STAT 472
EXAM #
9/30/0

(1) (a.) APV: $\boxed{\ddot{a}_1 = (1) + \left(\frac{4}{5}\right)e^{-\delta} + \left(\frac{4}{5}\right)\left(\frac{1}{3}\right)e^{-\delta(2)}}$

-1 (b.) SPP: $\boxed{100 A_1 = 100\left[\cancel{\left(\frac{1}{5}\right)e^{-\delta} + \left(\frac{4}{5}\right)\left(\frac{2}{3}\right)e^{-\delta(2)}}\right]}$

$100\left(1\left(\frac{1}{5}\right)\cdot e^{-\delta} + \frac{4}{5}\left(\frac{2}{3}\right)e^{-2\delta} + \frac{1}{3}\cdot\frac{4}{5}\cdot(1)\,e^{-3\delta}\right)$

(c.) $\boxed{\mathring{e}_1 = \left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{3}\right)}$

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(2) $\mu(x) = \frac{3}{2}\sqrt{x}$, $\delta = \text{FOI}$

(a) $s(x) = \exp\left[-\int_0^x \frac{3}{2}\sqrt{t}\,dt\right]$
$= \boxed{\exp\left[-(x)^{3/2}\right]}$

(b) $500\,A^{\,1}_{20:\overline{25}|} = 500\left[\int_0^{25}\left(\frac{\exp[-(20+t)^{3/2}]}{\exp[-(20)^{3/2}]}\right)\left(\frac{3}{2}\sqrt{t+20}\right)\left(e^{-\delta t}\right.\right.$

(c) APV for $500\,\bar{a}_{20:\overline{30}|} = 500\left[\int_0^{30}\left(\frac{\exp[-(20+t)^{3/2}]}{\exp[-(20)^{3/2}]}\right)\left(e^{-\delta t}\right)dt\right.$

(d) $\mathring{e}_{20} = \int_0^{\infty}\frac{\exp[-(20+t)^{3/2}]}{\exp[-(20)^{3/2}]}\,dt$

(e) $= ({}_{40}p_{20})\cdot({}_{30}q_{30}) + ({}_{30}p_{30})\cdot({}_{40}q_{20})$

$$= \left[\frac{\exp[-(60)^{3/2}]}{\exp[-(20)^{3/2}]}\right]\cdot\left[1 - \left[\frac{\exp[-(60)^{3/2}]}{\exp[-(30)^{3/2}]}\right]\right] + \left[\frac{\exp[-(60)^{3/2}]}{\exp[-(30)^{3/2}]}\right]\cdot\left[1 - \left(\frac{\exp[-(60)^{3/2}]}{\exp[-(20)^{3/2}]}\right)\right]$$

③ $l_x = \frac{1}{(2+x)^2}$, FoI $= \delta$

ⓐ $\mu_{20}(t)$: $s(x) = \frac{l_x}{l_0} = \frac{\frac{1}{(2+x)^2}}{.25} = \frac{4}{(2+x)^2}$

$$\frac{-d \log(s(x))}{dx} = \mu(x)$$

$$\log(s(x)) = \log(4(2+x)^{-2}) = -2[\log(4(2+x))]$$
$$= -2\log(8+4x)$$

$$\frac{-d \log(s(x))}{dx} = \frac{-8}{8+4x}$$

$$= \frac{-8}{8+4x} = -\left(\frac{-2}{2+x}\right)$$

O.K. so $\boxed{\mu(x) = \frac{2}{2+x}}$ $\mu_{20}(x)? = \frac{2}{2+x+20}$ ← You erase

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ⓑ $_{10|5}q_{20} = \frac{l(30) - l(35)}{l(20)} = \boxed{\frac{(32)^{-2} - (37)^{-2}}{(22)^{-2}}}$

$\boxed{= 0.119113518}$

ⓒ $_{30}E_{20} = \boxed{\frac{l(50)}{l(20)}} e^{-\delta(30)}$

$$= \boxed{\frac{(2+50)^{-2}}{(2+20)^{-2}} \left(e^{-\delta(30)}\right)}$$

$$= \boxed{0.178994083\, e^{-30\delta}}$$

4 a) $SPP = 500\left[(A_{20}) - ({}_{10}E_{20})(A_{30})\right]$ (From table)

b) $SPP = 500\left(\frac{0.06}{\ln(1.06)}\right)\left[(A_{20}) - ({}_{10}E_{20})(A_{30})\right]$ (from tak

c) $APV = \ddot{a}_{20} - ({}_{10}E_{20})(\ddot{a}_{30})$ (From table)

d) $APV = \left[\ddot{a}_{20} - ({}_{10}E_{20})(\ddot{a}_{30})\right] - 1 + {}_{10}E_{20}$ (From tabl

e) from (b) $\bar{A}^{\,1}_{20:10} = \left(\frac{0.06}{\ln(1.06)}\right)\left[(A_{20}) - ({}_{10}E_{20})(A_{30})\right]$ (From t

Since $\bar{A}_{20:\overline{10}|} = \bar{A}^{\,1}_{20:\overline{10}|} + {}_{10}E_{20}$,

& $\delta\,\bar{a}_{20:\overline{10}|} + \bar{A}_{20:\overline{10}|} = 1$

we have $\bar{a}_{20:\overline{10}|} = \dfrac{1 - \bar{A}_{20:\overline{10}|}}{\delta}$

OR $\bar{a}_{20:\overline{10}|} = \dfrac{1 - \left[\left(\frac{.06}{\ln(1.06)}\right)\left[A_{20} - ({}_{10}E_{20})(A_{30})\right] + {}_{10}E_{20}\right]}{\ln(1.06)}$

(A_{20}, ${}_{10}E_{20}$, A_{30} from ILT)

(5) $\mu_{[30]}(t) = \begin{cases} \mu_{30}(t) + \frac{3-t}{100} & 0 \le t < 3 \\ \mu_{30}(t) & t \ge 3 \end{cases}$

(a) $p_{[30]+1}$:

$$\boxed{{}_tp_{[30]} = e^{-\int \mu_{30}(t)\,dt - \int \frac{3-t}{100}\,dt} = {}_tp_{30} \cdot e^{-\left[\frac{3t - t^2/2}{100}\right]}}$$

$$p_{[30]+1} = \left[\frac{l(32)}{l(31)}\right]\exp\left[\frac{-3(2)+2}{100}\right]$$

note $\frac{2^2}{2} = 2$.

$\hookrightarrow$ ($l(32)$ + $l(31)$ from ILT)

(b) (From pt. A) ${}_tp_{[30]} = {}_tp_{30} \cdot e^{-\left[\frac{3t - t^2/2}{100}\right]}$

$$\boxed{{}_2p_{[30]} = \left[\frac{l_{(32)}}{l_{(30)}}\right] e^{-\left[\frac{6-2}{100}\right]}}$$

($l_{(32)}$ + $l_{(30)}$ from ILT.)

(c)
$$\boxed{\ddot{a}_{[30]} = 1 + \left[\frac{l(31)}{l(30)}\right] \cdot e^{-\left[\frac{3-1/2}{100}\right]}(1.06)^{-1} + \left[\frac{l(32)}{l(30)}\right] \cdot e^{-\left[\frac{6-2}{100}\right]}(1.06)^{-2} + \left[\frac{l(33)}{l(30)}\right](1.06)^{-3}(\ddot{a}_{33})}$$

(all $l(x)$ + $\ddot{a}_{33}$ taken from ILT

⑥ Mortality follows deMoivre.

$\mathring{e}(21) = 42$ & $\delta = .03$, find:

if $\mathring{e}(21) = 42$, $\quad \frac{\omega - 21}{2} = 42$

so $\omega = 105$

(a.) $\bar{A}^{1}_{25:\overline{50|}} = \int_0^{50} \frac{1}{80} e^{-.03t}\,dt$

$$= \frac{1}{(.03)(80)}\left[1 - e^{-.03(50)}\right]$$

$$= \boxed{0.323695767}$$

(b) $\ddot{a}_{25:\overline{50|}}$

$\Rightarrow$ Since deMoivre,

$$\left(\frac{\delta}{i}\right)\bar{A}^{1}_{25:\overline{50|}} = A^{1}_{25:\overline{50|}}$$

$$\text{so } A^{1}_{25:\overline{50|}} = \left(\frac{.03}{1-e^{.03}}\right)\left(\bar{A}^{1}_{25:\overline{50|}}\right) = .318864607$$

$e^{\delta} = 1.030454534$ so $i = .030454534$

$A_{25:\overline{50|}} = .318864607 + {}_{50}E_{25}$

$\hookrightarrow \left(\frac{105-25-50}{105-25}\right)\left(e^{-.03(50)}\right)$

$= .08367381$

$d = 1 - \frac{1}{1+i}$

& $\ddot{a}_{25:\overline{50|}}$: $d\ddot{a}_{25:\overline{50|}} + A_{25:\overline{50|}} = 1$

$$\frac{1 - .08367381}{.029554466} = 31.00466068$$

$$\boxed{\ddot{a}_{25:\overline{50|}} = 31.00466068}$$

(7) ${}_2p_{30}$ for ILT $= \frac{9{,}471{,}591}{9{,}501{,}381} = .996864666$

${}_2p_{30}$ for CFOM $= e^{-2\mu} = .996864666$

so $\mu = .00157013$

& $e_{30:\overline{2}|} = {}_1p_{30} + {}_2p_{30}$

${}_1p_{30} = e^{-.00157013} = 0.998431102$

so $e_{30:\overline{2}|} = .998431102 + .996864666$

$= \boxed{1.995295768}$

(8) $\bar{A}_0$ given $\delta = .03$ & $\mu(t) = .02,\ 0 \le t \le 10$

$l_x = 100 - x, \quad 10 \le x$

$$\bar{A}_0 = \int_0^{10} .02\, e^{-.05t}\, dt + {}_{10}E_0 \int \frac{1}{90} e^{-.03t}\, dt$$

First Integral: $\frac{0.02}{0.05}\left[1 - e^{-.05(10)}\right] = 0.1573877$

Second part: deMoivre's Law:

so ${}_tp_x \mu_x = \frac{1}{100-x} = \frac{1}{90}$

$$\int_0^{90} \frac{1}{90} e^{-.03t}\, dt = \frac{1}{(.03)(90)}\left[1 - e^{-.03(90)}\right] = 0.34547944$$

& ${}_{10}\bar{E}_0 = e^{-(.03)(10)} \cdot e^{-(.02)(10)} = 0.60653066$

so $\bar{A}_0 = 0.157387736 + (.60653066)(.34547944)$

$\boxed{\bar{A}_0 = 0.366931609}$