

Inverse Born approximation for two-dimensional nonlinear Schrödinger operator

Valery Serov*

*Department of Mathematical Sciences, University of Oulu,
P.O. Box 3000, FIN-90014, Oulu, Finland*

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This work deals with the inverse Born approximation for two-dimensional nonlinear Schrödinger equation with cubic nonlinearity

$$-\Delta u + q(x)u + \alpha(x)|u|^2u = k^2u,$$

where the real-valued unknown functions q and α belong to $L^p_{loc}(\mathbb{R}^2)$ with some behaviour at the infinity. This equation has the applications in nonlinear optics.

The following problem is studied: To estimate the smoothness of the terms from the Born sequence which corresponds to the scattering data with all arbitrary large energies and all angles in the scattering amplitude. These smoothness estimates allow us to conclude that the leading order singularities of the sum of unknown functions q and α can be obtained exactly by the Born approximation. Especially, for the functions from L^p - spaces the approximation agrees with the true sum up to the functions from the Sobolev spaces. In particular, for the sum being the characteristic function of a smooth bounded domain this domain is uniquely determined by this scattering data.

*electronic mail: vserov@cc.oulu.fi