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**Math 598K, Fall 2008**

# Homework 3

## Problems

Problem 1: Show that AB1 and AB2 imply that the 0 object is initial and final.

Problem 2: Fix an Abelian category $\mathcal{C}$ and let $\phi \in Hom(X, Y)$.

a) Show that $ker(ker(\phi)) = 0 = coker(coker(\phi))$.

b) Show that $ker(0 : X \to 0) = (X, id_x)$ and $coker(0 : 0 \to X) = (X, id_x)$.

c) Show that if $\phi$ is a monomorphism and an epimorphism then $\phi$ is an isomorphism. (Hint use the fact that $\phi = ker(coker(\phi))$ and $\phi = coker(\text{ker}(\phi))$ together with the fact that canonical decompositions are unique up to unique isomorphisms. Why is the last fact true?)