What to Trust: Reconciling Mathematical Work Done by Hand with Conflicting Graphing Calculator Solutions

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This paper reports on a mixed-methods study of 111 Advanced Placement calculus students' self-reports of their graphing calculator use, comfort, and rationale for trusting a solution produced with or without a graphing calculator when checking written work. It was found that there was no association between gender, teacher-reported mathematical ability, or comfort with the graphing calculator and students' trust in either a graphing calculator-produced solution or a solution produced without a graphing calculator. Furthermore, regardless of solution choice, the same four categories were evident in students' rationale for their solution choice: (a) an awareness of the possibility of careless errors, (b) the importance of checking over work, (c) a recognition of the limitations or affordances of the graphing calculator, and (d) a confidence (or lack thereof) in their own mathematical abilities. These results have implications for mathematics teaching as graphing calculators are used extensively in middle and high school mathematics classrooms and standardized tests in the United States.

Graphing calculators have taken on an important role in learning mathematics, especially at the secondary level. A survey investigating the status of calculator use (Dion et al., 2001) found that 99.9% of 4,568 U.S. high schools either require or allow calculators for their college preparatory classes, and graphing calculators in particular are required in 42% of Algebra II courses and 70% of precalculus courses. Furthermore, graphing calculators are currently allowed on more than 70% of U.S. states' mandated standardized tests; 100% of college entrance exams; and Advanced Placement (AP) calculus, statistics, physics and chemistry exams (College Board, 2010; Texas Instruments, n. d.; Weiss, Banilower, & Smith, 2001). This information points to the importance of taking a closer look at how graphing calculators are being used for teaching and learning mathematics both in and out of the classroom.

One way in which graphing calculator use is often promoted in the classroom is as a tool for checking or verifying work done by hand (Doerr & Zangor, 2000; Harskamp, Suhre, & Van Steun, 2000; Hennessy, Fung, & Scanlon, 2001; McCulloch, 2005, 2009; Quesada & Maxwell, 1994). The ability to use the calculator for checking has the potential to help students feel confident in their own work and provide motivation for continued work (McCulloch, 2009; Quesada & Maxwell, 1994). Studies have found that students often use the graphing calculator to check their work outside of class as well (Kenney, 2009; McCulloch, 2005, 2009). The fact that students use the tool to check their work is not surprising. However, the literature has neither considered the benefits and constraints for students when using the tool in this way, nor have they considered how students handle situations in which checking results in conflicting answers. In this article we report on a mixed-methods study of 111 high school AP calculus students' graphing calculator use, specifically their use of graphing calculators to check their work and their methods for reconciling any differences between solutions found by hand and with the technology.

Theoretical Framework

The theoretical lens through which we view graphing calculator use is called *instrumental genesis*. The premise that students' understandings are shaped by the tools that they use and by their relationship with those tools is consistent with socio-cultural theories of learning (Vygotsky, 1978). When studying how computer algebra system (CAS) tools (including graphing calculators) mediate learning, Artigue (2002) and her colleagues found it useful to turn to Chevallard's (1992) work in anthropology to better understand the ways in which students develop a relationship with a tool that takes into account the context of classroom learning. In our work, we consider the graphing calculator to be an artifact that is actually made up of many tools. For example, there are tools for both visualization and computation within a graphing calculator.

However, we believe that these tools are useless until they become "instruments" through a process that Artigue calls instrumental genesis.

The process of instrumental genesis involves one coming to understand the potentialities and constraints of a tool while interacting with it and new mathematical knowledge during the learning process. In other words, "users shape the artifacts they use and the artifacts shape the users, and that yields instruments" (Artigue & Kilpatrick, 2008, p. 6). Thus, while a student may own a graphing calculator, that alone does not make it an instrument. It is possible that particular modes (or tools) on the graphing calculator become instruments to a student before others. For example, in the context of linear functions, one student may have experienced instrumental genesis with the visualization tools of a graphing calculator and meaningfully use graphical representations to understand and engage in solving a linear function problem. That same student may not have developed the CAS capabilities as instruments with respect to linear functions and thus will not find them helpful in this context. It is important to note that the mathematical context matters, as the tool and the mathematical knowledge shape each other in the process of instrumental genesis. The same student just described might have developed the CAS capabilities in a different mathematical context, say function limits.

Kilpatrick (2009) noted that the theory of instrumental genesis appears to have considerable promise in research regarding the ways in which technology is and is not being used. In the context of this study, whether or not students have developed particular capabilities of the graphing calculator as instruments could have considerable impact on how and why they use them to check their work and reconcile differing solutions.

Background Literature

Confidence and the Graphing Calculator

Research on the use of graphing calculators as tools for learning has shown that the availability of graphing calculators has a positive impact on student assessment outcomes (Ellington, 2003). Some researchers have suggested that a possible reason for such improvements is that students are more comfortable or confident when they have a graphing calculator available (Dunham, 2000). In contrast, others have noted that, even when students feel that graphing calculators are useful, they may lack confidence in the calculator's ability to help them in problem solving and instead put more trust in their own work than in the calculator (Graham, Headlam, Honey, Sharp, & Smith, 2003). Research has also unveiled cases in which students' extreme confidence in their mathematical abilities contributes to a lack of perceived need for verification with the graphing calculator (Mesa, n. d.).

There is also a concern (in research and practice) that students can become overtrusting of the graphing calculator, using the device as a "black box" and blindly accepting calculator output (Doerr & Zangor, 2000; Forster & Taylor, 2000). Doerr and Zangor (2000) explain that this occurs when learners depend on calculators to produce answers without attending to the meaning, purpose, or interpretations of the problem situation. Using the graphing calculator as a black box is a concern also shared by Berry, Graham, and Smith (2005) who found that when using the graphing calculator to produce graphs, some students seemed to forget what they had learned when they first started plotting graphs and resorted to "mindless button pushing" (p. 24). Similarly, Goos, Galbraith, Renshaw, and Geiger (2003) have found that a graphing calculator can take on the role of "master" for the user and that students can become overly dependent on the tool when "lack of mathematical understanding prevents them from evaluating the accuracy of the output generated by the calculator" (p. 78). When used in these ways, the graphing calculator can become a source of mathematical authority for the user (Williams, 1993; Wilson & Krapfl, 1994) and be overused to the point that students rely on it with little critical analysis of the results (Burrill et al., 2002). Doerr and Zangor (2000), however, found that this was not the case when the teacher in a classroom emphasized the importance of checking and questioning the calculator results using one's own mathematical reasoning.

Graphing Calculator as a Checking Tool

Research on the use of graphing calculators to support mathematical learning during lessons has shown that they are often promoted as tools for validating written work (Berry, Graham, & Smith, 2006; Doerr & Zangor, 2000; Quesada & Maxwell, 1994), and that their use can encourage students to self-regulate their thinking (Hylton-Lindsay, 1998). In their case study of a precalculus class, Doerr and Zangor (2000) found that the use of the graphing calculator as a checking tool was an agreed-upon mode of use by the classroom community (i.e., both the students and the teacher). The graphing calculator was not only used to check nongraphing calculator-produced solutions, but was also used to "check conjectures made by students as they engaged with the problem investigations" (p. 156). Students were able to create mathematical meaning for and with the graphing calculator as a checking tool through their interactions with each other and with the teacher. Quesada and Maxwell (1994) stated that high school students believed that the ability of the graphing calculator to check answers was a positive aspect of the tool. From interviews with students, the researchers found:

> The statement "the ability to check their answers" was interpreted by some to mean not only the capability of confirming graphically or numerically the answers obtained algebraically, but also the ability (new for many of them) of thinking graphically about problems before trying to solve them algebraically. (p. 213)

Other researchers have seen that the use of the calculator in this way can be important for creating an understanding of graphing (Hennessy et al., 2001) and for allowing students to support their analytical work with graphs and tables (Waits & Demana, 1994). Researchers have not yet, however, clearly determined why a student chooses to use the graphing calculator as a checking tool (Berry et al., 2006).

While many students recognize the effectiveness of the calculator for confirming or checking the reasonableness of answers, difficulties can arise when answers obtained on the graphing calculator do not match their expectations or work they have done on paper (Kenney, 2009; McCulloch, 2005, 2009). This is a common dilemma that students face and is one that could conceivably add stress in a testing setting. The notion of *how* students handle such a conflict is a facet of using the graphing calculator as a checking tool that has not been studied but, given the prevalence of graphing calculator use in U.S. high school classrooms and standardized tests, could contribute to teachers', researchers', and test makers' understandings of how the calculator serves students in learning and assessments. In this study, we focus on this issue by examining where students place their trust when conflicts arise when checking with a graphing calculator. Specifically, we address these three research questions: What do students do when a solution produced without technology does not match one produced with technology? How do they reconcile this situation? Is there any association between their choices and (a) gender, (b) teacher of student, (c) teacher-rated mathematical ability, or (d) student-reported comfort with the graphing calculator?

Methods

Setting

AP Calculus classes were chosen as the focus for this study because the curriculum and expectation of calculator use in these courses is set by The College Board and, as a result, is relatively consistent nationwide. This study was set in four high schools in the northeastern United States. High School A is located in a low-income urban community that serves approximately 2,000 students in grades 9 through 12, High School B serves approximately 2,800 students in grades 9 through 12 from both suburban and rural communities, High School C serves approximately 1,100 students in grades 9 through 12 in an affluent suburban community, and High School D serves approximately 1,700 students in grades 9 through 12 in a middle-class suburban community. These high schools were purposefully selected based on access and presence of an AP Calculus program that uses graphing calculator technology. All four of these schools provided their AP Calculus students with a graphing calculator to use at home and at school. High Schools A, B, and D provided their students with a TI-83+, while High School C provided the TI-89 (which has CAS capabilities).[1]

Data Sources

All AP Calculus students at these four schools ($n = 111$; 49 females; 62 males) completed a survey designed (based on pilot studies) to identify the ways that they typically use a graphing calculator both in and out of the classroom and their comfort in doing so. In particular, the survey was designed to provide data on student (a) demographics, (b) frequency of graphing calculator use, and (c) comfort with the tool(s). Comfort with the tool(s) was included to provide insight into where students' were in the process of instrumental genesis with respect to different graphing calculator tools. The students' teachers were asked to evaluate the students' mathematical ability, rating each with respect to their peers in AP Calculus as high, average, or lower ability. In addition, an open-ended item was included that read:

> Imagine the following situation: You solved a problem on your own and then used your graphing calculator to check your solution. The calculator gave you a different solution than the one you got when you worked the problem on your own. Which answer do you trust? Why?

All 111 respondents provided answers to this question on the survey, in many cases surprising the researchers with the amount of detail included in their responses. The responses to this open-ended item are the focus of this paper.

Data Analysis

The categorical data and open-ended responses from the survey instrument were entered into a Microsoft Excel file.

This included school/teacher, gender, relative mathematical ability, and student-reported comfort using the graphing calculator. The students' relative mathematical ability was determined by their teachers and included the labels: high, average, and low as compared with their peers. Comfort with the tool was measured by assigning a "degree of comfort" based on responses to a Likert-scale item that asked students to rate their comfort on a scale of 1–5 (1 very uncomfortable, 5 very comfortable) with using the graphing calculator in each of five modes (i.e., numeric computation, using tables, using graphs, symbolic computation, and statistics). The mean comfort level for all students across all five modes was 4.22 with a standard deviation of .5. Based on this, it was decided that average comfort scores across the five modes within one standard deviation of the mean would be considered "comfortable." Those more than one standard deviation below the mean would be classified as "uncomfortable" and those more than one standard deviation above the mean would be classified as "very comfortable." This overall comfort rating for each student was added to the Excel file.

In terms of the written responses to the open-ended item, each answer was first coded for which kind of solution was chosen in the reconciliation process (i.e., graphing calculator [GC], nongraphing calculator [non-GC], or *neither*). We then examined all possible associations between descriptive student characteristics collected in the survey and the solution chosen. These associations were examined because in a pilot study conducted with a small sample of students, there was an association between gender and solution choice. As the descriptive data were categorical in nature, bivariate associations were analyzed using the Pearson's chi-square test of independence. Associations were examined between solution choice and each of gender, teacher, teacher-rated mathematics ability, and student-reported comfort using a graphing calculator.

We further analyzed the written responses to the open-ended item using a thematic content analysis process (Coffey & Atkinson, 1996). After coding each response based on the solution ultimately chosen in the reconciliation process, we examined students' written responses for emerging themes within each solution group. This resulted in the development of a codebook with 12 data-driven codes. The research team determined that the codes that emerged within each group of solution choices (GC, non-GC, and neither) directly corresponded with each other, regardless of the ultimate solution choice, and fell into four larger reconciliation categories: (a) careless errors, (b) check work, (c) graphing calculator affordances/limitations, and (d) confidence in mathematics ability (Table 1). For example, in the careless errors category, the GC solution choosers assumed they had made a careless error in their written work (e.g., "I would trust the calculator because it is easy to make a careless mistake in computation") while the non-GC choosers assumed the careless error was in their button pressing (e.g., "I would trust my own work because I often push the wrong button on the calculator"). In the careless errors category, the GC solution choosers noted that they make mistakes in their handwritten work (e.g., "The calculator solution because I probably made a mistake") and the non-GC choosers noted the mistakes made when using the calculator as a tool (e.g., "I would trust my own work because I often push the wrong button on the calculator"). It is important to note that the assignment of codes to student responses was not discrete. For example, the response "If I'm not sure, the calculator. If I'm sure, my answer. I probably plugged something wrong into the calc" was coded as both confidence in math ability and careless errors. In the next section, we provide further details, interpretations, and examples from the coding results.

Table 1
Categories for Rationale of Solution Choice

Category	Definition
Careless errors	Student notes that "careless errors" (either arithmetic or syntactical) are possibly the cause of any discrepancies between GC and non-GC solutions.
Check work	Student notes that either the GC or non-GC (or both) solution(s) must be checked for small errors and, barring any small errors, ultimately accepted.
Recognition of GC affordances and limitations	Student notes either affordances or limitations of the GC in their reasons for accepting or rejecting a GC solution.
Confidence in math ability	Student notes that acceptance or rejection of a GC solution is based on confidence (or lack thereof) in own math ability.

Findings

Considering the open-ended item asking students to reconcile a GC-produced solution with a different non-GC produced solution, 60 out of 111 students (54%) wrote that they would ultimately choose a GC-produced solution, 39 (35%) said they would choose their own work (non-GC-produced solution), and 12 (11%) did not make a definitive choice between the two. No significant associations were

Table 2
Code Frequency in Each Solution Preference Category

	GC solution (n = 60)	Non-GC solution (n = 39)	Neither (n = 12)	Total (n = 111)
Careless errors	36	18	4	58
Check work	22	14	7	43
Recognition of GC affordances and limitations	13	6	1	20
Confidence in math ability	5	6	1	12

Note. Rationale codes within a column are not discrete.

found between the solution choice and gender (χ^2[2, N = 111] = 2.649, p = .266), teacher (χ^2[6, N = 111] = 8.231, p = .222), teacher-rated mathematics ability (χ^2[4, N = 111] = 2.603, p = .626), or student-reported comfort using a graphing calculator (χ^2[4, N = 111] = 4.051, p = .399). Though there was no significant correlation between solution choice and student characteristics, we were interested in examining students' reasons for their choices in more detail. The two most commonly provided explanations for deciding which solution to trust, regardless of solution choice, were students' concerns about making careless errors (n = 58, 52%) and students' suggestions that they would check their work before choosing which solution they trust (n = 43, 38%). A summary of the frequency of codes within each of the reconciliation categories appears in Table 2.

We present the remaining results in three sections organized by solution choice: graphing calculator, nongraphing calculator, and neither. Within each section, the rationale codes are discussed and examples that typify the student responses are provided.

Trust the Graphing Calculator Solution

Careless errors. As noted above, 54% of the students responded that they would choose a GC-produced solution over a non-GC-produced one. A majority of these students (36 out of 60) reasoned that they would choose the GC solution because it is easy to make *careless errors* when working by hand. The following statements by students in this category exemplify their responses:

> S66: I trust the calculator because of human error.
>
> S89: I would trust the calculator because it is easy to make a careless mistake in computation when without a calculator and the numbers are large.
>
> S54: The calculator, as long as I entered my data correctly, it should be accurate, while my work may have errors.

Several students in this category were also assigned a secondary code of *check work* due to their claims that they would use the situation to help identify their errors (e.g., "I would trust the calculator because everyone makes mistakes, so I would use that proposed answer and work back and see my mistake and fix it"). Students were also assigned this secondary code if they indicated that they trusted their calculator solutions so much that they would try to identify how to fix their written work to match the calculator (e.g., "Calculator. I am a very confident calculator user. I'd try to change my work to match the calculator answer").

Check work. Students were given check work as the primary code if, before making the ultimate decision to trust the graphing calculator, they would first check their calculations in some way. Sixteen students noted that they would first check both their written and GC-produced work before choosing the GC solution. The following responses are examples of this:

> S57: I would re-check both. If still different answers, I'd check to make sure I plugged into calc right. If I did, then I would go w/ calculator answer, more likely I made a human error.
>
> S65: The calculator even though I would check both again.
>
> S45: Well I would actually make sure I plugged in everything correctly into the calculator. If that was right, then I would doubt my own solution. So I'd trust the calculator answer.

Although these responses might imply a similar feeling to those in the careless error category (i.e., that students feel they are more likely to have made a mistake by hand), we see these students' willingness to check all work first, before choosing their GC solution, as demonstrating more than just blind trust in the technology.

Recognition of GC affordances/limitations. Justification for choosing a GC-produced solution was also attributed to a belief in the infallibility of the GC by 13 students in their explanation of why they would choose the GC-produced solution to trust. This reason for trusting the GC was evident in responses such as:

> S38: Calculator. The only possible error made by a calculator occurs when a wrong number, equation, etc. is entered. Room for error on the calculator is restricted.

> S9: Unlike humans, calculators don't make computational mistakes for no apparent reason.

> S2: I trust the calculator's answer b/c I know that factors like lack of study/sleep may influence the correctness of my procedure & answer, but a calculator won't make the same mistakes. Thus, I will trust the calculator b/c it doesn't err.

We have identified this reasoning under the broader category of limitations and affordances of the graphing calculator due to students' beliefs that one of the affordances of the calculator is that it does not make (or is not capable of making) errors.

Confidence in math ability. In this last category for why students trust the GC solutions, we found 5 of the 60 students reported that they trusted the GC due to a lack of confidence in their own mathematics abilities. For example, students answered:

> S6: The calculator. It is better at algebra than me.

> S52: I would trust the calculator because I am usually wrong and I have had success in the past with trusting the calculator.

> S44 I would trust the calculator – I'd trust my data inputting abilities over my actual math skills.

Interestingly, only the first of these students (S6) was rated as a relatively lower-ability student by the teacher, one other (S52) was rated average, and the final three (S44, S58, and S92) were considered to be among the strong students according to the teachers. Thus, these student responses indicate a possible disconnect between confidence in their ability to do mathematics and their actual performance in class.

Trust the Nongraphing Calculator Solution

Careless errors. Thirty-five percent of the calculus students reported that they would choose a non-GC-produced solution (i.e., work done by hand) in the situation that it did not match a GC-produced one. Like those that chose GC-produced solutions, many in the non-GC category also considered the possibility of careless errors when making their decisions ($n = 17$). The difference, however, was that these students were not as concerned about errors in their written work as they were about errors they may make pressing the calculator buttons. For example, students suggested:

> S82: My work is probably more correct. I probably entered it incorrectly on the calculator. I'd try to punch it in again.

> S83: My own answer is probably right; I have finger-calculator problems. Usually the reason for the disparity between my value and the calculator value is the lack of parentheses or wrong decimal place.

> S108: I would check my answer to see if my answer was reasonable, because it is usually more likely that I mistyped something in my calculator.

Unlike the GC choosers, none of those who used the careless error reasoning suggested using the situation to help identify their errors. They were not as concerned with determining how to get the correct solution on the calculator if they trusted the work they had done by hand.

Check work. Thirteen of the non-GC choosers responded that they would trust their non-GC-produced solution after checking their work for mistakes. For example, students stated:

> S69: I'll check my work again and if I didn't do anything wrong then I'll trust my work.

> S13: I never trust the graphing calculator's answer if it doesn't match mine. However, if they don't match, I check both my work and what I input in the calculator.

> S4: I would double-check my work, and then use my answer because I could have easily input something wrong in the calculator.

As with the GC group, these students were not willing to blindly accept one answer over the other, but did demonstrate more confidence in their own work. Many of the reasons in both this and the careless error category were related primarily to students' personal experiences, as evidenced by their responses of what they "often," "easily," or "usually" do.

Recognition of GC affordances/limitations. Like the GC choosers, some non-GC-produced solution choosers noted their beliefs about the affordances or constraints of the GC in their justifications. However, unlike the GC-produced solution choosers, these students placed the authority in the situation with themselves. For example, six responses for the non-GC choosers focused on the limitations of the GC as a reason for not trusting its solution sharing:

> S72: I trust the answer I would have gotten on my own. The calculator does not show all the steps and it is easy to make mistakes when putting information into the calculator.

> S48: I trust my own; sometimes the graphing calculator comes up with weird answers using trig functions or does not find the right answer.

> S103: I would trust myself because the calculator tends to make mistakes sometimes in graphing when the equation isn't entered properly.

It is interesting to note that, in the last two examples, students place the blame for mistakes on the calculator, when in fact mistakes may be a result of their own careless errors. However, according to these students this is a limitation of the graphing calculator, not themselves.

Confidence in math ability. Six students noted their confidence in their mathematical abilities (rather than their lack of confidence) as their reasons for trusting the non-GC solution. Responses included:

> S81: I would trust my own because I went thru [sic] a procedure to get the answer.

> S102: Myself, because I am able to do the problem on my own, I wouldn't even need to use the calculator.

> S79: As long as I am confident with the answer I got, and was not very unsure with it in the first place, then I would trust my own answer not the calculator's.

The first two responses above allude to a confidence in proven procedures. It is important to note that none of the students who noted confidence in their mathematical abilities as justification for choosing a non-GC-produced solution were considered by their teacher to be among the highest ability students as compared with their peers.

Neither

Twelve of the 111 students that participated in the study did not make a definitive choice between either the GC or non-GC-produced solutions. The responses of these students indicate that they do not necessarily value or rely on one solution over the other. Instead, they all noted the importance of rechecking their work, both on the GC and on paper, to identify errors and to understand why the solutions differed. For example,

> S76: I recheck the calculator first and then my own answer. I check both and trust neither.

> S16: Neither answer fully; I would go back and check my work. (In this case, we assume that the student means both the work on the GC and by hand.)

In addition to the check work rationale, these students' responses included explanations that fell into each of the other categories as well. Student S33 wrote, "Well, I would compute the answer twice with each method. Then I identify what I did wrong on paper/calculator screen. Sometimes I write wrong signs, forget numbers, etc. on paper, but I also forget parentheses and other such items on the calculator so I trust the two answers equally" showing concern about careless errors in both written and GC work. Student S59 noted, "Neither. Some problems can't be solved with a calculator or the calculator gives long complicated answers," which exhibits recognition of GC affordances and limitations. Showing a consideration of confidence in math ability, S18 wrote, "I check the way I did both, both the written process and the calculator . . . if I find not problems, I'd take my answer if I was confident, if not then I'd take the calculator." These students are considering the same things as the GC and non-CG choosers, but they appear to place somewhat equal value with each solution method. Most importantly, they do not exhibit blind trust in either. However, the responses do suggest that, as long as the two solutions do match, students would feel fairly confident in their work and with the answer.

Discussion

Our findings in this study show that, whether they would choose the GC or non-GC solution when a conflict in answers arises, students are considering similar factors:

- The possibility of careless errors
- The importance of checking their work

- The constraints and/or affordances of the graphing calculator
- Confidence in their mathematical abilities

More than half of the students chose the GC solution over the non-GC-produced solution.

Many of the students' reasons for doing so were attributed to lack of confidence in their mathematical ability (Berry et al., 2005; Goos et al., 2003) or to an overconfidence in the infallibility of the GC (Doerr & Zangor, 2000; Forster & Taylor, 2000). These findings are consistent with Goos et al. (2003) who found that students sometimes develop relationships with graphing calculators in which the graphing calculator is viewed as the "master," and suggests that it is truly important for teachers to be aware of the issue of mathematical authority. The students surveyed here are high school calculus students—often the best and brightest at their schools—and we find that more than half of them are handing the authority in a mathematical situation to the tool over themselves. This raises concerns for other groups of students, especially those who struggle with mathematics when using graphing calculators in high-stakes assessment situations.

Looking across the solution choices, we see that 39% of the students noted that they would not immediately choose one solution over the other, but would check their work for careless errors before determining which to trust. However, if the error was not immediately evident, 22 of those students said they would choose the GC solution while 14 chose the non-GC solution. The number of students that noted they would choose the GC because they are concerned about making careless errors in their written work gives credence to the literature that has suggested that students' attitudes toward mathematics increases when GCs are available because having the tool increases their levels of confidence (Dunham, 2000). More generally, researchers have found that students' use of strategies and efforts to strategically regulate their work directly depend on self-perceptions of academic efficacy (Schunk & Zimmerman, 2003). If having a GC available can decrease students' concerns about the occurrence of careless errors, it is possible that students may spend less time worrying about small mistakes and more time focusing on thinking deeply about the mathematics.

It is promising to see that more than a third of the students, regardless of solution choice, noted the need to check their work. This suggests that these students are thinking critically about what might have caused the difference in answers, rather than just accepting one as true. On the other hand, it is possible that those students who did not mention checking might have assumed that the situation was placed in a testing situation, one in which they may have assumed they would not have time to go back and check their work and would thus be forced to choose a solution and move on. Such interpretations should be examined more carefully in future studies.

A small percentage of students did not make a definitive choice between either the GC or non-GC solutions. Many of these students noted the importance of rechecking their work, both on the GC and on paper, to identify errors and to understand why the solutions differed. This is the type of checking behavior that we ideally want to see because it suggests that students are reflecting on both solution strategies. Such reflection requires understanding of solutions and representations.

Though the results are quite compelling, this study did have some limitations, the most prominent limitation being the phrasing of the open-ended survey question. We asked students to respond in writing which solution they would choose and why. However, we did not specifically ask how they would reconcile the situation nor did we make the context in which the situation was to be imagined clear. Though we did not ask about the reconciliation process, all of the students addressed it, some in more detail than others. In future studies, we suggest not only that the question about reconciliation be added, but also that a stratified subsample of participants be chosen for follow-up interviews to help understand in more detail what may be influencing these decisions. The context of the question could have been interpreted as occurring during class time, homework, or even during a high-stakes test. As this was not made clear, it is likely that students imagined different contexts and the context influenced their responses. Future studies should be clear about the context, preferably studying student responses in multiple contexts. Finally, the results suggest that for some students, confidence in their mathematical abilities influences the way they handle checking situations in which they are required to reconcile differing solutions. While we had a measure of students' abilities, it was from their teachers' point of view. Future studies should include students' own reports of self-efficacy and confidence in mathematics.

The results here have important implications for mathematics teachers. First, having a tool with which to check is important to students. The findings here point to the confidence it may provide in potentially stressful situations. Second, having a graphing calculator to check work not only provides students with a way to ensure a solution is correct, but also provides different representations with which to do so. As noted previously, such actions require deep understanding of solutions and representations.

We suggest that tasks that promote good checking practice be regularly incorporated into classroom practice (McCulloch, Kenney, & Keene, 2012).

In conclusion, given the prevalence of graphing calculator use in U.S. high school classrooms and standardized tests, these tools are likely going to be a mainstay in school mathematics for quite some time (College Board, 2010; Texas Instruments, n. d.; Weiss et al., 2001). The results of this study suggest that while such promotion is beneficial, it needs to be handled carefully so that students do not blindly place mathematical authority with the tool when reconciling differing solutions. To build a better understanding of these phenomena, further research is needed on how graphing calculator use and decision making is being promoted by teachers as well as how students perceive this promotion. In addition, it is important to investigate if these results hold true for other technology tools such as computer algebra systems, spreadsheets, and dynamic geometry systems.

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Authors' Note

[1] For the purposes of this study, the term graphing calculator refers to both calculators with and without CAS capabilities.