AN INVERSE PROBLEM FOR A MODEL ELECTROMAGNETOELASTIC SYSTEM

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Introduction. The interaction of electromagnetic fields with deformable media is a subject of many theoretical and experimental investigations in the field of continuum mechanics and geophysics in the recent decades.

For description of simple enough interactions, the theories of magnetohydrodynamics, electro-elasticity, and magneto-elasticity were developed. These theories are, basically, a combination of objects and phenomena considered in continuum mechanics and electrodynamics.

Investigation of more complex electromagnetoelastic interactions in a continuous medium requires to consider complex models. For a more profound acquaintance with the modern state of the theory of electromagnetoelastic interactions the reader is referred to, e.g., [1].

Statement of the problem. Consider a simple model consisting from two differential equations, one of them is a hyperbolic equation (an analog of the Lamé system) and another one is a parabolic equation (an analog of the diffusion approximation of Maxwell's system) coupled by nonlinear terms in both equations. In the case $\rho = \text{const}$, $\mu = \text{const}$ we can form the following 1D non-dimensional model system (cf. [2])

$$h_t = (rh_z)_z - (hu_t)_z - (rj)_z,$$
$$u_{tt} = (\nu^2 u_z)_z - phh_z + f,$$

where h, u, j, f are dimensionless scalar analogues of the magnetic and elastic fields, electromagnetic and elastic forces, correspondingly; $r^{-1} = \mu L V_0 \sigma$ is the magnetic Reynolds number, $p = \mu H_0^2 / \rho V_0^2$, $\nu = \sqrt{(\lambda + 2\kappa)/\rho V_0^2}$ is dimensionless velocity of elastic waves propagation; ρ is the density of the medium, μ is the magnetic permeability, σ is the electroconductivity, λ, κ are the parameters of Lamé, and L, V_0, H_0 are characteristic values of length, seismic velocity and magnetic field, respectively. In this paper we assume that f admits the following representation

$$f(z,t) = \phi(t) g(z,t).$$

Inverse Problem. *Determine a set of the functions*

$$h : \overline{Q}_T \to \mathcal{R}, \quad u : \overline{Q}_T \to \mathcal{R}, \quad \phi : [0,T] \to \mathcal{R}$$

such that

$$h_t = (rh_z)_z - (hu_t)_z - (rj)_z, \quad (z,t) \in Q_T, \tag{1}$$
$$u_{tt} = (\nu^2 u_z)_z - phh_z + \phi g, \quad (z,t) \in Q_T, \tag{2}$$
$$h(\pm l, t) = 0, \quad u(\pm l, t) = 0, \quad t \in (0,T), \tag{3}$$
$$h(z,0) = h_0(z), \quad u(z,0) = u_0(z), \quad u_t(z,0) = u_1(z), \quad z \in \Omega, \tag{4}$$
$$\int_\Omega \rho(z) h h_z \, dz = -\frac{1}{2} \int_\Omega \rho_z h^2 \, dz = \psi(t), \quad t \in [0,T], \tag{5}$$

where $\rho \in \overset{o}{W}{}_2^1(\Omega)$ *and* $Q_T = \Omega \times (0,T), \Omega = (-l, l)$. *The functions* r, ν, g, j *are supposed to be piecewise smooth functions with jumps in the points* $z_m : -l < z_1 < z_2 < \cdots < z_m < l$; $0 < r_0 \le r(z) \le r_1 < \infty$, $0 < \nu_0 \le \nu(z) \le \nu_1 < \infty$; $\phi \in L_2(0,T)$, p *is a positive number and*

$$\int_\Omega \rho(z) g(z,t) \, dz \ge \rho_0 > 0, \quad t \in [0,T]. \tag{6}$$

At the points of discontinuity we assume the fulfilment of the following transmission conditions

$$[h]_{z=z_i} = 0, \quad [u]_{z=z_i} = 0, \tag{7}$$

$$[r(h_z - j)]_{z=z_i} = 0, \quad [\nu^2 u_z]_{z=z_i} = 0, \quad i = 1, 2, \ldots, m. \tag{8}$$

Basic results. Assume that the functions r, ν, the free members $\phi g, j$, the constant p and initial data h_0, u_0, u_1 in problem (1)–(4), (7)–(8) enjoy the properties

a) the functions r, ν, g, j are supposed to be piecewise smooth functions with jumps in the points z_m: $-l < z_1 < z_2 < \cdots < z_m < l$; $0 < r_0 \le r(z) \le r_1 < \infty$, $0 < \nu_0 \le \nu(z) \le \nu_1 < \infty$; $\phi \in L_2(0,T)$ and p is a positive number;

b) $h_0 \in C^\alpha(\overline{\Omega}), \alpha \in (0,1), h_0(\pm l) = 0$, and $u_0 \in \overset{o}{W}{}_2^1(\Omega)$ and $u_1 \in L_2(\Omega)$.

For direct problem (1)–(4), (7)–(8), the following existence and uniqueness theorem holds under the assumption that function ϕ is known.

Theorem 1. *In the conditions, formulated above, problem* (1)–(4), (7)–(8) *has a unique solution*

$$h(z,t) \in \overset{o}{V}_2(Q_T), \quad u(z,t) \in \overset{o}{W}{}_2^{1,1}(Q_T).$$

For inverse problem (1)–(8) the following existence and uniqueness theorem holds .

Theorem 2. *For sufficiently small values* $T > 0$ *inverse problem* (1)–(8) *has a unique solution, which can be obtained by the method of successive approximations.*

References

1. *A. C. Eringen and G. A. Maugin* Electrodynamics of Continua. Vols. I, II // Springer-Verlag, New York—Berlin, 1990.
2. *Viatcheslav Priimenko and Mikhail Vishnevskii* An inverse problem of electromagnetoelasticity in the case of complete nonlinear interaction // J. of Inverse and Ill-Posed Problems 2005, Vol.13(3), pp.277-301.