1. Below is a life table for frumious bandersnatches. 10 pts.

(x)	q_x	l_x
0	$\frac{1}{4}$	100
1	$\frac{1}{5}$	75
2	$\frac{2}{3}$	60
3	$\frac{4}{5}$	20
4	1	4

$60 \times (1-\frac{2}{3})$

Assume that $i = .05$. Find the APV of a whole life annuity that pays \$1 at the beginning of each year sold to a 1 year old bandersnatch.

$$APV = \sum_{n=0}^{\infty} v^n \, {}_nP_1$$

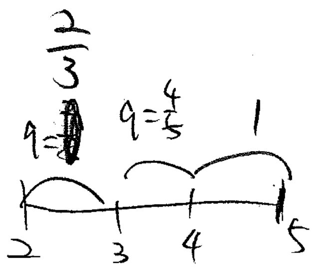
$$= 1 + v \cdot (1-\tfrac{1}{5}) + v^2(1-\tfrac{1}{5})(1-\tfrac{2}{3}) + v^3(1-\tfrac{1}{5})(1-\tfrac{2}{3})(1-\tfrac{4}{5})$$

$$= 1 + 1.05^{-1} \cdot \tfrac{4}{5} + 1.05^{-2} \cdot \tfrac{4}{5} \cdot \tfrac{1}{3} + 1.05^{-3} \cdot \tfrac{4}{5} \cdot \tfrac{1}{3} \tfrac{1}{5}$$

$$= 2.04985$$

100

2. Let data be as in problem 1. We sell a \$100 whole life PYD policy to a 2 year old bandersnatch. The beginning of year costs (BOY), end of year costs (EOY), and annual premiums (P) are as listed below. (These were NOT determined by the equivalence principle.) **Use the recursion relation** to find the gross reserves ${}_kV$ for $k = 2, 1,$ and 0. **The purpose of this question is to test your knowledge of the recursion relation. Other techniques will not be accepted.** 10 pts.



Time After Issue	BOY	At Death	P
0	4	2	40
1	1	2	50
2	3	2	~~60~~
3	3	2	70
4	3	2	70

Because a 2 year old bandersnatch could only live three more years at most,

so: $({}_2V + P - E)(1+i) = 100 + 2$

$({}_2V + \cancel{60}\,70 - 3) \times 1.05 = 102$

${}_2V = 40.1429$

$({}_1V + P - E)(1+i) = q_3 \times 102 + p_3 \times {}_2V$

$({}_1V + 50 - 1) \times 1.05 = \frac{4}{5} \times 102 + \frac{1}{5} \times 40.1429$

${}_1V = 36.3606$

$({}_0V + P - E)(1+i) = q_2 \times 102 + p_2 \times {}_1V$

$({}_0V + 40 - 4) \times 1.05 = \frac{2}{3} \times 102 + \frac{1}{3} \times 36.3606$

${}_0V = 40.30495$

100

$d = 1-(1+i)^{-1} = 1-\frac{1}{1+i} = \frac{i}{1+i}$

3. Assume ILT data. We sell a temporary annuity to (70) that pays \$100,000 each year at the beginning of the year for 10 years.

   (a) Find the APV of this annuity.

   (b) Find $Var(Y)$ where Y is the present value random variable for this annuity.

12 pts.

a)

$APV = 100{,}000\,\ddot{a}_{70:\overline{10}|} = 100{,}000 \times (\ddot{a}_{70} - {}_{10}E_{70}\,\ddot{a}_{80})$

$= 100{,}000 \times (8.5693 - 0.33037 \times 5.9050)$

$= 661846.515$ ✓

b) $Var(Y) = 100{,}000^2 \dfrac{{}^2A_{70:\overline{10}|} - A_{70:\overline{10}|}^2}{d^2}$

${}^2A_{70:\overline{10}|} = {}^2A_{70} - {}_{10}E_{70} \cdot v^{10} \cdot {}^2A_{80} + {}_{10}E_{70} \cdot v^{10}$

$= 0.30642 - 0.33037 \times 1.06^{-10} \cdot 0.47359 + 0.33037 \times 1.06^{-10}$

$= 0.40353048$ ✓

$A_{70:\overline{10}|} = A_{70} - {}_{10}E_{70} A_{80} + {}_{10}E_{70}$

$= 0.51495 - 0.33037 \times 0.66575 + 0.33037$

$= 0.625376$

$\therefore Var(Y) = 100{,}000^2 \cdot \dfrac{0.40353048 - 0.625376^2}{\left(\frac{0.06}{1.06}\right)^2}$ ✓

$= 3.881207 \times 10^{10}$

100

4. Assume ILT data and UDD. Richard age 50 intends to retire at age 70, at which time he hopes to receive \$100,000 each year at the beginning of the year for 10 years. What should his **monthly** contribution to his 401k plan, made at the begging of each month for 20 years, be to insure this annual income? 10 pts.

APV of the income $= 100{,}000 \times \ddot{a}_{70:\overline{10}|} \cdot {}_{20}E_{50}$

$= 100{,}000 \times (\ddot{a}_{70} - {}_{10}E_{70}\, \ddot{a}_{80}) \times {}_{20}E_{50}$

$= 100{,}000\,(8.5693 - 0.33037 \times 5.905) \times 0.23047$

$= 152535.7663$

so $P \times \ddot{a}^{(12)}_{50:\overline{20}|} = 152535.7663$

$\ddot{a}^{(12)}_{50:\overline{20}|} = \ddot{a}^{(12)}_{50} - {}_{20}E_{50}\, \ddot{a}^{(12)}_{70}$

$\ddot{a}^{(12)}_{50} = \alpha(12)\, \ddot{a}_{50} - \beta(12)$

$= 1.00028 \cdot 13.2668 - 0.46812$

$= 12.8023947$

$\ddot{a}^{(12)}_{70} = \alpha(12)\, \ddot{a}_{70} - \beta(12)$

$= 1.00028 \times 8.5693 - 0.46812$

$= 8.1035794$

So $\ddot{a}^{(12)}_{50:\overline{20}|} = 12.8023947 - 0.23047 \times 8.1035794$

$= 10.934763$

so $P = \dfrac{152535.7663}{10.934763} = 13949.6183$

So monthly contribution $= \dfrac{P}{12} = 1162.47$

100

5. Assume ILT data. Penney Life sells a special whole life PYD policy to (30). The annual payments are P for years 1-10 and $2P$ thereafter. The benefit is \$200,000 in years 1-20 and \$100,000 thereafter. Find P. 10 pts.

APV of Payment $= P\ddot{a}_{30} + {}_{10}E_{30} \cdot P \cdot \ddot{a}_{40}$

$= P \cdot 15.8561 + 0.54733 \times P \times 14.8166$

$= 23.96567P$

APV of benefit $= 100{,}000 A_{30} + 100{,}000 A^{1}_{30:\overline{20}|}$

$= 100{,}000 A_{30} + 100{,}000 (A_{30} - {}_{20}E_{30} A_{50})$

$= 100{,}000 \times 0.10248 + 100{,}000 (0.10248 - 0.29374 \times 0.24905)$

$= 13180.4053$

So $23.96567 P = 13180.4053$

$P = 549.97$

100

6. Assume ILT data. STAT 490 Life sells a $100,000 whole life PTD policy to (30) that is paid with annual payments. We know: 14 pts.

   (i) Commissions are 60% of the premium in the first year and 10% of the premium there after.

   (ii) There is a $100 expense to close the policy upon death of the insured.

   (a) Find the gross premium P^g.

   (b) Find the net premium P^n.

   (c) Find the expense premium P^e.

(a) $B + E = P$

$100{,}000\bar{A}_{30} + 0.1 \times P^g \cdot \ddot{a}_{30} + 0.5 \times P^g + 100\bar{A}_{30} = P^g \ddot{a}_{30}$

$100{,}100\bar{A}_{30} = (0.9 \times \ddot{a}_{30} - 0.5) \times P^g$

$\bar{A}_{30} = \frac{i}{\delta} A_{30} = \frac{0.06}{\ln 1.06} \times 0.10248 = 0.10552454$

So $100{,}100 \cdot 0.10552454 = (0.9 \cdot 15.8561 - 0.5) \times P^g$

$P^g = 767.0756$

(b) $100{,}000\bar{A}_{30} = P^n \cdot \ddot{a}_{30}$

$P^n = \frac{100{,}000 \cdot 0.10552454}{15.8561}$

$P^n = 665.51387$

(c) $P^e = P^g - P^n$

$= 767.0756 - 665.51387$

$= \cancel{110}.5617$

101

100

7. For the policy described in problem 6, assume that STAT 490 Life charges a gross annual premium of \$1000. 14 pts.

   (a) Give an expression for the loss at issue random variable L_0.

   (b) What is the value of the loss if the insured dies at age 40?

   (c) What is the probability that $L_o \leq 0$?

   (d) Find $Var(L_o)$.

(a) $L_0 = 100{,}100 \cdot V^{T_{30}} + 0.5 \times 1000 - 0.9 \times 1000 \cdot \frac{1 - V^{k_{30}+1}}{d}$

$= 100{,}100 \cdot V^{T_{30}} + 500 - 900 \times \frac{1 - V^{k_{30}+1}}{d}$

(b) Die 40

$L_0 = 100{,}100 \cdot 1.06^{-10} + 500 - 900 \times \frac{1 - 1.06^{-11}}{\frac{0.06}{1.06}}$

$= 48871.23882$

(c) $P(L_0 \leq 0)$

$= P\left(100{,}100\, V^{T_{30}} + 500 - 900 \times \frac{1 - V^{k_{30}+1}}{d} < 0\right)$

assume UDD

does not work

$= P\left(100{,}100 \times \cancel{\frac{i}{\delta}}\, V^{k_{30}+1} + 500 - 900 \times \frac{1 - V^{k_{30}+1}}{d} < 0\right)$ $\quad \cancel{V^{k_{30}+1}}\ \ V^{T} \neq \frac{i}{\delta} V^{K}$

$= P(118973.8381\, V^{k_{30}+1} < 15400)$

$= P(k_{30} > 34.0879)$ $\underline{\underline{\text{assume UDD}}}$ $\frac{(1 - 0.0879) \times l_{64} + 0.0879 \times l_{65}}{l_{30}} = 0.80733$

(d) Assume UDD

$L_0 = 100100 \frac{i}{\delta} V^{k_{30}+1} + 500 - 900 \times \frac{1 - V^{k_{30}+1}}{d} = \left(100100 \frac{i}{\delta} + \frac{900}{d}\right) V^{k_{30}+1} + 500 - \frac{900}{d}$

~~$var(L_0) = \left(100100 \frac{i}{\delta} + \frac{900}{d}\right)^2 \left({}^2A_{30} - A_{30}^2\right)$~~

~~$= \left(100100 \frac{i}{\delta} + \frac{900}{d}\right)^2 \times (0.02531 - 0.1048^2)$~~

~~$= 20960766.6$~~

100

8. STAT 490 Life charges a gross annual premium of \$1000 for the policy described in problem 6. (This was not determined by the equivalence principle.) Find the gross reserves at 20 years ${}_{20}V^g$ for this policy. 10 pts.

$${}_{20}V^g = 100{,}000\,\bar{A}_{50} + 100\,\bar{A}_{50} + 0.1 \times 1000\,\ddot{a}_{50} - 1000\,\ddot{a}_{50}$$

$$= 100{,}100\,\bar{A}_{50} - 900\,\ddot{a}_{50}$$

assume UDD

$$= 100{,}100\,\frac{i}{\delta}A_{50} - 900\,\ddot{a}_{50}$$

$$= 100100 \cdot \frac{0.06}{\ln(1.06)} \cdot 0.24905 - 900 \cdot 13.2668$$

$$= 13730.419$$

100

9. Assume ILT data. STAT 490 Life sells 500, \$100,000 whole life PTD policies to (30) that are paid with annual payments. *Unlike problem 6, this problem does not consider costs.* 10 pts.

Use the normal approximation to find the premium P' for which STAT 490 Life is 95% certain of not having a loss.

For a policy

$$E(L_0) = 100{,}000\,\bar{A}_{30} - P\ddot{a}_{30}$$

assume UDD

$$L_0 = 100{,}000 \cdot \frac{i}{\delta} A_{30} - P\ddot{a}_{30} = \frac{100{,}000\,i}{\delta} A_{30} - P\,\frac{1-A_{30}}{d} \rightarrow \left(\frac{100{,}000\,i}{\delta} + \frac{P}{d}\right)A_{30} - \frac{P}{d}$$

$$E(L_0) = 100{,}000 \times \frac{0.06}{\ln 1.06} \times 0.10248 - 15.8561P$$

$$= 10552.45447 - 15.8561P$$

$$\text{var}(L_0) = \left(\frac{100{,}000 \cdot i}{\delta} + \frac{P}{d}\right)^2 \cdot \left({}^2A_{30} - A_{30}^2\right) \text{ ~~doesn't work.~~}$$

$$= \cancel{\left(\frac{100{,}000 \cdot i}{\delta} + \frac{P}{d}\right)^2 \cdot \left(0.02531 - 0.10248^2\right)}$$

$$= \cancel{0.01480785\left(\frac{100{,}000\,i}{\delta} + \frac{P}{d}\right)^2}$$

$$P(L<0) = 0.95$$

so

$$\frac{500 \times (10552.45447 - 15.8561P)}{\sqrt{500 \times 0.01480785} \times \left(\frac{100{,}000\,i}{\delta} + \frac{P}{d}\right)} = -1.645$$

$$7928.05P - 5276227.235 = 460904.8498 + 79.07724338P$$

$$P = 730.94$$