A SUPPORT THEOREM FOR GEODESIC RAY TRANSFORM ON REAL-ANALYTIC RIEMANNIAN MANIFOLDS

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We consider a compact simple Riemannian manifold M with boundary and with a real-analytic metric. Suppose we have a function f on M which satisfies the condition that the integral of f along geodesics γ is zero for all γ belonging to a certain set $\mathcal{A}$. We show that if the set $\mathcal{A}$ satisfies a topological condition, then $f = 0$ on the set of points filled by these geodesics. Using this result we show that if we consider a geodesically convex subset K contained in inetrior of M and if we let $\mathcal{A}$ be the set of geodesics in M not intersecting K then $f = 0$ on the set $M \setminus K$. This provides a generalization of the classical support theorem to the Riemannian manifold setting. We use analytic microlocal analysis to prove these results. The main idea we use comes from [SU]. There Stefanov and Uhlmann use Sjöstrand's complex stationary phase method coupled with his FBI transform approach, [S], to study the integral geometry problem for a class of non-simple Riemannian manifolds. We combine these ideas with a theorem of Kawai-Kashiwara-Hörmander [H, S] first used in this setting by Boman and Quinto [BQ]. We also generalize this result for the case of symmetric tensor fields. Replacing the function f above by a symmetric tensor field and under the same assumptions as above, we show that such a tensor field is potential near the boundary.

References

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