STAT 472, Test 1, Fall 2009

Show all work! An answer without supporting work will receive no credit!

1. Below is a life table for male frumious bandersnatches. 15 pts.

(x)	p_x
0	$\frac{1}{2}$
1	$\frac{3}{5}$
2	$\frac{7}{10}$
3	$\frac{1}{3}17$
4	$\frac{21}{26}$
5	$\frac{1}{-}$

[2pt]

   (a) Give an explicit formula using the given data for the actuarial present value of a whole life annuity paid at the beginning of the year, issued to a 2 year old bandersnatch in terms of the force of interest δ.

   (b) Give an explicit formula for the single payment pure premium for a \$100 whole life insurance policy issued at age 2, payable at the end of the year of death in terms of the force of interest δ.

   (c) Give an explicit formula for the expected remaining years of life (curtate expectancy of life) of a 2 year old.

2. Given that $\mu(x) = \frac{3}{2}\sqrt{x}$, find an explicit expression in terms of the force of interest δ for the following quantities. **You may leave all integrals, except the integral of μ, unevaluated.** 25 pts.

   (a) $s(x)$ (the survival function)

   (b) The single payment pure premium for a 25 year term insurance policy issued at age 20 paying \$500 at the time of death,

   (c) The actuarial present value of a continuous 30 year temporary life annuity paying \$500 issued at age 20.

   (d) The complete expectation of life of an individual aged 20,

   (e) A 20 year old is married to a 30 year old. What is the probability that exactly one of them is still alive at age 60?

3. Given that $l_x = \frac{1}{(2+x)^2}$, find an explicit expression for the following quantities in terms of the force of interest δ. **You may leave all sums and integrals unevaluated.** 10 pts.

   (a) $\mu_{20}(t)$,

   (b) ${}_{10|5}q_{20}$,

   (c) ${}_{30}E_{20}$,

4. Find an expression that computes the following quantities from data found on the ILT (assuming that $i = 0.06$).

   In this problem, rather than copying values from the ILT, you may simply use the appropriate actuarial symbol with the explanation "from table." For example you could write "$Y = A_{20} + \frac{l_{31}}{l_{30}}$ (from table)" rather than "$Y = 0.06528 + \frac{9501381}{9486854}$". But you could not write "$Y = \overline{A}_{20}$(from table)" since this is not on the table. You may also use quantities computed in one part of the question in other parts. 16 pts.

   (a) The single payment pure premium of a 10 year \$500 term policy, payable at the end of the year of death, issued to a 20 year old.

   (b) The single payment pure premium of a 10 year \$500 term policy, payable at the time of death, issued to a 20 year old. (Assume UDD.)

   (c) The actuarial present value of a 10 year temporary annuity that pays 1 at the beginning of the year sold to a 20 year old.

   (d) The actuarial present value of a 10 year temporary annuity that pays 1 at the **end** of the year sold to a 20 year old.

   (e) The actuarial present value of a 10 year temporary annuity that pays 1 continuously sold to a 20 year old. (Assume UDD).

5. An individual has mortality described by the ILT–i.e. for integral values of x, $s(x) = l_x/l_0$ where l_x is the corresponding entry if the ILT. At age 30 this individual has a 3 year long medical problem that increases his force of mortality according to the following formula:

$$\mu_{[30]}(t) = \begin{cases} \mu_{30}(t) + \frac{3-t}{100} & 0 \le t < 3 \\ \mu_{30}(t) & t \ge 3 \end{cases}$$

where $\mu_{30}(t)$ was his original force of mortality.

In this problem, rather than copying values from the ILT, you may simply use the appropriate actuarial symbol with the explanation "from table." For example you could write "$Y = A_{20} + \frac{l_{30}}{l_{31}}$ (from table)" rather than $Y = 0.06528 + \frac{9501381}{9486854}$. But you could not write "$Y = \overline{A}_{20}$(from table)" since this is not on the table. You may also use quantities computed in one part of the question in other parts.

Find an expression that computes the following quantities from data found on the ILT (assuming that $i = 0.06$). 10 pts.

(a) $p_{[30]+1}$

(b) ${}_2p_{[30]}$.

(c) $\ddot{a}_{[30]}$.

6. Mortality follows a De Moivre distribution. Given that $\mathring{e}_{(21)} = 42$ and $\delta = .03$ find: 8 pts.

(a) $1xn2550$

(b) $\ddot{a}_{25:\overline{50}|}$.

I want a number–not a formula!

7. You are given two mortality assumptions: ILT (Illustative Life Table) and CF, constant force of mortality. You also know that ${}_2p_{30}$ is the same for both mortality assumptions. Compute $e_{2:\overline{30}|}$ under the constant force assumption. 8 pts.

I want a number–not a formula!

8. Find $\overline{A}_0$ given that $\delta = .03$ and 8 pts.

$$\mu(t) = .02 \quad 0 \le t \le 10$$
$$l_x = 100 - x \quad 10 \le x.$$

I want a number–not a formula!