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My Answers Don't Match! Using the Graphing Calculator to Check

Imagine the following situation: You have worked out a problem by hand and decide to check your solution by doing the problem a second time using your graphing calculator. The solutions you get don't match. Which do you choose to trust? Why?

The prevalence of graphing calculators in high school mathematics classes and the common use of graphing calculators as checking tools has landed many high school mathematics students in this situation. What do they do? And why? We asked high school AP calculus students these questions, and in this article we share their responses and suggest some ways to address this issue in the classroom.

Graphing calculators play an important role during classroom instruction and assessment in secondary school mathematics (Dion et al. 2001). They are currently allowed on more than 70 percent of U.S. states' mandated standardized tests, 100 percent of college entrance exams, and a majority of AP mathematics and science exams (College Board 2010; Texas Instruments n.d.). Research has shown that they are commonly promoted as tools for checking or verifying work done by hand (e.g., Hennessy, Fung, and Scanlon 2001; Doerr and Zangor 2000; Harskamp, Suhre, and van Streun 2000; McCulloch 2005, 2009; Quesada and Maxwell 1994) and that students greatly value being able to use them in this way (e.g., McCulloch 2009; Kenney 2008; Quesada and Maxwell 1994).

That students use graphing calculators to check their mathematics answers is not surprising. However, in our research we have seen students do some surprising (or maybe not so surprising) things when their graphing-calculator solution differs from their written one. For example, students may change their written work to force their solution to match that of the graphing calculator, even when they know that the changes are mathematically incorrect. Other students may completely ignore the graphing-calculator solution and just move on to the next problem. Observations such as these prompted us to ask, What happens when students use their graphing calculators to check their work and get a conflicting solution?

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WHAT THE STUDENTS SAY

To address this question, we administered a survey to AP calculus students. This population was selected because the curriculum and expectation of calculator use in this course is set by the College Board and, as a result, is relatively consistent nationwide. All AP calculus students at four schools ($n = 111$; 49 female, 62 male) completed a survey related to graphing-calculator use that included this open-ended item:

Table 1 Categories and Frequencies for Rationale of Solution Choice

		Frequencies*		
Category	**Definition**	**GC**	**Non-GC**	**Neither**
Careless errors	Student notes that "careless errors" (either arithmetic or syntactical) are possibly the cause of discrepancies between GC and non-GC solutions.	36	18	4
Checking work	Student notes that either the GC or non-GC (or both) solution(s) must be checked for small errors and, barring any small errors, ultimately accepted.	22	14	7
Recognition of GC affordances and limitations	Student notes either affordances or limitations of the GC in the reason for accepting or rejecting a GC solution.	13	6	1
Confidence in mathematical ability	Student notes that acceptance or rejection of a GC solution is based on confidence (or lack thereof) in own mathematical ability.	5	6	1

*Note: Some responses fell into more than one category.

> Imagine the following situation: You solved a problem on your own and then used your graphing calculator to check your solution. The calculator gave you a solution different from the one you got when you worked the problem on your own. Which answer do you trust? Why?

More than half the students (60 out of 111, 54 percent) responded that they would ultimately choose a graphing calculator–produced solution ("GC-produced solution" in **table 1**), 39 (35 percent) said that they would choose their own work ("non-GC-produced solution" in **table 1**), and 12 (11 percent) did not make a definitive choice between the two.

Students were surprisingly detailed in their responses, allowing us to look beyond their solution choice and examine their thinking when faced with such a decision. Regardless of the ultimate solution choice (graphing-calculator solution or non-graphing-calculator solution), students' explanations for their choices fell into four categories: concern about making careless errors, needing to check work (their own written work or the calculator's work) before making a choice, beliefs about graphing calculator affordances and limitations, and confidence in their own mathematical ability (these are referred to in **table 1** as "careless errors," "checking work," "recognition of GC affordances and limitations," and "confidence in mathematical ability," respectively).

Careless Errors

More than half the students (52 percent) noted that their solution choice was based on a concern about making careless errors. Students who chose the graphing-calculator solution assumed that they had made a careless error in their written work (e.g., "I would trust the calculator because it is easy to make a careless mistake in computation"), whereas students who did not choose the graphing-calculator solution assumed that the careless error was in their button pressing (e.g., "I would trust my own work because I often push the wrong button on the calculator").

Checking Work

Thirty-nine percent of students noted that they would not immediately choose one solution over the other but would check their work for careless errors before determining which to trust. However, if the error were not immediately evident, 22 of these students said they would choose the graphing-calculator solution, whereas 14 chose the non-graphing-calculator solution. For example, a student who chose the graphing-calculator solution noted, "I would double-check my work and also how I entered the problem into the calculator. If they still don't match, I would trust the calculator's answer." Similarly, a student who did not choose the graphing-calculator solution wrote, "I'll check my work again, and if I didn't do anything wrong, then I'll trust my work."

Recognition of Affordances and Limitations

Justification was also attributed to beliefs about the affordances and limitations of the technology. Students in this category who chose the graphing-calculator solution pointed to the infallibility of the graphing calculator (e.g., "Unlike humans, calculators don't make computational mistakes for no apparent reason"). Students who did not choose the graphing-calculator solution were wary about the limitations of the graphing calculator, especially with respect to form (e.g., "I trust my own. Sometimes the graphing calculator comes up with weird answers using trig functions or does not find the right answer").

Confidence in Mathematical Ability

Finally, a relatively small set of students (11 percent) said that their decisions about which solution to trust were based on their confidence in their mathematical abilities. Students who chose the graphing-calculator solution were not confident in their own abilities (e.g., "The calculator. It is better at algebra than me"), whereas students who chose their own solution were (e.g., "I would trust my own because I went through a procedure to get the answer").

REFLECTING ON THE RESULTS

A positive finding from these data is that more than one-third of the students (including those who chose both the graphing-calculator solution and their own work) noted a need to check their work before choosing a solution. However, many of the AP calculus students, arguably among the strongest mathematics students in their schools, said that they would trust a graphing-calculator solution over their own work without further reflection. This response raises concerns for other groups of students, especially those who struggle with mathematics, when using graphing calculators in high-stakes assessment situations.

A small percentage of students did not make a definitive choice between the graphing-calculator solution and their own work. These students all noted the importance of rechecking their work, both on the calculator and on paper, to identify errors and to understand why the solutions differed. For example, one student clearly explained,

> Well, I would compute the answer twice with each method. Then I identify what I did wrong on paper/calculator screen. Sometimes I write wrong signs, forget numbers, etc., on paper, but I also forget parentheses and other such items on the calculator, so I trust the two answers equally.

This is the ideal type of checking behavior, the checking behavior that we want to see, because it suggests that students are reflecting on both solution strategies. Such reflection requires understanding of solutions and representations. So the question that remains is, How do we help all students become better checkers?

PROMOTING GOOD CHECKING PRACTICES

The survey results show that checking mathematical work with the graphing calculator is not a straightforward process, especially when it results in conflicting solutions. Students' choices in these situations are not always grounded in the mathematics. We believe that the process of checking is very important and so should be discussed explicitly in the classroom. Checking activities should develop critical thinking with respect to graphing calculator use.

We offer three suggestions to support the development of good checking practices: modeling good checking practices in the classroom, discussing graphing calculator limitations, and providing ample opportunities for students to check their work and reason about their processes.

Modeling Good Checking Practices with Students

Models of good practices in the classroom include demonstrating the use of multiple representations to make sense of solutions, thinking aloud in front of the class (or asking students to do so) as checking strategies are chosen and carried out, and creating explicit situations in which it would be useful to check work. A further method of modeling is to include checking tasks in class work, homework, and assessments. Tasks such as the one shown in

Checking a Solution

A student's solution to a problem is given below. How many different ways can you check the student's solution using a graphing calculator? If you believe that the student is correct, explain how your methods on the calculator convince you of this. If you believe that the student is incorrect, how would you use your calculator solutions to convince that student that he or she is wrong?

Student A was asked to solve the following system of equations:

$$\begin{aligned} 3x - y &= 7 \\ -2x + 4y &= 5 \end{aligned}$$

Student A's solution:

$$\begin{aligned} 4(3x - y) = 4(7) \rightarrow 12x - 4y &= 28 \\ \underline{-2x + 4y} &\underline{= 5} \\ 10x &= 33 \\ x &= 3.3 \\ y &= 16.9 \end{aligned}$$

Fig. 1 Students were asked to corroborate this solution (or demonstrate that it was incorrect) using the graphing calculator.

Rounding Error and Limited Precision

Enter the expression **(10^15+3–10^15)*100** into your calculator. What does it give you? Next, try varying the order for the same expression and enter **(10^15–10^15+3)*100**. What do you find? What do you think is going on? (Howell n.d.)

Bizarre Graph Behavior

Use **RADIAN** mode and change the **WINDOW** settings so that ***X*min** = –10, ***X*max** = 10, ***Y*min** = –2, ***Y*max** = 2, and both scales are 1. Graph ***y* = sin(28*x*)**. Note the y-intercept and the slope as it crosses the y-axis. Now change ***X*min** to –5 and ***X*max** to 15 (leave the other values the same). Do you notice anything interesting? Change these values again to make ***X*min** = –8 and ***X*max** = 12. Now what happened? (Benson n.d.)

Fig. 2 These two tasks illustrate the limitations of the graphing calculator.

figure 1 highlight possible methods for checking and also promote understanding of both solutions and representations.

Discussing the Limitations of Graphing Calculators

It is very important for students to be aware of the limitations of their calculators, both those related to representations and those related to programming, so that they can make sense of graphing calculator–produced solutions and errors. Researchers have noted the importance of discussing limitations of the screen resolution explicitly when exploring graphical representations (see, e.g., Vonder Embse and Engebretsen 1996). However, programming errors can cause problems as well.

Consider the tasks (see **fig. 2**) that we found online (Benson n.d.; Howell n.d.) and shared with a large group of teachers at a national conference. These tasks demonstrate a range of errors that can occur on a TI calculator. (Note that a calculator's limitations depend on the model. These examples were tested on the TI-83+ and TI-84+. The TI-85 and TI-89 can often handle larger calculations; thus, numbers may need to be increased to get the same results.) Carefully chosen examples like these can help bring students' attention to the fact that they must analyze all graphing-calculator solutions, especially those that could be affected by the software programming.

After trying these tasks themselves, the teachers at the conference were eager to find more such examples and share them with their students. One teacher noted, "I have never thought about the importance of discussing checking before. I can't wait to try these examples with my students."

Providing Opportunities for Checking Work

The majority of students in our survey talked about their tendencies to make careless mistakes and their desire to check their steps to try to find and correct these mistakes. Many students may indeed have a deep understanding of the mathematical concepts, but they often do not take the time (or have the time) to work carefully.

Finding a discrepancy on the graphing calculator can make students aware that they have made a mistake in their work, but careful and thoughtful checking practices take time. For example, students may not check their work if they are worried about time constraints. Responses to the survey suggest that, given time, students do want to check and should be encouraged to be thoughtful in their checking with graphing calculators. It is important to allow time for students to check their solutions in class and during assessments.

CONCLUSION

The use of technology, specifically graphing calculators, as a teaching and learning tool in high school mathematics is not going to go away. Graphing calculators have become a fixture in classrooms and on standardized assessments. As these tools become more sophisticated, it becomes even more important to help students learn to reason about calculator-produced solutions and not assume that all calculator-produced solutions are correct.

The research that we have presented here indicates that using graphing calculators to check written work can be a thoughtful and valuable process for students. Further, explicitly including checking activities for calculator-produced solutions during instruction has the potential not only to increase the number of correct solutions that students produce but also to deepen students' understanding of the mathematics.

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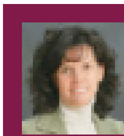

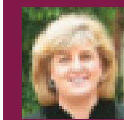

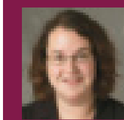

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