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Math 572, Spring 20‘0

HOMEWORK 4. EXTRA QUESTION.

EXTRA PROBLEM 1: Let K be a chain complex with finitely many p–simplices in each degree, so that the chain groups $C_p(K)$ have finite rank. Define the Euler characteristic of K to be

$$\chi(K) = \sum_i (-1)^i b_i$$

where b_i is the Betti number of $H_i(K)$.

Using the fact that *if G is torsion free* $H_i(K;G) \simeq H_i(K) \otimes G$ show that for any field k of characteristic 0

(1)
$$b_i = dim_k(H_i(K;k))$$

(2)
$$\chi(K) = \sum_i (-1)^i dim_k(C_i(K;k))$$

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