Test 1

1. LS, WL, 10,000 on (20) ; (30)

$_tp^*_{30} = \frac{70-t}{70}$, $_tp^*_{20} = \frac{80-t}{80}$, $\mu_z = .03$, $\delta = 0.05$

$_tp_{30} = \left(\frac{70-t}{70}\right)e^{-.03t}$, $\mu_{30}(t) = \frac{1}{70-t} + .03$

$\Rightarrow \bar{A}_{30} = \int_0^{70}\left(e^{-.05t}\right)\left[\left(\frac{70-t}{70}\right)e^{-.03t}\right]\left(\frac{1}{70-t} + .03\right)dt$

$_tp_{20} = \frac{80-t}{80}e^{-.03t}$, $\mu_{20}(t) = \frac{1}{80-t} + .03$

$\Rightarrow \bar{A}_{20} = \int_0^{80}\left(e^{-.05t}\right)\left[\left(\frac{80-t}{80}\right)e^{-.03t}\right]\left(\frac{1}{80-t} + .03\right)dt$

$_tp_{20:30} = \left(\frac{70-t}{70}\right)\left(\frac{80-t}{80}\right)e^{-.03t}$, $\mu_{20:30}(t) = \frac{1}{70-t} + \frac{1}{80-t} + .03$

$\Rightarrow \bar{A}_{20:30} = \int_0^{70}\left(e^{-.05t}\right)\left[\left(\frac{70-t}{70}\right)\left(\frac{80-t}{80}\right)e^{-.03t}\right]\left(\frac{1}{70-t} + \frac{1}{80-t} + .03\right)dt$

$\bar{A}_{\overline{20:30}} = \bar{A}_{20} + \bar{A}_{30} - \bar{A}_{20:30}$ and APV of insurance is $10,000\,\bar{A}_{\overline{20:30}}$

2. $\mu^{(1)}(t) = \frac{1}{10-t}$, $\mu^{(2)}(t) = .03$, $\mu^{(3)}(t) = 3t^2$

$_tp_{62}^{(\tau)} = e^{-\int_0^t\left(\frac{1}{10-s} + .03 + 3s^2\right)ds} = e^{-(-\ln(10-s) + .03s + s^3)|_0^t} = e^{-[-\ln(10-t) + .03t + t^3 + \ln(10)]}$

$= e^{\ln\left(\frac{10-t}{10}\right) - .03t - t^3} = \frac{10-t}{10}e^{-.03t-t^3}$

P(cease work due to decrement 2 before $t=3$) $= \int_0^3 {}_tp_{62}^{(\tau)}\,\mu^{(2)}(t)\,dt = \int_0^3\left(\frac{10-t}{10}e^{-.03t-t^3}\right)(.03)\,dt$

3. 1000 lives age 60; disability = decrement 1, death = decrement 2

$q_{60}^{(1)} = q_{60}'^{(1)}\left(1 - \frac{q_{60}'^{(2)}}{2}\right) = .02\left(1 - \frac{.04}{2}\right) = .0196$; $q_{60}^{(2)} = .04\left(1 - \frac{.02}{2}\right) = .0396$

$q_{61}^{(1)} = .04\left(1 - \frac{.06}{2}\right) = .0388$; $q_{61}^{(2)} = .06\left(1 - \frac{.04}{2}\right) = .0588$

Expected number of people disabled before age 62 $= 1000\left[0.0196 + (1 - .0196 - .0396)(.0388)\right] = 56.103$

4. a) $_2p_{37}^{(\tau)} = p_{37}^{(\tau)}\,p_{38}^{(\tau)} = (1 - .02 - .02 - .05)(1 - .05 - .05 - .1) = (.91)(.8) = .728$

b) $_{2|}q_{37}^{(2)} = {}_2p_{37}^{(\tau)}\,q_{39}^{(2)} = (.728)(.05) = .0364$

5. $G\ddot{a}_x = 25,000\bar{A}_x + .23G + 3\left(\frac{25,000}{1000}\right) + 14 + .07G(\ddot{a}_x - 1) + 0.47(25)(\ddot{a}_x - 1) + 2(\ddot{a}_x - 1) + 15\bar{A}_x$

$\Rightarrow G\ddot{a}_x = 25,015\bar{A}_x + .23G + 75 + 14 + .07G\ddot{a}_x - .07G + 11.75\ddot{a}_x - 11.75 + 2\ddot{a}_x - 2$

$\Rightarrow G\ddot{a}_x = 25,015\bar{A}_x + .16G + 75.25 + .07G\ddot{a}_x + 13.75\ddot{a}_x$

$\Rightarrow .93G\ddot{a}_x - .16G = 25,015\bar{A}_x + 13.75\ddot{a}_x + 75.25$

$$\Rightarrow G = \frac{25,015\bar{A}_x + 13.75\ddot{a}_x + 75.25}{.93\ddot{a}_x - .16}$$

6. WL (30), DB = 3000 for decrement 1, 2000 for decrement 2; ${}_tp_{30}^{\prime(1)} = e^{-t^2}$, ${}_tp_{30}^{\prime(2)} = e^{-.03t}$; $\delta = .05$

$${}_tp_{30}^{(\tau)} = e^{-t^2} e^{-.03t} = e^{-t^2-.03t}, \quad \mu^{(1)}(t) = 2t, \quad \mu^{(2)}(t) = .03$$

$$\text{APV ben} = 3000\int_0^\infty e^{-.05t}\left(e^{-t^2-.03t}\right)(2t)\,dt + 2000\int_0^\infty e^{-.05t}\left(e^{-t^2-.03t}\right)(.03)\,dt$$

$$\text{APV prem} = \int_0^\infty e^{-.05t}\left(e^{-t^2-.03t}\right)dt$$

then the benefit premium, $\pi = \dfrac{\text{APV ben}}{\text{APV prem}}$

7. $1650(1-.03-.04) = (1670 + .95G - 80)(1.06) - 10{,}000(.04) - 1000(.03)$

this expression can be solved to find G

8. $\text{APV} = .2Bv + (.7)(.2)Bv^2 + \left[(.7)^2(.2) + (.2)(.5)(.2)\right]Bv^3$

9. $\text{APV prem} = P + .7Pv + \left((.7)^2 + (.2)(.5)\right)Pv^2$

Exam 1

1. Last Survivor, WL, 10,000, x = age 20 y = Age 30

Age 20 : $\frac{80-t}{80} = {}_tp_x'^{(1)}$, Age 30 : $\frac{70-t}{70} = {}_tp_x'^{(2)}$, $\mu_z = .03$, $\delta = .05$

$APV = A_{\overline{xy}} = A_x' + A_y' - A_{xy}'$

$A_x' = \int_0^{80} e^{-.05t}\left(\frac{80-t}{80}\right)e^{-.03t}\left(\frac{1}{80-t}+.03\right)dt = \int_0^{80} e^{-.08t}\left(\frac{80-t}{80}\right)\left(\frac{1}{80-t}+.03\right)dt$

$A_y' = \int_0^{70} e^{-.05t}\left(\frac{70-t}{70}\right)e^{-.03t}\left(\frac{1}{70-t}+.03\right)dt = \int_0^{70} e^{-.08t}\left(\frac{70-t}{70}\right)\left(\frac{1}{70-t}+.03\right)dt$

$A_{xy}' = \int_0^{70} e^{-.05t}\left(\frac{70-t}{70}\right)\left(\frac{80-t}{80}\right)e^{-.03t}\left(\frac{1}{70-t}+\frac{1}{80-t}+.03\right)dt$

-0 $= \int_0^{70} e^{-.08t}\left(\frac{(70-t)(80-t)}{(70)(80)}\right)\left(\frac{1}{70-t}+\frac{1}{80-t}+.03\right)dt$

$\therefore \boxed{APV = A_x' + A_y' - A_{xy}'}$

2. $\int_0^3 {}_tp_x^{(\tau)}\mu_x^{(2)}\,dt$ age = x = 62

${}_tp_x'^{(1)} = \frac{10-t}{10}$, ${}_tp_x'^{(2)} = e^{-.03t}$

${}_tp_x'^{(3)} = e^{-\int_0^t 3x^2dx} = e^{-t^3}$

${}_tp_x^{(\tau)} = {}_tp_x'^{(1)} \cdot {}_tp_x'^{(2)} \cdot {}_tp_x'^{(3)} = \left(\frac{10-t}{10}\right)e^{-.03t} \cdot e^{-t^3}$

$\mu_{x+t}^{(2)} = .03$

-0 $\boxed{\therefore \int_0^3\left(\frac{10-t}{10}\right)\left(e^{-.03t}\right)\left(e^{-t^3}\right)(.03)\,dt}$

3.

x	l_x	$q_x'^{(1)}$ (disability)	$q_x'^{(2)}$	${}_tp_x^{(\tau)}$	${}_tq_x^{(\tau)}$	$q_x^{(1)}$	$d_x^{(1)}$
60	1000	.02	.04	.9408	.0592	.0196	19.5986
61	940.8	.04	.06	.9024	.0976	.0388	36.4991

-0

$q_x^{(1)} = \frac{q_x^{(\tau)}}{\ln(p_x^{(\tau)})} \cdot \ln(1-q_x'^{(1)})$, $q_{60}^{(1)} = -.97(\ln(.98)) = .019598625$

$d_x^{(1)} = l_x(q_x^{(1)})$ $q_{61}^{(1)} = -.95(\ln(.96)) = .03879579$

$d_{60}^{(1)} = 1000(.0196) = 19.5986$ $d_{61}^{(1)} = (940.8)(.03879579) = 36.4991$

$\therefore d_{60}^{(1)} + d_{61}^{(1)} = 19.5986 + 36.4991 = 56$

E(number of people disabled before age 62) $\boxed{= 56}$

8. $Q = \begin{bmatrix} .7 & .2 & .1 \\ .5 & .3 & .2 \\ 0 & 0 & 1 \end{bmatrix}$ Paid if moves from 1 to 2

$\pi_0 = [1 \ 0 \ 0]$

$\pi_1 = \pi_0 Q = [.7 \ .2 \ .1]$

$\pi_2 = \pi_1 Q = [.7 \ .2 \ .1]\begin{bmatrix} .7 & .2 & .1 \\ .5 & .3 & .2 \\ 0 & 0 & 1 \end{bmatrix} = [.59 \ .2 \ .21]$

$APV = \sum_{t=0}^{2} \pi_t(1)\, \overset{(.2)}{Q_t^{(1,2)}}\, v^{t+1}$

$= B\left((1)(.2)v + (.7)(.2)v^2 + (.59)(.2)v^3\right)$

$\boxed{= B\left(.2v + .14v^2 + .118v^3\right)}$

9. $APV = \sum_{t=0}^{2} \pi_t(1)\, v^t$ $\pi_t(1)$ from above Problem

$= P\left((1)(v^0) + (.7)(v^1) + (.59)(v^2)\right)$

$\boxed{= P\left(1 + .7v + .59v^2\right)}$