Lesson 13 Section 7.4

Partial Fractions

$$\frac{1}{2}+\frac{1}{3}=\frac{3+2}{6}=\frac{5}{6}$$

Review of Algebra:

$$\frac{1}{x+1}-\frac{1}{x+2}=\frac{x+2-(x+1)}{(x+1)(x+2)}=\frac{1}{(x+1)(x+2)}$$

$$\frac{1}{x+1}-\frac{1}{x+2}=\frac{1}{(x+1)(x+2)}.$$

$$\int\frac{dx}{(x+1)(x+2)}=\int\frac{dx}{x+1}-\int\frac{dx}{x+2}=\ln|x+1|-\ln|x+2|+C$$

Example 1: Compute $\int \frac{dx}{x^2+5x+6}$.

$$x^2+5x+6 = (x+2)(x+3)$$

$$\frac{1}{x^2+5x+6} = \frac{1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$= \frac{A(x+3) + B(x+2)}{(x+2)(x+3)}$$

We want: $\frac{A(x+3) + B(x+2)}{(x+3)(x+2)} = \frac{1}{(x+2)(x+3)}$.

So we must have

$$A(x+3) + B(x+2) = 1$$ for all values of x

$$Ax + 3A + Bx + 2B = 1$$

$$(A+B)x + 3A + 2B = 1.$$

$$A + B = 0 \qquad \underline{A = -B}$$

$$3A + 2B = 1. \qquad 3A - 2A = 1$$

$$A = 1$$

$$B = -1.$$

Set $x = -2$. $\boxed{A = 1}$

$x = -3$ $\quad -B = 1$

$\boxed{B = -1}$

Conclusion: $\dfrac{1}{x^2+5x+6} = \dfrac{1}{(x+2)(x+3)} = \dfrac{1}{x+2} - \dfrac{1}{x+3}$.

$$\int \frac{dx}{x^2+5x+6} = \int \frac{dx}{x+2} - \int \frac{dx}{x+3}$$

$$= \ln|x+2| - \ln|x+3| + C.$$

Compute $\int_1^2 \frac{x+3}{x^2+5x+4}\,dx$.

Step 1: Factor the denominator

$$x^2+5x+4 = (x+1)(x+4)$$

$$\frac{x+3}{x^2+5x+4} = \frac{x+3}{(x+1)(x+4)} = \frac{A}{x+1} + \frac{B}{x+4}.$$

$$= \frac{A(x+4)+B(x+1)}{(x+1)(x+4)}$$

So we must have:

$$x+3 = A(x+4) + B(x+1)$$

$$= Ax + 4A + Bx + B$$

$$x+3 = x(A+B) + 4A + B.$$

$$\begin{cases} A+B = 1. \\ 4A+B = 3. \end{cases}$$

Let $x=-1$: $\quad 2 = 3A \quad A = \frac{2}{3}.$

$x=-4$ $\quad -1 = -3B$; $\quad B = \frac{1}{3}.$

$$\frac{x+3}{x^2+5x+4} = \frac{2}{3(x+1)} + \frac{1}{3(x+4)}$$

$$\int \frac{x+3}{x^2+5x+4} = \frac{2}{3}\int \frac{dx}{x+1} + \frac{1}{3}\int \frac{dx}{x+4}$$

$$= \frac{2}{3}\ln|x+1| + \frac{1}{3}\ln|x+4|.$$

Compute $\int \frac{x^2+3x+1}{x^2+5x+6}\,dx.$

Step 1: Get rid of the x^2 term in the numerator.

$$x^2+3x+1 = (x^2+5x+6) - 2x - 5.$$

$$\frac{x^2+3x+1}{x^2+5x+6} = 1 - \frac{2x+5}{x^2+5x+6}$$

$$\int \frac{x^2+3x+1}{x^2+5x+6}\,dx = \int \left(1 - \frac{2x+5}{x^2+5x+6}\right) dx$$

$$\int \left(1 - \frac{2x+5}{(x+2)(x+3)}\right) dx.$$

$$\frac{2x+5}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}.$$

$$\int \frac{dx}{(x+1)(x+2)^2}.$$

$$\frac{1}{(x+1)(x+2)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$= \frac{A(x+2)^2 + B(x+1)(x+2) + C(x+1)}{(x+1)(x+2)^2}.$$

$$1 = A(x+2)^2 + B(x+1)(x+2) + C(x+1)$$

$x=-2$: $\quad 1=-C, \quad C=-1.$

$x=-1$. $\quad 1=A.$

$x=0$. $\quad 1=4A+2B+C$

$1=4+2B-1$

$1=3+2B; \quad 2B=-2, \quad B=-1.$

$$\frac{1}{(x+1)(x+2)^2}=\frac{1}{x+1}-\frac{1}{x+2}-\frac{1}{(x+2)^2}.$$

$$\int \frac{dx}{(x+1)(x+2)^2} = \int \left[\frac{1}{x+1} - \frac{1}{x+2} - \frac{1}{(x+2)^2} \right] dx$$

$$= \ln|x+1| - \ln|x+2| + \frac{1}{x+2} + C.$$