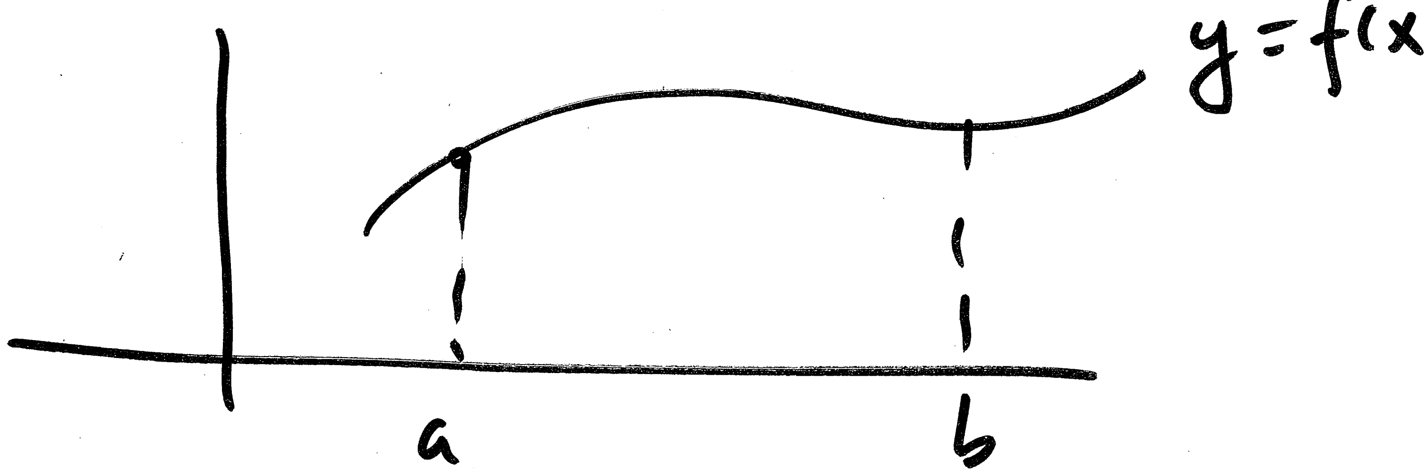
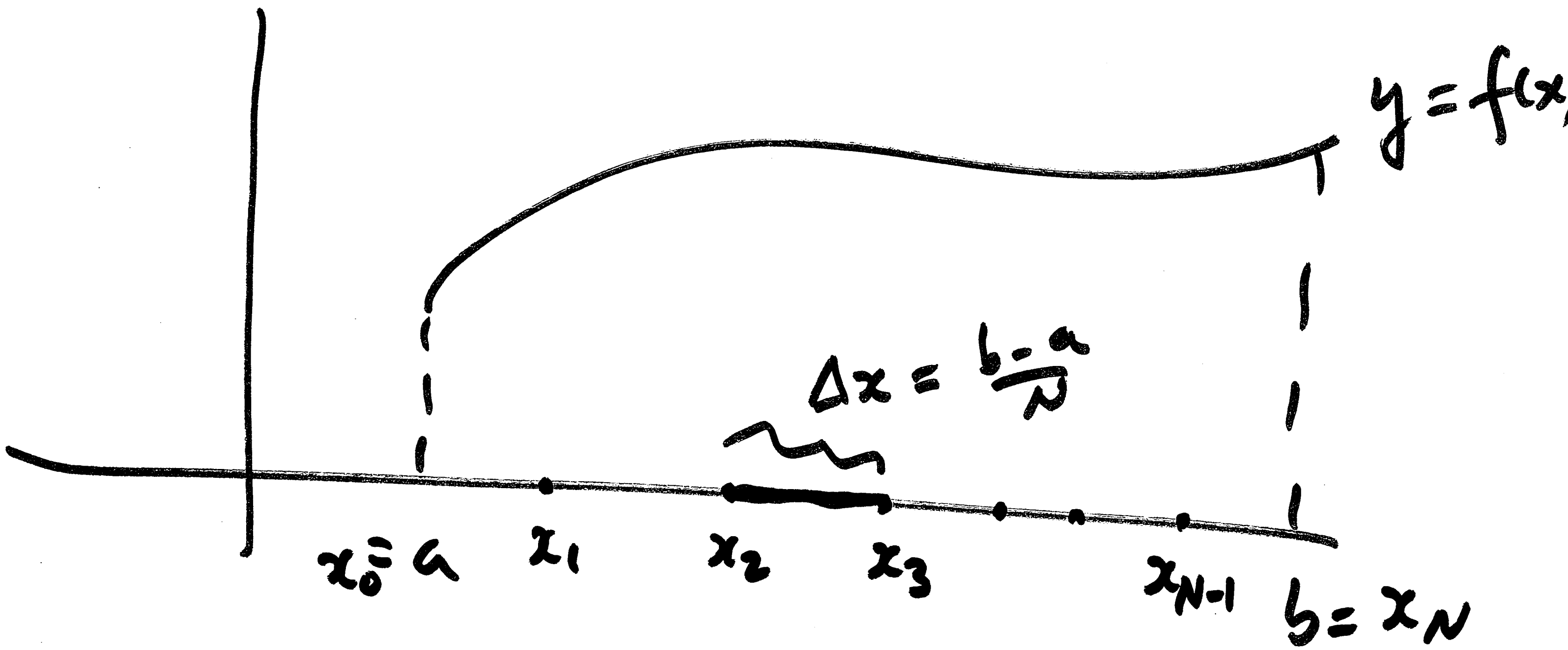
Lesson 15. Sections 7.6 and 7.7

Section 7.7. Approximate Integration.

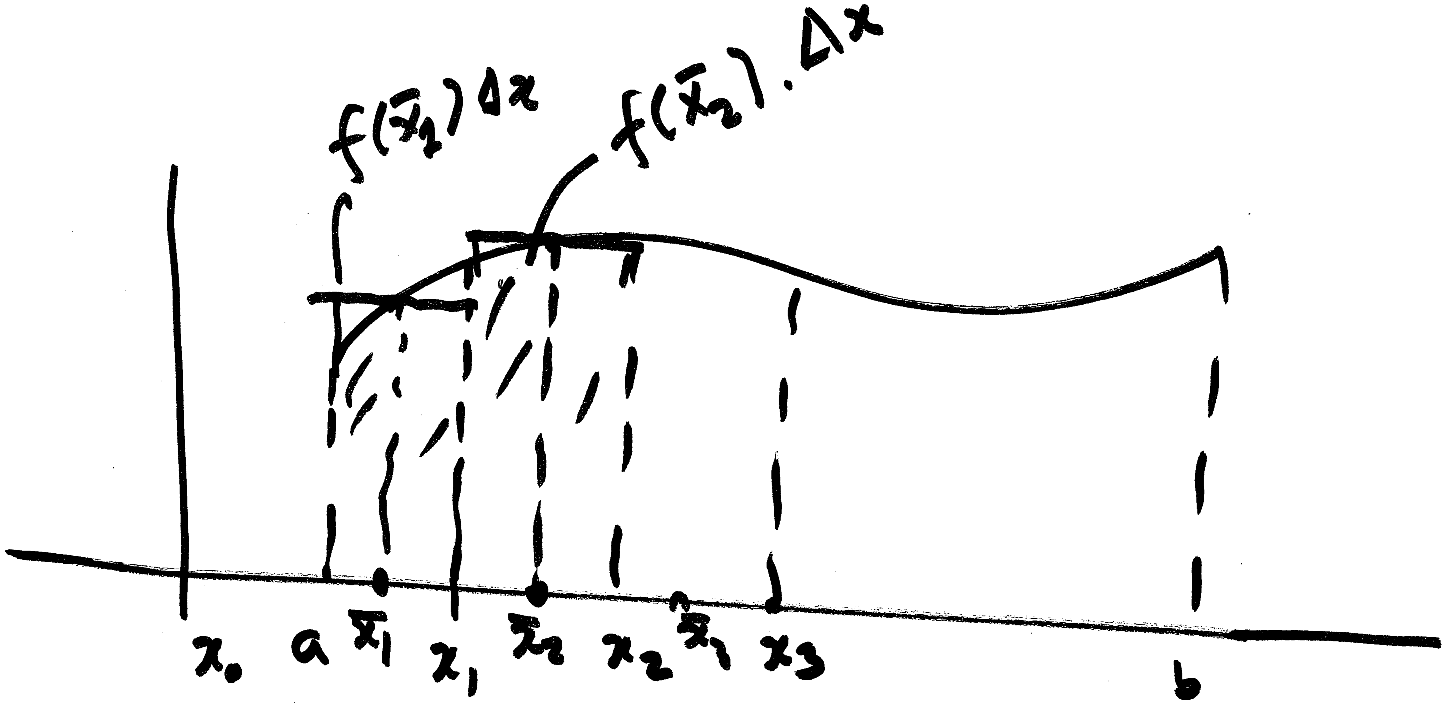


Write a computer program to compute $\int_a^b f(x)\,dx$ approximately.



<u>Step 1</u>: Divide $[a,b]$ into N equal parts.

$\Delta x = \frac{b-a}{N}$ = length of each of the small intervals.



<u>Step 2</u>: Pick the middle point of each interval

$$\bar{x}_j = \text{middle of } [x_{j-1}, x_j]$$

$$\bar{x}_j = \frac{x_j + x_{j-1}}{2}$$

Step 3:

$$S_N = f(\bar{x}_1)\Delta x + f(\bar{x}_2)\Delta x + \dots + f(\bar{x}_N)\Delta x$$

$$\boxed{S_N = \Delta x\left(f(\bar{x}_1) + f(\bar{x}_2) + \dots + f(\bar{x}_N) \right)}$$

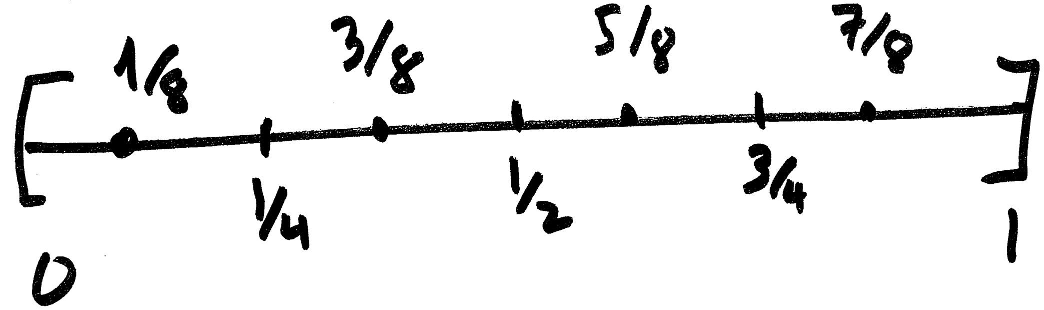
This is called the mid-point approximation

<u>Example:</u> Approximate $\int_0^1 x^2\,dx$

with $N = 4$.

$$\int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}.$$

<u>Approximation:</u> (i)



$\Delta x = 1/4$

(ii) Find the mid-points

(iii) $S_4 = \frac{1}{4}\left(\left(\frac{1}{8}\right)^2 + \left(\frac{3}{8}\right)^2 + \left(\frac{5}{8}\right)^2 + \left(\frac{7}{8}\right)^2 \right)$

$$S_4 = \frac{1}{4} \cdot \frac{1}{64} \left(1 + 9 + 25 + 49 \right)$$

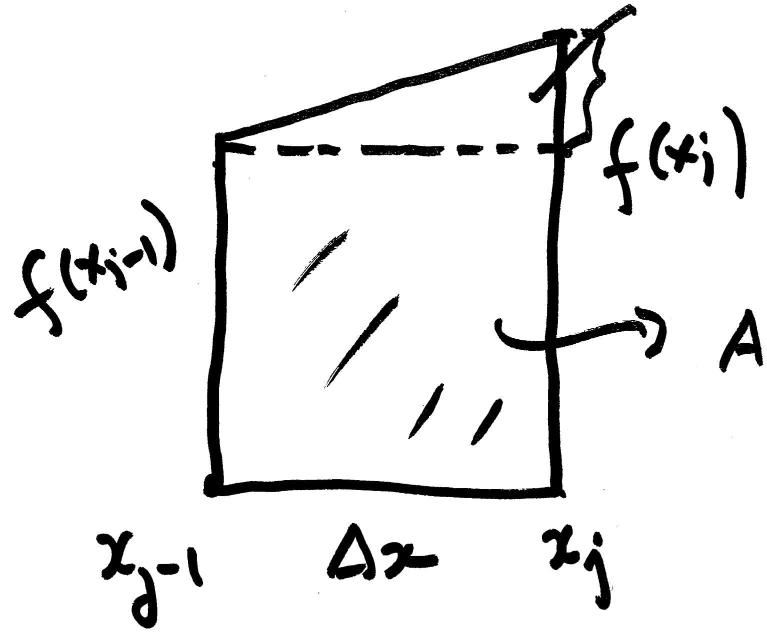
$$= \frac{84}{4 \times 64} = \frac{21}{64}$$

The Trapezoidal Rule

Step 1: Divide $[a,b]$ into N equal parts

$$\Delta x = \frac{b-a}{N}.$$

$$A = \frac{f(x_i) - f(x_{i-1})}{2} \cdot \Delta x$$

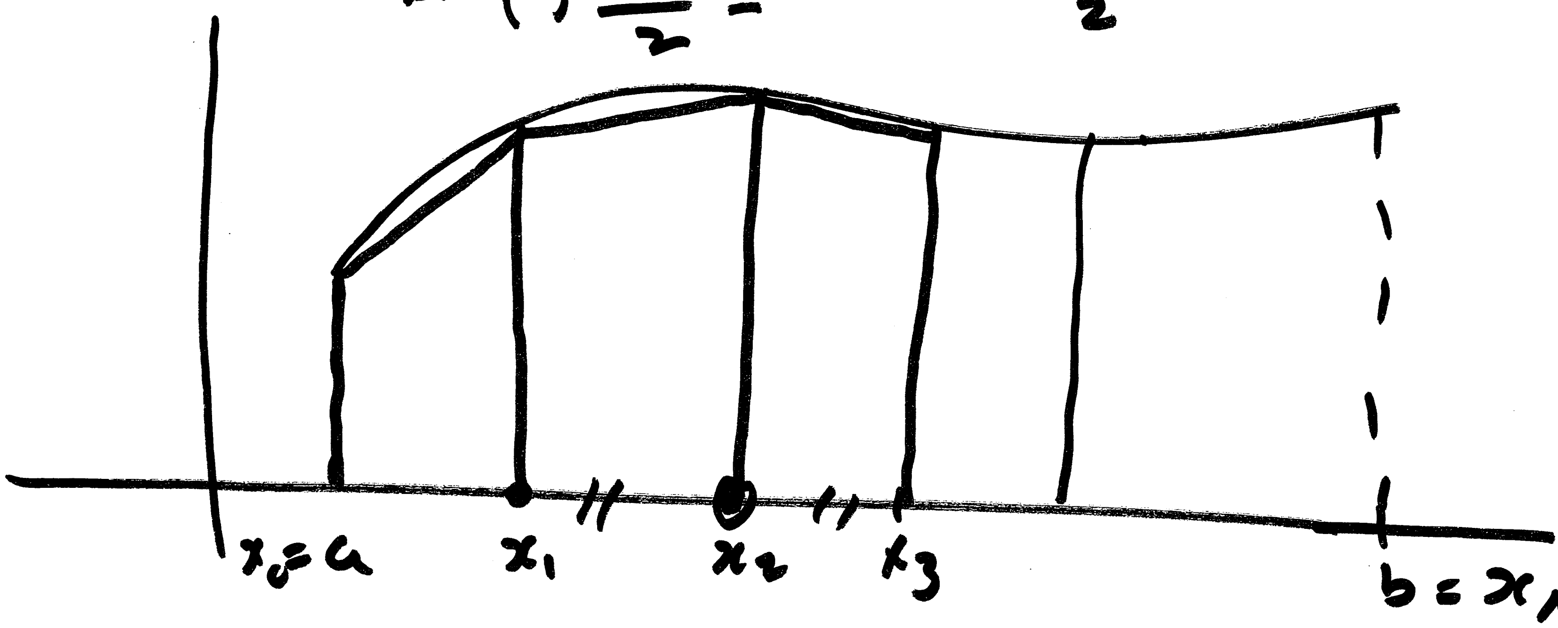


$$A = f(x_{j-1}) \cdot \Delta x$$

Area of trapezoid $= f(x_{j-1}) \cdot \Delta x + \frac{f(x_i) - f(x_{i-1})}{2} \Delta x$

$$= \frac{f(x_{j-1}) + f(x_i)}{2} \Delta x .$$

$$\Delta x \left(\frac{f(x_1) + f(x_2)}{2} \right) + \left(\frac{f(x_2) + f(x_3)}{2} \right) \Delta x$$



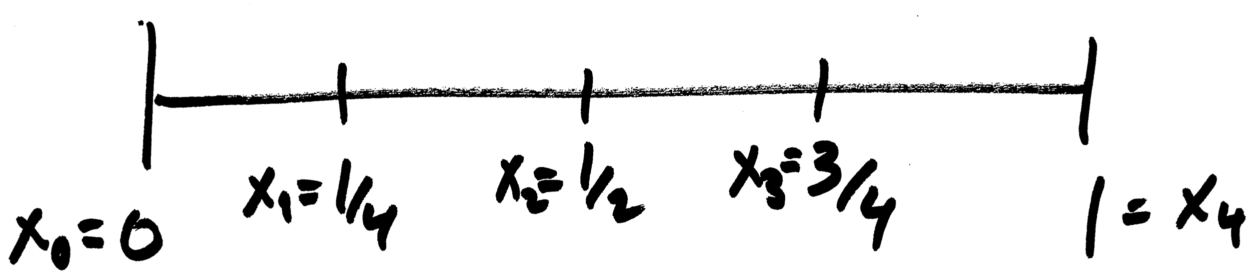
Step 2 Add the areas of the trapezoids

$$T_N = \left(\frac{f(x_1) + f(x_0)}{2} + \frac{f(x_2) + f(x_1)}{2} + \dots + \frac{f(x_{N-1}) + f(x_N)}{2} \right) \Delta x$$

$$T_N = \Delta x \left(\frac{f(x_0)}{2} + f(x_1) + f(x_2) + \dots + f(x_{N-1}) + \frac{f(x_N)}{2} \right)$$

$$\int_0^1 x^2\,dx = 1/3. \qquad \Delta x = 1/4$$

Use trapezoidal rule with $N=4$.

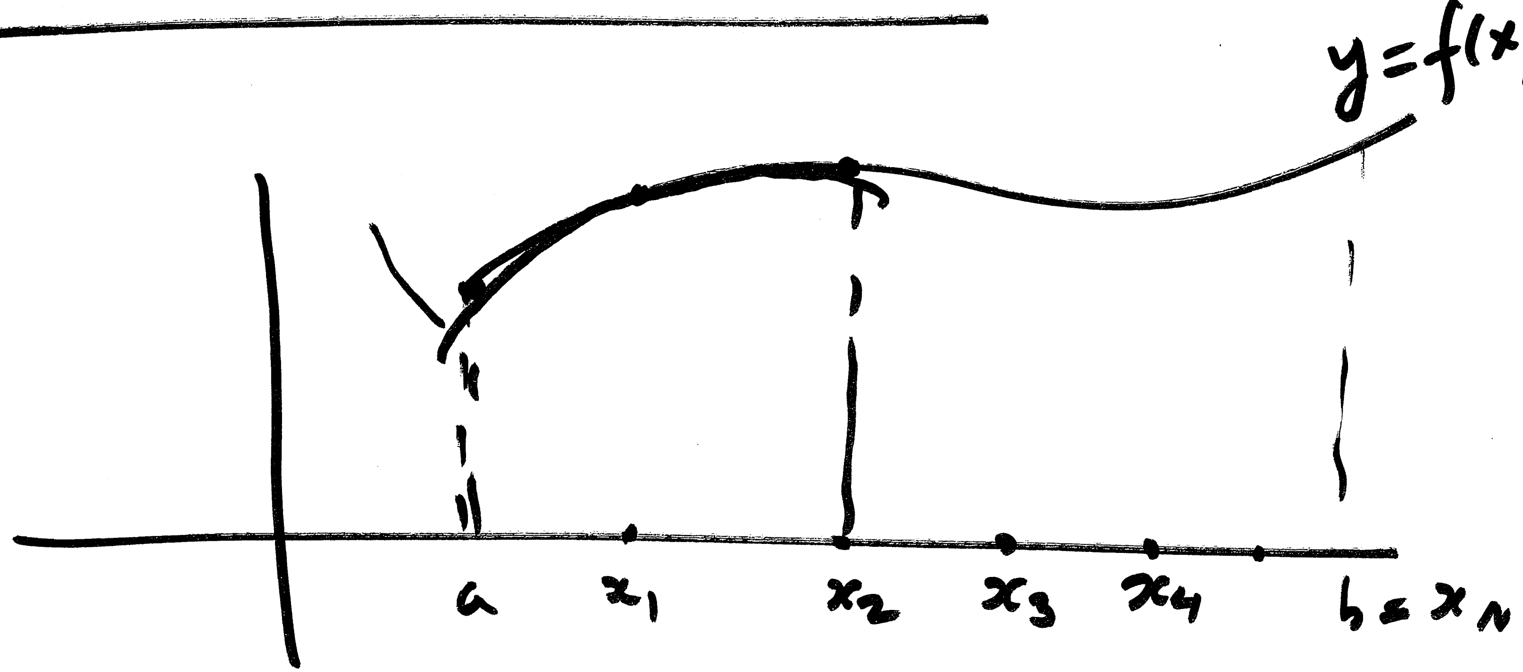


$$T_4 = \frac{1}{4}\left(0 + \left(1/4\right)^2 + \left(1/2\right)^2 + \left(3/4\right)^2 + \frac{1}{2}\right)$$

$$= 1/4\left(\frac{1}{16} + \frac{1}{4} + \frac{9}{16} + \frac{1}{2}\right).$$

$$= \frac{1}{4} \quad \frac{1+4+9+8}{16} = \frac{22}{64}$$

The Simpson's Rule



Step 1: The same, N even

Step 2, Find the parabola which goes through

$$S_N = \frac{\Delta x}{3}\Big(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 4f(x_{N-1}) + f(x_N) \Big)$$

Section 7-6: The use of a ~~T~~ Table of Integrals

Example: Compute

$$\int \frac{\sec^2 x}{\tan^2 x - 4}\, dx$$

Set $\underline{u = \tan x}$, $du = \sec^2 x \, dx$

$a = 2$

$$\int \frac{du}{u^2 - 4} = \frac{1}{4} \ln\left|\frac{u-2}{u+2}\right| + C$$

$$\int \frac{\ln(10+\sqrt{x})}{\sqrt{x}}\,dx$$

$$u = 10+\sqrt{x}, \quad du = \frac{1}{2\sqrt{x}}\,dx$$

$$= 2\int \ln|u|\,du.$$