1) a) $10{,}000\,\bar{A}_{20} = 10{,}000\,(A_{20})\frac{i}{\delta} = 10{,}000\,(.06525)\left(\frac{.06}{\ln(1.06)}\right) = 671.8849084$ (assuming UDD)

b) $P = \frac{10{,}000\,\bar{A}_{20}}{\ddot{a}_{20}} = \frac{671.8849084}{16.5133} = 40.69$

c) $L = v^T - P\left(\frac{1-v^{K+1}}{d}\right)$

Where $P = 40.69$ $\quad d = \frac{.06}{1.06}$ $\quad v = (1.06)^{-1}$

2) a) $10{,}000\,\bar{A}'_{20:10|}$ $\quad A_{20} = .06525$ $\quad A_{30} = .10248$ $\quad l_{30} = 9{,}501{,}381$ $\quad l_{20} = 9{,}617{,}802$

$\bar{A}'_{20:10|} = \bar{A}_{20} - {}_{10}E_{20}\,\bar{A}_{30}$

Assuming UDD $\quad \bar{A}'_{20:10|} = \left(\frac{.06}{\ln(1.06)}\right)A_{20} - (1.06)^{-10}\left(\frac{l_{30}}{l_{20}}\right)\left(\frac{.06}{\ln(1.06)}\right)A_{30}$

$\bar{A}'_{20:10|} = .008977$

$10{,}000\,\bar{A}'_{20:10|} = 89.77400285$

b) $P = \frac{10{,}000\,\bar{A}'_{20:10|}}{\ddot{a}_{20:10|}} = \frac{89.77400285}{\ddot{a}_{20:10|}} = \frac{89.77400285}{7.766511\,51} = 11.55911541$

$\ddot{a}_{20:10|} = \ddot{a}_{20} - {}_{10}E_{20}\,\ddot{a}_{30}$

$\ddot{a}_{20:10|} = 16.5133 - (1.06)^{-10}\left(\frac{l_{30}}{l_{20}}\right)(15.8561)$

$\ddot{a}_{20:10|} = 7.766511\,51$

c) $P = \frac{10{,}000\,\bar{A}'_{20:10|}}{\ddot{a}^{(12)}_{20:10|}} = \frac{89.77400285}{7.578369377} = 11.84608435$

$\ddot{a}^{(12)}_{20:10|} = \ddot{a}^{(12)}_{20} - {}_{10}E_{20}\,\ddot{a}^{(12)}_{30}$

$\ddot{a}^{(12)}_{20} = \alpha(12)\,\ddot{a}_{20} - \beta(12) = (1.0028)(16.5133) - .46812$

$= 16.09141724$

$\ddot{a}^{(12)}_{30} = \alpha(12)\,\ddot{a}_{30} - \beta(12) = (1.0028)(15.8561) - .46812$

$= 15.43237708$

$\ddot{a}^{(12)}_{20:10|} = 16.09141724 - (1.06)^{-10}\left(\frac{l_{30}}{l_{20}}\right)(15.43237708)$

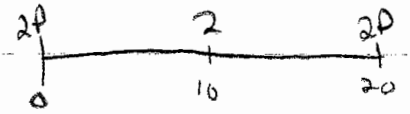
$\ddot{a}^{(12)}_{20:10|} = 7.578369377$

3 a) $1{,}000\bar{A}_{20}$ Because of Demoivre, UDD can be used throughout #3

$$\bar{A}_{20} = \frac{1-e^{-\delta n}}{\delta n} = \frac{1-e^{-(.05)(100-20)}}{(80)(.05)} = .24542109$$

$$1{,}000\bar{A}_{20} = 245.42109\,03$$

b)



$$2P + v^{10}{}_{10}p_{20}\,2 + v^{20}{}_{20}p_{20}\,2P = 1{,}000\bar{A}_{20} \qquad v = e^{-.05}$$

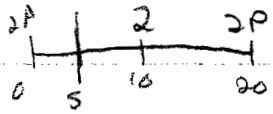
$$2P + e^{-.05(10)}\left(\frac{100-30}{100-30}\right)2 + e^{-.05(20)}\left(\frac{100-40}{100-20}\right)2P = 245.42109\,03$$

$$2P\left(1 + e^{-.05(20)}\left(\frac{6}{8}\right)\right) = 245.42109\,03 - e^{-.05(10)}\left(\frac{70}{100-20}\right)2$$

$$2P(1.27590958\,1) = 244.3596616$$

$$P = 95.75900411$$

c)



$${}_5V = 1{,}000\bar{A}_{25} - \left(v^5{}_5p_{25}\,2 + v^{15}{}_{15}p_{25}\,2P\right)$$

$$\bar{A}_{25} = \frac{1-e^{-.05(75)}}{.05(75)} = .26039526\,8 \qquad v = e^{-.05} \qquad P = 95.75900411$$

$${}_5p_{25} = \frac{\ell_{30}}{\ell_{25}} = \frac{70}{75} = \frac{14}{15}$$

$${}_{15}p_{25} = \frac{\ell_{40}}{\ell_{25}} = \frac{60}{75} = \frac{4}{5}$$

$${}_5V = 260.39526\,78 - \left(e^{-.05(5)}\left(\frac{14}{18}\right)2 + e^{-.05(15)}\left(\frac{4}{5}\right)(2)(95.75900411)\right)$$

$${}_5V = 186.56814\,52$$

4) a) $L(t) = v^T - \bar{P}\left(\frac{1-v^T}{\delta}\right)$

where $v = e^{-.04}$ $\delta = .04$

$$\bar{P} = \frac{1{,}000\bar{A}_{20}}{\ddot{a}_{20}} = 20.18884049$$

$$\bar{A}_{20} = \frac{1-e^{-\delta n}}{\delta n} = \frac{1-e^{-(.04)(90-20)}}{.04(70)} = .33542\,4978$$

$$\ddot{a}_{20} = \frac{1-\bar{A}_{20}}{\delta} = \frac{1-.33542\,4978}{.04} = 16.61437556$$

1000 - PV

c) $Var[L] = \left({}^2\bar{A}_{20} - (\bar{A}_{20})^2\right)\left(1 + \frac{\bar{P}}{\delta}\right)^2$

${}^2\bar{A}_{20} = \frac{1 - e^{-2\delta w}}{2\delta w} = \frac{1 - e^{-2(.04)(20)}}{2(.04)(20)} = .17791096$

$\bar{A}_{20} = .33542497 8$

$\bar{P} = 20.18884049 \quad \delta = .04$

$Var[L] = 16726.59657$

d) ${}_{10}L = v^{T-10} - \bar{P}\left(\frac{1 - v^{T-10}}{\delta}\right) \quad t \geq 10$

$v = e^{-.04} \quad \delta = .04$

$\bar{P} = \frac{1{,}000\,\bar{A}_{30}}{\bar{a}_{30}} = 24.39849999$

$\bar{A}_{30} = \frac{1 - e^{-(.04)(60)}}{(.04)(60)} = .37886751 9$

$\bar{a}_{30} = \frac{1 - \bar{A}_{30}}{\delta} = \frac{1 - .37886751 9}{.04} = 15.52831 2 c$

e) $Var[{}_{10}L \mid T(20) \geq 10] = \left({}^2\bar{A}_{30} - (\bar{A}_{30})^2\right)\left(1 + \frac{\bar{P}}{\delta}\right)^2$ (1000.)

${}^2\bar{A}_{30} = \frac{1 - e^{-2(.04)(60)}}{2(.04)(60)} = .20661880 3$

$\bar{A}_{30} = .37886751 9$

$\bar{P} = 24.39849999 \quad \delta = .04$

$Var[{}_{10}L \mid T(20) \geq 10] = (.06307820 6)(610.9624975)^2 = 23545.52828$

5) $q_x = 1 - \frac{l_{x+1}}{l_x} = 1 - \frac{20}{100} = \frac{3}{10} \quad q_{x+1} = 1 - \frac{l_{x+2}}{l_{x+1}} = 1 - \frac{50}{70} = \frac{2}{7}$

$p_x = \frac{7}{10} \quad p_{x+1} = \frac{5}{7}$

${}_0V = 0$

${}_1V = \frac{(1.04)(0 + 100) - 1{,}000(.3)}{\frac{7}{10}} = 280$

${}_2V = \frac{(1.04)(-280 + 50) - 1{,}000\left(\frac{2}{7}\right)}{\frac{5}{7}} = -734.88$

6 $1{,}000\,\bar{A}_{50} = 250$

$\bar{A}_{50} = .25$ $\quad 1 - \frac{\bar{a}_{50}}{\bar{a}_{40}} = .11765$

$\frac{B\,\bar{A}_{40}}{\ddot{a}_{40}} = 25.93$

$\bar{a}_{50} = \frac{1 - .25}{.05} = 15$ $\quad 1 - \frac{15}{\bar{a}_{40}} = .11765$

$.88235\,\bar{a}_{40} = 15$

$\bar{a}_{40} = 17.000056607$

$1 - 17.000056607(.05) = \bar{A}_{40}$

$\bar{A}_{40} = .149997167$ ✓

$\ddot{a}_{40} = \frac{1 - \bar{A}_{40}}{d} = \frac{1 - \frac{i}{\delta}\bar{A}_{40}}{d}$; assuming UDD

$\ddot{a}_{40} = \frac{1 - \left(\frac{.05}{e^{.05} - 1}\right)(.149997167)}{1 - e^{-.05}}$ $\quad 1 + i = e^{.05}$ $\quad i = e^{.05} - 1$

$\ddot{a}_{40} = 17.504184806$ $\quad 1 - d = e^{-.05}$

$d = 1 - e^{-.05}$

$\frac{B(.149997167)}{17.504184806} = 25.93$ ✓

$B = 3{,}026.061887$