1. a) \$10,000 whole life (30), PTD w/ annual premiums , $i = 6\%$, ILT mortality

Single payment premium $= 10{,}000\,\bar{A}_{30} = 10{,}000\,\frac{i}{\delta} A_{30} = 10{,}000\,\frac{.06}{\ln(1.06)}\left(\frac{102.48}{1000}\right) = 1{,}055.25$ (UDD)

b) Annual premium $= \dfrac{10{,}000\,\bar{A}_{30}}{\ddot{a}_{30}} = \dfrac{1055.25}{15.8561} = 66.55$ (Again, UDD)

c) $L = 10{,}000\,v^{T} - 66.55\,\dfrac{1 - v^{K+1}}{d} = 10{,}000(1.06)^{-T} - 66.55\,\dfrac{1-(1.06)^{-(K+1)}}{\frac{.06}{1.06}}$

where T is the time until death and $K = \lfloor T \rfloor$

2. a) \$10,000, 20-yr term PTD to (30) , $i = .06$, ILT mortality

Single payment premium $= 10{,}000\,\bar{A}^{1}_{30:\overline{20|}} = 10{,}000\left(\bar{A}_{30} - {}_{20}E_{30}\,\bar{A}_{50}\right) = 10{,}000\,\frac{i}{\delta}\left(A_{30} - {}_{20}E_{30}\,A_{50}\right)$

$= 10{,}000\,\frac{.06}{\ln(1.06)}\left(\frac{102.48}{1000} - \frac{293.74}{1000}\cdot\frac{249.05}{1000}\right) = 301.95$ (UDD)

b) $\ddot{a}_{30:\overline{20|}} = \ddot{a}_{30} - {}_{20}E_{30}\,\ddot{a}_{50} = 15.8561 - \frac{293.74}{1000}\cdot 13.2668 = 11.96$

Annual premium $= \dfrac{10{,}000\,\bar{A}^{1}_{30:\overline{20|}}}{\ddot{a}_{30:\overline{20|}}} = 25.25$ (UDD)

c) $\ddot{a}^{(12)}_{30} = \alpha(12)\,\ddot{a}_{30} - \beta(12) = 1.00028\,(15.8561) - .46812 = 15.39$

$\ddot{a}^{(12)}_{50} = \alpha(12)\,\ddot{a}_{50} - \beta(12) = 1.00028\,(13.2668) - .46812 = 12.80$

$\ddot{a}^{(12)}_{30:\overline{20|}} = \ddot{a}^{(12)}_{30} - {}_{20}E_{30}\,\ddot{a}^{(12)}_{50} = 15.39 - \frac{293.74}{1000}(12.80) = 11.63$

Annualized monthly premium $= \dfrac{10{,}000\,\bar{A}^{1}_{30:\overline{20|}}}{\ddot{a}^{(12)}_{30:\overline{20|}}} = \dfrac{301.95}{11.63} = 25.96$ => monthly premium $= 2.16$ (UDD)

3. a) \$1,000 whole life PTD to (30) , $\delta = .05$, $l_x = 100 - x$, $0 \le x \le 100$ (Note: De Moivre = UDD throughout)

single payment premium $= 1000\,\bar{A}_{30} = 1000\int_0^{70} e^{-.05t}\,\frac{1}{70}\,dt = 1000\left(\frac{1}{70}\right)\left(\frac{1}{.05}\right)\left(1 - e^{-.05(70)}\right) = 277.09$

b) $277.09 = P + 2e^{-.05(10)}\left(\frac{60}{70}\right) + P\,e^{-.05(20)}\left(\frac{50}{70}\right)$ => $P = 218.60$

c) ${}_5V = 1000\,\bar{A}_{35} - 2e^{-.05(5)}\left(\frac{60}{65}\right) - P\,e^{-.05(15)}\left(\frac{50}{65}\right)$

$= 1000\left(\frac{1}{65}\right)\left(\frac{1}{.05}\right)\left(1 - e^{-.05(65)}\right) - 2e^{-.05(5)}\left(\frac{60}{65}\right) - 218.60\,e^{-.05(15)}\left(\frac{50}{65}\right) = 214.89$

4. a) \$1000 whole life PTD to (30) , $\delta = .05$, $l_x = 100 - x$, paid w/ a continuous annuity (De Moivre = UDD)

$L = 1000\,v^{T} - \bar{P}(\bar{A}_{30})\,\dfrac{1 - v^{T}}{\delta}$, $\bar{a}_{30} = \dfrac{1 - \bar{A}_{30}}{\delta} = \dfrac{1 - .27709}{.05} = 14.46$ (using values from 3a)

$\bar{P}(\bar{A}_{30}) = \dfrac{1000\,\bar{A}_{30}}{\bar{a}_{30}} = \dfrac{277.09}{14.46} = 19.16$

Thus, $L = 1000\,e^{-.05T} - 19.16\,\dfrac{1 - e^{-.05T}}{.05}$

b) - NIXED -

c) $Var(L) = Var\left(1000\, v^T - 19.16\, \frac{1-v^T}{.05}\right) = Var\left(\left(1000 + \frac{19.16}{.05}\right) v^T\right) = \left(1000 + \frac{19.16}{.05}\right)^2 Var(v^T)$

$^2\bar{A}_{30} = \int_0^{70} e^{-.1t} \frac{1}{70}\, dt = \left(\frac{1}{70}\right)\left(\frac{1}{.1}\right)\left(1 - e^{-.1(70)}\right) = .1427$, $\bar{A}_{30} = .27709$ (from 3a)

$\Rightarrow Var(L) = \left(1000 + \frac{19.16}{.05}\right)^2 \left({}^2\bar{A}_{30} - (\bar{A}_{30})^2\right) = \left(1000 + \frac{19.16}{.05}\right)^2 \left(.1427 - (.27709)^2\right) = 126,194.92$

d) ${}_{10}L = 1000\, v^{T-10} - 19.16\, \frac{1 - v^{T-10}}{\delta} = 1000\, e^{-.05(T-10)} - 19.16\, \frac{1 - e^{-.05(T-10)}}{.05}$ where T is the future lifetime of (30)

e) $Var({}_{10}L \mid T(30) \geq 10) = \left(1000 + \frac{19.16}{.05}\right)^2 \left({}^2\bar{A}_{40} - (\bar{A}_{40})^2\right)$

$\bar{A}_{40} = \int_0^{60} e^{-.05t} \frac{1}{60}\, dt = \frac{1}{60}\left(\frac{1}{.05}\right)\left(1 - e^{-.05(60)}\right) = .3167$, ${}^2\bar{A}_{40} = \int_0^{60} e^{-.1t} \frac{1}{60}\, dt = \frac{1}{60}\left(\frac{1}{.1}\right)\left(1 - e^{-.1(60)}\right) = .1663$

$\Rightarrow Var({}_{10}L \mid T(30) \geq 10) = \left(1000 + \frac{19.16}{.05}\right)^2 \left(.1663 - (.3167)^2\right) = 126,158.25$

5. $l_x = 100$, $l_{x+1} = 90$, $l_{x+2} = 60$, $i = .04$.

${}_0V + 50 = v\, q_x (1000) + v\, p_x ({}_1V) \Rightarrow 50 = (1.04)^{-1}(.1)(1000) + (1.04)^{-1}(.9)({}_1V) \Rightarrow {}_1V = -53.33$

${}_1V + 100 = v\, q_{x+1} (1000) + v\, p_{x+1} ({}_2V) \Rightarrow -53.33 + 100 = (1.04)^{-1}\left(\frac{1}{3}\right)(1000) + (1.04)^{-1}\left(\frac{2}{3}\right)({}_2V) \Rightarrow {}_2V = -427.2$

6. $\delta = .05$, $\bar{A}_{50} = .25$, ${}_{10}\bar{V}(\bar{A}_{40}) = .11765$, $B\, \bar{A}_{40} = 25.93\, \ddot{a}_{40}$. Find B .

${}_{10}\bar{V}(\bar{A}_{40}) = \frac{\bar{A}_{50} - \bar{A}_{40}}{1 - \bar{A}_{40}} \Rightarrow .11765 = \frac{.25 - \bar{A}_{40}}{1 - \bar{A}_{40}} \Rightarrow .11765 - .11765\, \bar{A}_{40} = .25 - \bar{A}_{40} \Rightarrow \bar{A}_{40} = .15$

$A_{40} = \frac{\delta}{i} \bar{A}_{40} = \frac{.05}{e^{.05} - 1} (.15) = .146$. $\ddot{a}_{40} = \frac{1 - A_{40}}{d} = \frac{1 - .146}{1 - e^{-.05}} = 17.505$

$\Rightarrow B(.15) = 25.93(17.505) \Rightarrow B = 3026.06$ (UDD)

~~assume UDD~~