}$ as computed in 2b, $\overline{a}_{30} = (1 - \overline{A}_{30})/\delta$, $\delta = \ln(1.05)$, and $\nu = (1.05)^{-1}$.

(3)

$$ 1 - \frac{\ddot{a}_{45}}{\ddot{a}_{40}} = .3 \Rightarrow \frac{\ddot{a}_{45}}{\ddot{a}_{40}} = .7 $$

$$ 1 - \frac{\ddot{a}_{50}}{\ddot{a}_{40}} = .6 \Rightarrow \frac{\ddot{a}_{50}}{\ddot{a}_{40}} = .6 $$

Hence

$$ \frac{\ddot{a}_{50}}{\ddot{a}_{45}} = \frac{.6}{.7} \Rightarrow {}_5V(A_{45}) = \frac{1}{7}. $$

(4) See "Practice Question Solutions" on the web page.

(5) See "Practice Question Solutions" on the web page.