Solutions to Practice Questions, Test 2

1. (a) (From ILT)

```
> A25:=.08165;A45:=.20120;add25:=16.2242;add45:=14.1121;i:=.06;
  del:=ln(1+i);d:=i/(1+i);v:=(1+i)^(-1);Ab25:=(i/del)*A25;Ab45:=
  (i/del)*A45;e20E25:=.29873;
```

$$A25 := 0.08165$$
$$A45 := 0.20120$$
$$add25 := 16.2242$$
$$add45 := 14.1121$$
$$i := 0.06$$
$$del := 0.05826890812$$
$$d := 0.05660377358$$
$$v := 0.9433962264$$
$$Ab25 := 0.08407571307$$
$$Ab45 := 0.2071773848$$
$$e20E25 := 0.29873 \quad \mathbf{(1)}$$

```
> Ab125_20:=Ab25-e20E25*Ab45;
```

$$Ab125\_20 := 0.02218561291 \quad \mathbf{(2)}$$

```
> P1:=1000*Ab125_20;
```

$$P1 := 22.18561291 \quad \mathbf{(3)}$$

1(b)

```
> add25_20:=add25-e20E25*add45;
```

$$add25\_20 := 12.00849237 \quad \mathbf{(4)}$$

```
> P:=P1/add25_20;
```

$$P := 1.847493609 \quad \mathbf{(5)}$$

1(c) (From ILT)

```
> alph4:=1.00027;bet4:=.38424;
```

$$alph4 := 1.00027$$
$$bet4 := 0.38424 \quad \mathbf{(6)}$$

WARNING: The alpha and beta do NOT work for temporary annuities. The following calculation is NOT CORRECT!

```
> add4_25_20:=alph4*add25_20-bet4;
```

$$add4\_25\_20 := 11.62749466 \quad \mathbf{(7)}$$

Here is how to do it:

```
> add4_25:=alph4*add25-bet4;add4_45:=alph4*add45-bet4;
```

$$add4\_25 := 15.84434053$$
$$add4\_45 := 13.73167027 \quad \mathbf{(8)}$$

```
> add4_25_20:=add4_25-e20E25*add4_45;
```

$$add4\_25\_20 := 11.74227867 \quad \mathbf{(9)}$$

```
> P4:=P1/add4_25_20;
```

$$P4 := 1.889378845 \quad \mathbf{(10)}$$

1(d) (First solution)

```
> Ab25_20:=Ab125_20+e20E25;
```

$$Ab25\_20 := 0.3209156129 \quad (11)$$

```
> ab25_20:=(1-Ab25_20)/del;
```

$$ab25\_20 := 11.65431804 \quad (12)$$

```
> Pcont:=P1/ab25_20;
```

$$Pcont := 1.903638877 \quad (13)$$

1(d) (Second solution) From ILT

```
> alphInf:=1.00028;betInf:=.50985;
```

$$alphInf := 1.00028$$
$$betInf := 0.50985 \quad (14)$$

```
> ab25:=alphInf*add25-bet4;ab45:=alphInf*add45-bet4;
```

$$ab25 := 15.84450278$$
$$ab45 := 13.73181139 \quad (15)$$

```
> ab25_20:=ab25-e20E25*ab45;
```

$$ab25\_20 := 11.74239876 \quad (16)$$

```
> Pcont:=P1/ab25_20;
```

$$Pcont := 1.889359522 \quad (17)$$

Remark: The second answer is perhaps more accurate than the first. In the second we compute ab25_20 using vlaues of ab25 and ab45 from the table and then finally apply UDD. In the first answer we use values of A25, along with UDD, to get Ab25 to COMPUTE ab25, rather than use the more accurate values from the table. Applying UDD sooner introduces additional roundoff error in subsequent calculations.

```
>
```

1(e) From ILT

```
> A30:=.10248;A45:=.20120;Ab30:=(i/del)*A30;Ab45:=(i/del)*A45;
  add30:=15.8561;add45:=14.1121;L30:=9501381;L45:=9164051;
```

$$A30 := 0.10248$$
$$A45 := 0.20120$$
$$Ab30 := 0.1055245447$$
$$Ab45 := 0.2071773848$$
$$add30 := 15.8561$$
$$add45 := 14.1121$$
$$L30 := 9501381$$
$$L45 := 9164051 \quad (18)$$

```
> e15E30:=v^15*L45/L30;
```

$$e15E30 := 0.4024507907 \quad (19)$$

```
> Ab130_15:=Ab30-e15E30*Ab45;
```

$$Ab130\_15 := 0.02214584237 \quad (20)$$

```
> add30_15:=add30-e15E30*add45;
```

$$add30\_15 := 10.17667420 \quad (21)$$

```
> r5V:=1000*Ab130_15-P*add30_15;
```
$$r5V := 3.34450182 \quad (22)$$

1(f)
```
> add4_30:=alph4*add30-bet4;
```
$$add4\_30 := 15.47614115 \quad (23)$$

```
> add4_30_15:=add4_30-e15E30*add4_45;
```
$$add4\_30\_15 := 9.949819592 \quad (24)$$

```
> r5V:=1000*Ab130_15-P*add4_30_15;
```
$$r5V := 3.76361426 \quad (25)$$

1(g), (h), (i) See Practice sheet.

2. We begin by computing the basic actuarial functions:
```
> del:=.03;v:=exp(-del);i:=exp(del)-1;d:=i/(i+1);l:=x->115-x;
```
$$del := 0.03$$
$$v := 0.9704455335$$
$$i := 0.030454534$$
$$d := 0.02955446650$$
$$l := x \rightarrow 115 - x \quad (26)$$

```
> Ab25:=(1-exp(-del*(115-25)))/(del*(115-25));
```
$$Ab25 := 0.3454794398 \quad (27)$$

```
> Ab30:=(1-exp(-del*(115-30)))/(del*(115-30));
```
$$Ab30 := 0.3615366015 \quad (28)$$

```
> Ab45:=(1-exp(-del*(115-45)))/(del*(115-45));
```
$$Ab45 := 0.4178778913 \quad (29)$$

```
> A25:=(del/i)*Ab25;A30:=(del/i)*Ab30;A45:=(del/i)*Ab45;
```
$$A25 := 0.3403231583$$
$$A30 := 0.3561406668$$
$$A45 := 0.4116410627 \quad (30)$$

```
> add25:=(1-A25)/d;add30:=(1-A30)/d;add45:=(1-A45)/d;
```
$$add25 := 22.32071561$$
$$add30 := 21.78551703$$
$$add45 := 19.90761489 \quad (31)$$

We now proceed as in Exercise 1:
2(a)(a)
```
> e20E25:=v^20*l(45)/l(25);
```
$$e20E25 := 0.4268534942 \quad (32)$$

```
> Ab125_20:=A25-e20E25*Ab45;
```
$$Ab125\_20 := 0.1619505202 \quad (33)$$

```
> P1:=1000*Ab125_20;
```
$$P1 := 161.9505202 \quad (34)$$

Alternatively
```
> Ab125_20:=int(exp(-del*t)/(115-25), t=0.. 20);
```

$$Ab125\_20 := 0.1671068014 \quad (35)$$

```
> A125_20:=(del/i)*Ab125_20;
```

$$A125\_20 := 0.1646127320 \quad (36)$$

2(a)(b)

```
> add25_20:=add25-e20E25*add45;
```

$$add25\_20 := 13.82308063 \quad (37)$$

```
> P:=P1/add25_20;
```

$$P := 11.71594991 \quad (38)$$

2(a)(c) Unfortunately, i is not .06 so we cannot use the values of alpha and beta from the ILT. Since UDD holds, we can obtain an exact answer using formulas (5.4.2), p. 149 and the formula directly above (5.4.8), p. 150. Yet more formulas to memorize?!!! NO WAY! I will not demand that you learn these. Instead we use the approximations alpha=1, beta=(m-1)/2m

```
> add4_25:=add25-(4-1)/(2*4);add4_45:=add45-(4-1)/(2*4);
```

$$add4\_25 := 21.94571561$$

$$add4\_45 := 19.53261489 \quad (39)$$

```
> add4_25_20:=add4_25-e20E25*add4_45;
```

$$add4\_25\_20 := 13.60815069 \quad (40)$$

```
> P4:=P1/add4_25_20;
```

$$P4 := 11.90099404 \quad (41)$$

2(a)(d)

```
> Ab25_20:=Ab125_20+e20E25;
```

$$Ab25\_20 := 0.5939602956 \quad (42)$$

```
> ab25_20:=(1-Ab25_20)/del;
```

$$ab25\_20 := 13.53465681 \quad (43)$$

```
> Pcont:=P1/ab25_20;
```

$$Pcont := 11.96561704 \quad (44)$$

2(a)(e)

```
> e15E30:=v^15*l(45)/l(30);
```

$$e15E30 := 0.5251055362 \quad (45)$$

```
> Ab130_15:=Ab30-e15E30*Ab45;
```

$$Ab130\_15 := 0.1421066073 \quad (46)$$

```
> add30_15:=add30-e15E30*add45;
```

$$add30\_15 := 11.33191824 \quad (47)$$

```
> r5V:=1000*Ab130_15-P*add30_15;
```

$$r5V := 9.3424208 \quad (48)$$

2(a)(f)

```
> add4_30:=add30-(4-1)/(2*4);;
```

$$add4\_30 := 21.41051703 \quad (49)$$

```
> add4_30_15:=add4_30-e15E30*add4_45;
```

$$add4\_30\_15 := 11.15383281 \quad (50)$$

```
> r5V:=1000*Ab130_15-P*add4_30_15;
```

(51)

$r5V := 11.4288608$ **(51)**

2(a)(g), (h), (i) See Practice sheet.

2(b)

```
> Ab30:=(1-exp(-del*(115-30)))/(del*(115-30));
```

$Ab30 := 0.3615366015$ **(52)**

```
> P1:=1000*Ab30;
```

$P1 := 361.5366015$ **(53)**

2(c) The probability of surviving t years beyond 30 is $_tp_{30}$ =l(30+t)/l(30) =(85-t)/85. At the beginning of the first year, t=0, at the beginning of the 5th year, t=4, and at the beginnning of the 15th year, t=14. Since the premiums are paid at the beginning of the year, the APV of the payments are P, 2Pv^4$_4p_{30}$ and 3Pv^14$_{14}p_{30}$

```
> PP:=solve(W+v^4*2*W*((85-4)/85)+v^(14)*3*W*((85-14)/85)=P1,W);
```

$PP := 83.36390815$ **(54)**

2(d) For t=5, i.e. age 35, we expect to pay benifits B where

```
> Ab35:=(1-exp(-del*(115-35)))/(del*(115-35));
```

$Ab35 := 0.3788675195$ **(55)**

```
> B:=1000*Ab35;
```

$B := 378.8675195$ **(56)**

t=5 corresponds to age 35. We receive only the payment at t=14 which is 14-5=9 years away.

```
> R:=v^9*3*PP*(115-(35+9))/(115-35);
```

$R := 169.4369685$ **(57)**

Hence

```
> r5V:=B-R;
```

$r5V := 209.4305510$ **(58)**

2(d) t=10, i.e. age 40, is still befor the 15th year. We receive the pament at t=14 which is 14-10=4 years away.

```
> R:=v^4*3*PP*(115-44)/(115-40);
```

$R := 209.9815168$ **(59)**

For t=10, we expect to pay B where

```
> Ab40:=(1-exp(-del*(115-40)))/(del*(115-40));B:=1000*Ab40;
```

Hence

```
> r5V:=B-R;
```

$Ab40 := 0.3976003447$

$B := 397.6003447$

$r5V := 187.6188279$ **(60)**

2(e)(i)

```
> ab30:=(1-Ab30)/del;
```

$ab30 := 21.28211328$ **(61)**

Compute premium:

```
> Pb:=1000*Ab30/ab30;
```

$Pb := 16.98781492$ **(62)**

```
> L:=t->1000*v^t-Pb*(1-v^t)/del;
```

$$L := t \rightarrow 1000\, v^t - \frac{Pb\,(1 - v^t)}{del} \quad \textbf{(63)}$$

```
> v;del;Pb;
```

0.9704455335

0.03

16.98781492 **(64)**

2(e)(ii)

```
> T:=solve(1000*v^t-Pb*(1-v^t)/del=0,t);
```

$T := 33.91306639$ **(65)**

```
> sT:=(115-(30+T))/(115-30);
```

$sT := 0.6010227484$ **(66)**

2(e)(iii)

```
> Ab230:=(1-exp(-2*del*(115-30)))/(2*del*(115-30));
```

$Ab230 := 0.1948829909$ **(67)**

```
> VarL:=(1000+Pb/del)^2*(Ab230-(Ab30)^2);
```

$VarL := 1.574305351\; 10^5$ **(68)**

2(e)(iv)

```
> ab35:=(1-Ab35)/del;
```

$ab35 := 20.70441602$ **(69)**

```
> r5V:=1000*Ab35-Pb*ab35;
```

$r5V := 27.1447321$ **(70)**

2(e)(v) See problem sheet.

2(e)(vi) Same as 2(e)(iv)

2(e)(vii)

```
> Ab235:=(1-exp(-2*del*(115-35)))/(2*del*(115-35));
```

$Ab235 := 0.2066188027$ **(71)**

```
> Var5L:=(1000+Pb/del)^2*(Ab235-(Ab35)^2);
```

$Var5L := 1.547416838\; 10^5$ **(72)**

3. See problem sheet.
4.

```
> i:=.03;r0V:=0;P0:=10;P1:=12;B1:=1000;B2:=1000;q0:=.004;q1:=.003;
  p0:=1-q0;p1:=1-q1;
```

$i := 0.03$

$r0V := 0$

$P0 := 10$

$P1 := 12$

$B1 := 1000$

$B2 := 1000$

$q0 := 0.004$

$$q1 := 0.003$$
$$p0 := 0.996$$
$$p1 := 0.997 \quad \textbf{(73)}$$

```
> r1V:=((1+i)*(r0V+P0)-B1*q0)/p0;;
```

$$r1V := 6.325301205 \quad \textbf{(74)}$$

```
> r2V:=((1+i)*(r1V+P1)-B2*q1)/p1;
```

$$r2V := 15.92282873 \quad \textbf{(75)}$$

5.

```
> Ab50:=.375;del:=.05;r10V40:=.075;
```

$$Ab50 := 0.375$$
$$del := 0.05$$
$$r10V40 := 0.075 \quad \textbf{(76)}$$

```
> ab50:=(1-Ab50)/del;
```

$$ab50 := 12.50000000 \quad \textbf{(77)}$$

```
> solve(r10V40=Ab50-p*ab50,p);
```

$$0.02400000000 \quad \textbf{(78)}$$

```
>
>
```