Lesson 19 Section 11.1

Sequences

Exam 2: March 9, 8:00 – 9:00 Elliott.

Lessons 9 to 21.

Sequences: $\{a_1, a_2, a_3, \dots\}$

A sequence is a list of numbers displayed in a particular order.

Examples: $a_n = \frac{1}{n}$, $n = 1, 2, 3, \ldots$

$a_1 = 1$, $a_2 = 1/2$, $a_3 = 1/3$, $a_4 = 1/4$, ...

Recursive formulas: $a_1 = 1$, $a_n = 2a_{n-1}$

$a_1 = 1$, $a_2 = 2 \cdot a_1 = 2$, $a_3 = 2a_2 = 4$

$a_4 = 2a_3 = 8$, ...

Sequence: 1, 2, 4, 8, 16, 32, ...

The Fibonacci Sequence:

$$a_1 = 1, \quad a_2 = 1, \quad a_n = a_{n-1} + a_{n-2} \text{ for } n \geq 2$$

$$1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

$$a_1 = 1, \quad a_2 = 0$$

$$1, 0, \underline{1, 1, 2, 3}, 5, 8, \ldots$$

$$a_1 = 1, \quad a_2 = 5$$

$$1, 5, 6, 11, 17, 28, \ldots$$

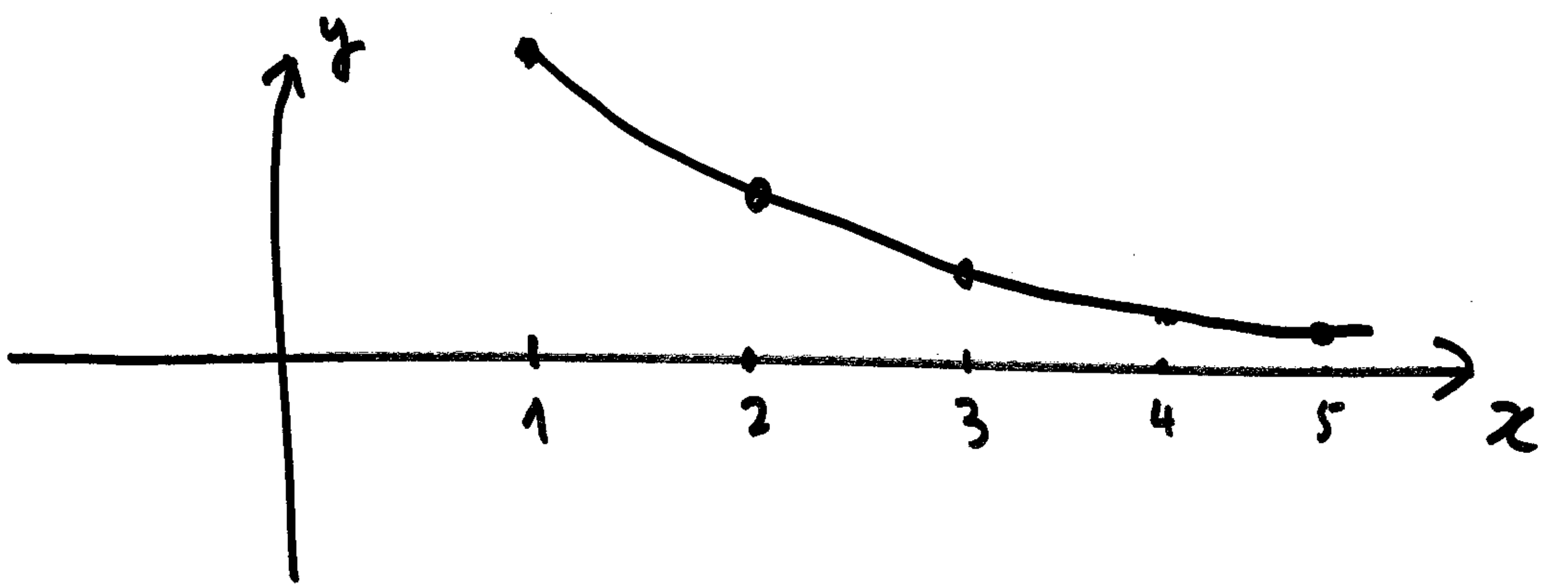
Our goal is to understand the behavior of a sequence $\{a_1, a_2, a_3, a_4, \dots\} =$

$$= \{a_n\}_{n=1}^{\infty}$$

as n goes to infinity.

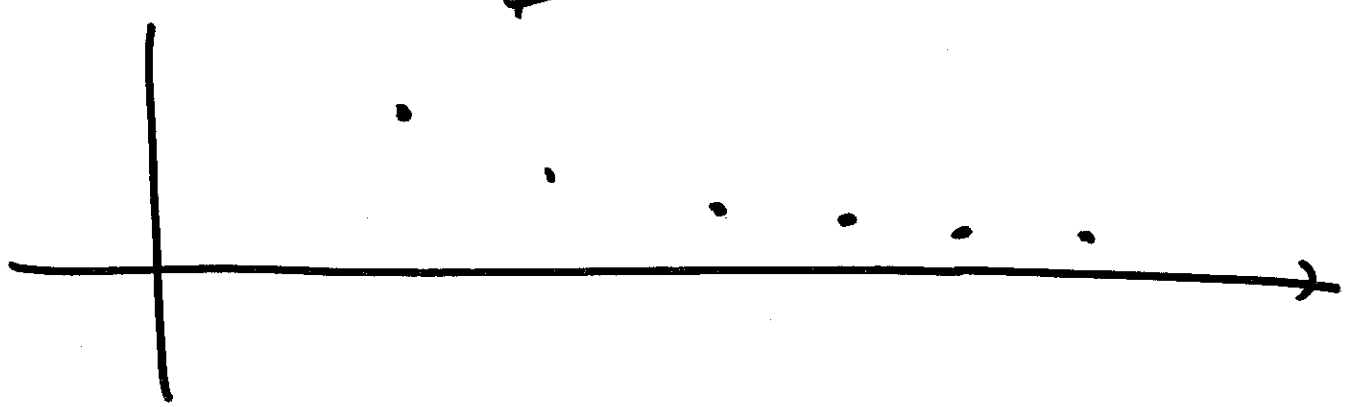
Limits of Functions and Sequences

$$f(x) = \frac{1}{x} \quad , \; x \geq 1$$

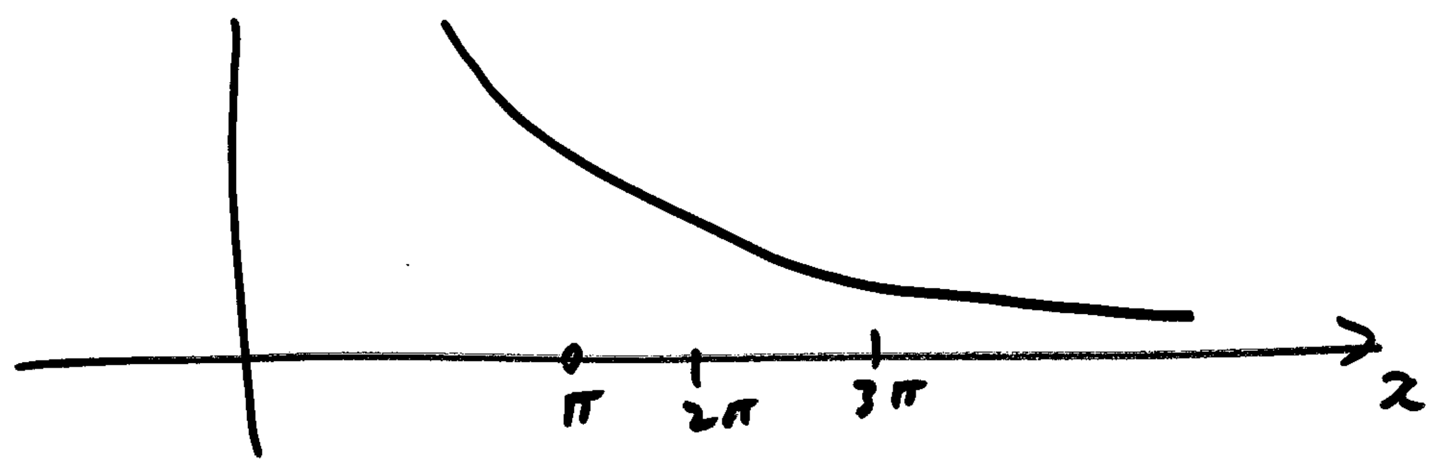


$$f(n) = a_n = \frac{1}{n} .$$

graph of the sequence



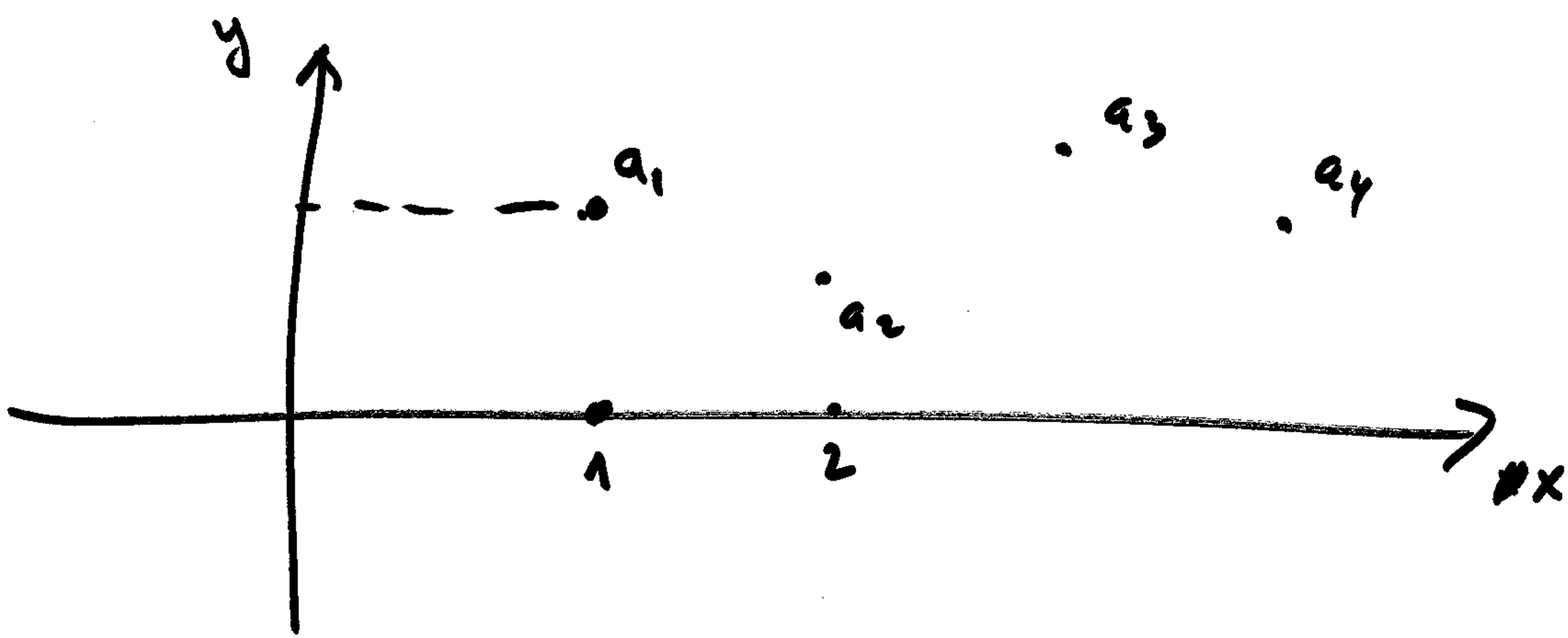
$$\boxed{\lim_{\underline{x} \to \infty} \frac{1}{x} = 0}$$



$\lim_{n \to \infty} \frac{1}{n} = 0$ is a particular case of

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

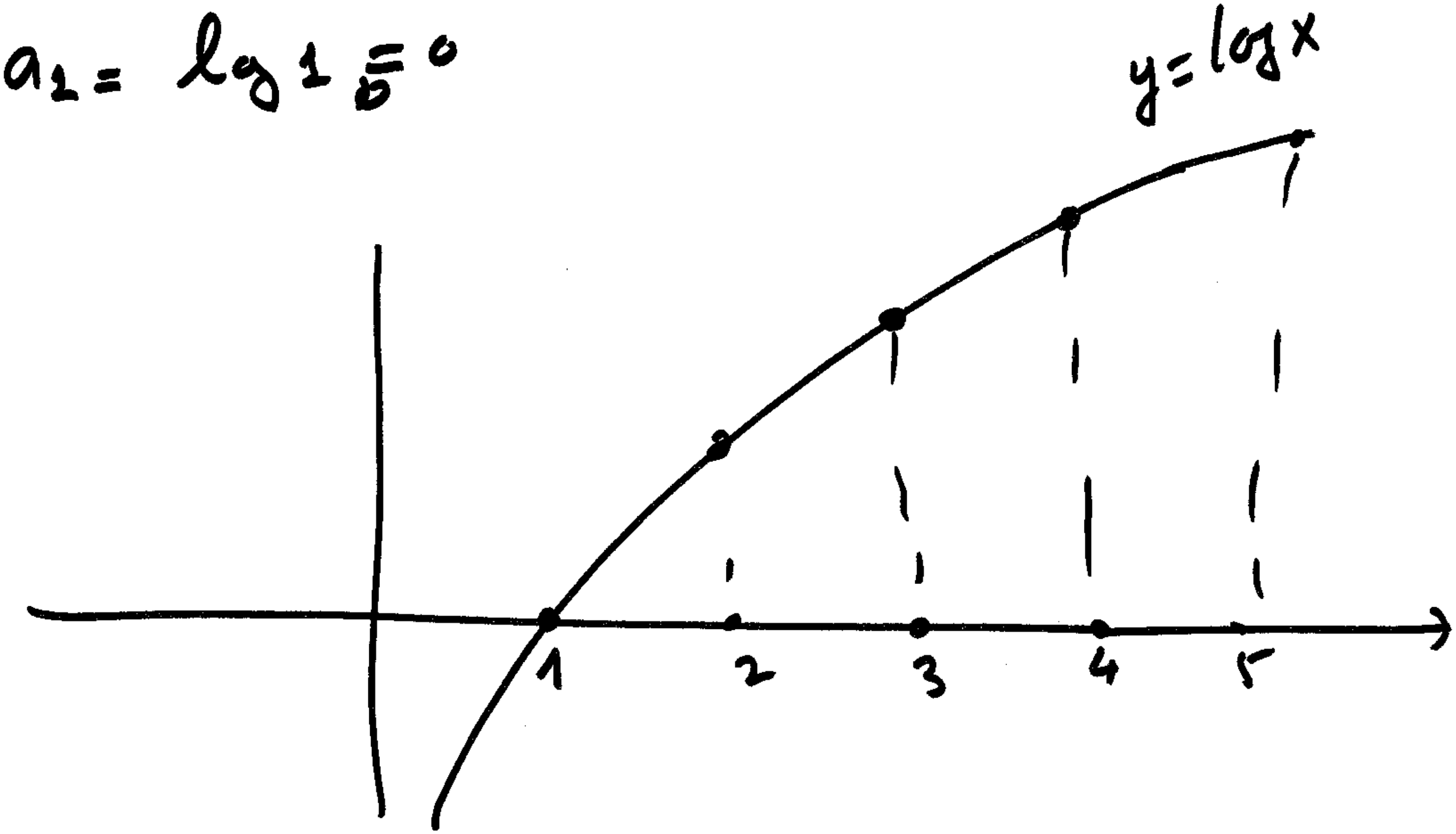
A sequence $\{a_n\}$ also has a graph.



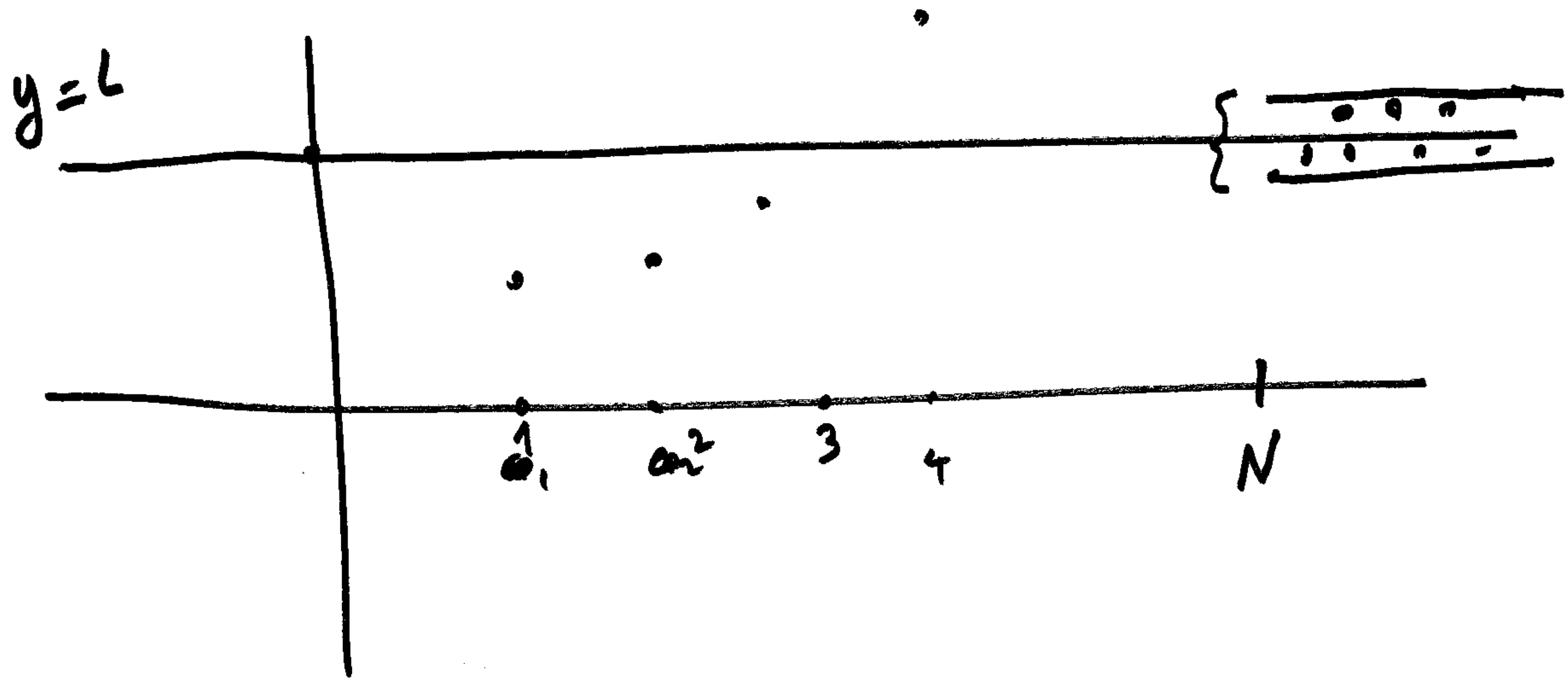
We say that $\lim_{n \to \infty} a_n = L$ if.

Example: $a_n = \log n$

$a_1 = \log 1 = 0$



$$\lim_{n\to\infty} \log n = \infty$$



Given a strip about the line $y = L$ one can find a number N such that $\underbrace{(n, a_n)}$ is inside the strip for $n \geq N$.

Example: $a_n = \dfrac{n+1}{n+3}$

$a_1 = \dfrac{2}{4} = 1/2$ $\qquad a_3 = \dfrac{4}{6}$

$a_2 = \dfrac{3}{5}$

$$a_n = \frac{n+1}{n+3} = \frac{\cancel{n}(1+1/n)}{\cancel{n}(1+3/n)} = \frac{1+1/n}{1+3/n}$$

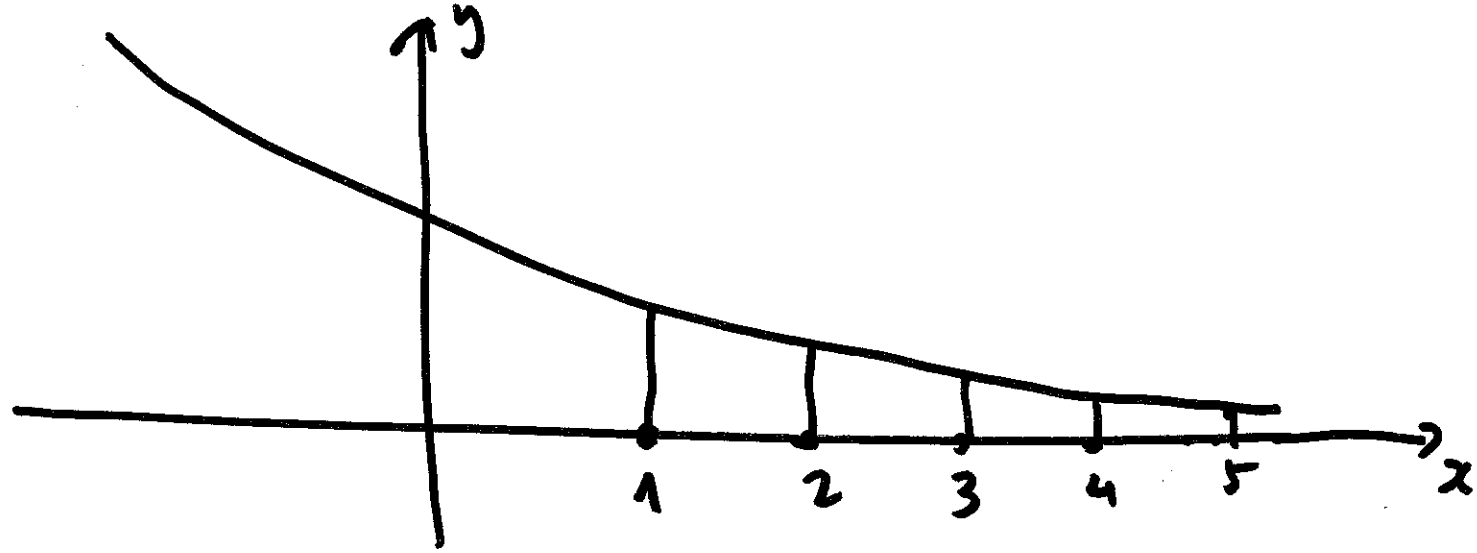
$$a_n = \frac{1+1/n}{1+3/n} \qquad a_n \to 1.$$

$$\boxed{\lim_{n\to\infty} \frac{n+1}{n+3} = 1}$$

$$\lim_{n\to\infty} \frac{1}{2^n} = 0$$

$a_n = 1/2^n$, $\quad a_1 = \frac{1}{2}$, $a_2 = 1/4$, $a_3 = 1/8$, ...

$f(x) = \frac{1}{2^x} = 2^{-x}$ $\qquad \lim_{x\to\infty} \frac{1}{2^x} = 0$



The Factorial:

$$n! = 1.2.3.4.5 \ldots n.$$

$$1! = 1$$

$$2! = 1.2 = 2$$

$$3! = 1.2.3 = 6$$

$$4! = 1.2.3.4 = 24$$

$$5! = 1.2.3.4.5 = 120$$

$$6! = 1.2.3.4.5.6 = 720$$

$$\boxed{\lim_{n \to \infty} n! = \infty}$$

$\lim_{n \to \infty} a_n = \infty$ if a_n gets arbitrarily large as $n \to \infty$.

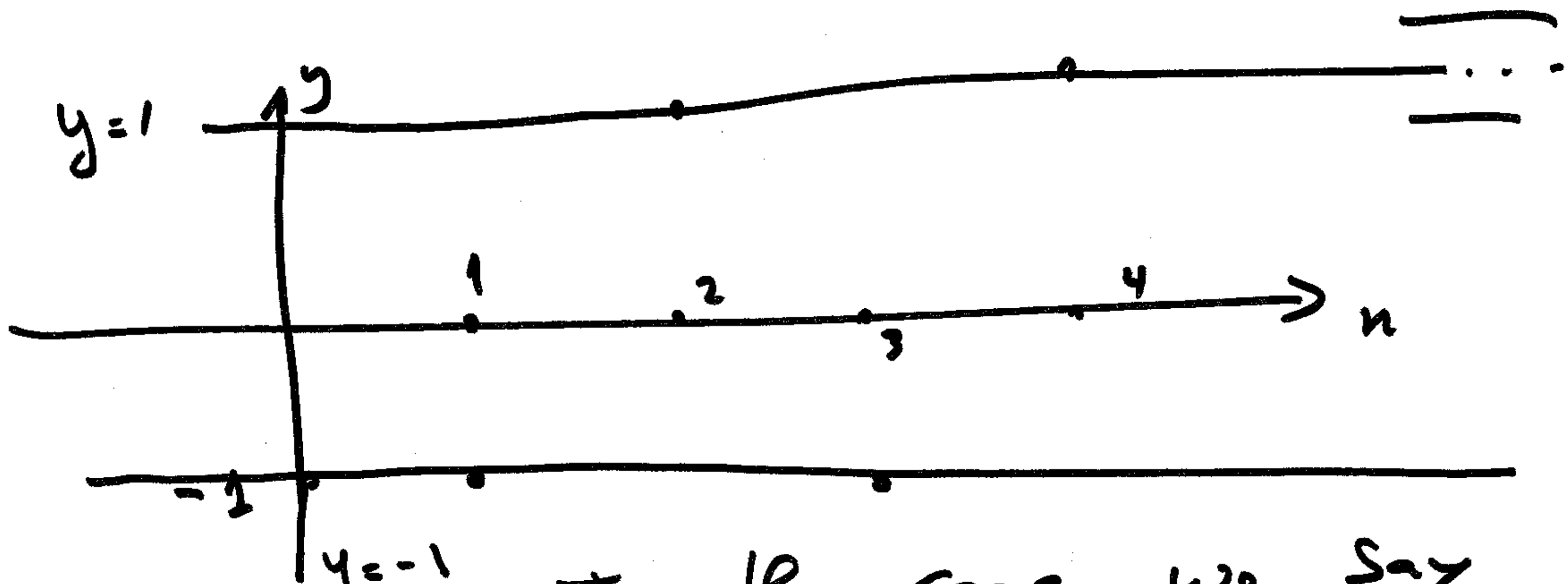
$\lim_{n \to \infty} a_n = -\infty$ if a_n gets arbitrarily large, but negative as $n \to \infty$.

Example $a_n = (-1)^n$.

$a_1 = -1$, $a_2 = 1$, $a_3 = -1$,

$a_n = -1$, when n is odd

$a_n = 1$, when n is even.



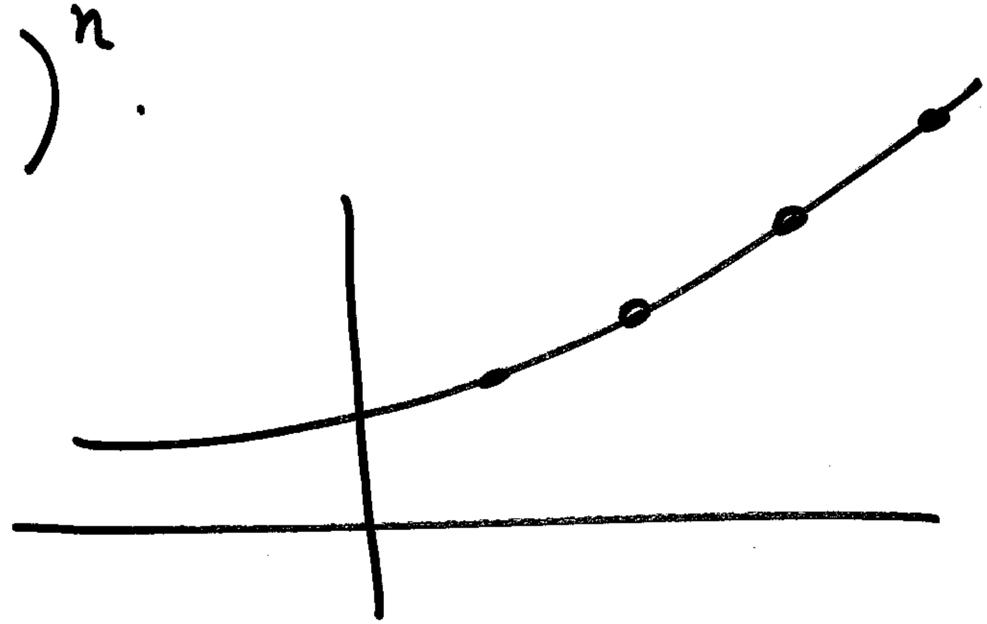
In this case we say that $\lim_{n\to\infty} a_n$ does not exist.

$a_n = r^n$, where r is a real number.

<u>Ex:</u> $a_n = 2^n$, $a_n = \frac{1}{3^n}$; $a_n = (-2)^n$

$a_n = \left(-\frac{1}{4}\right)^n$.

$a_n = 2^n$

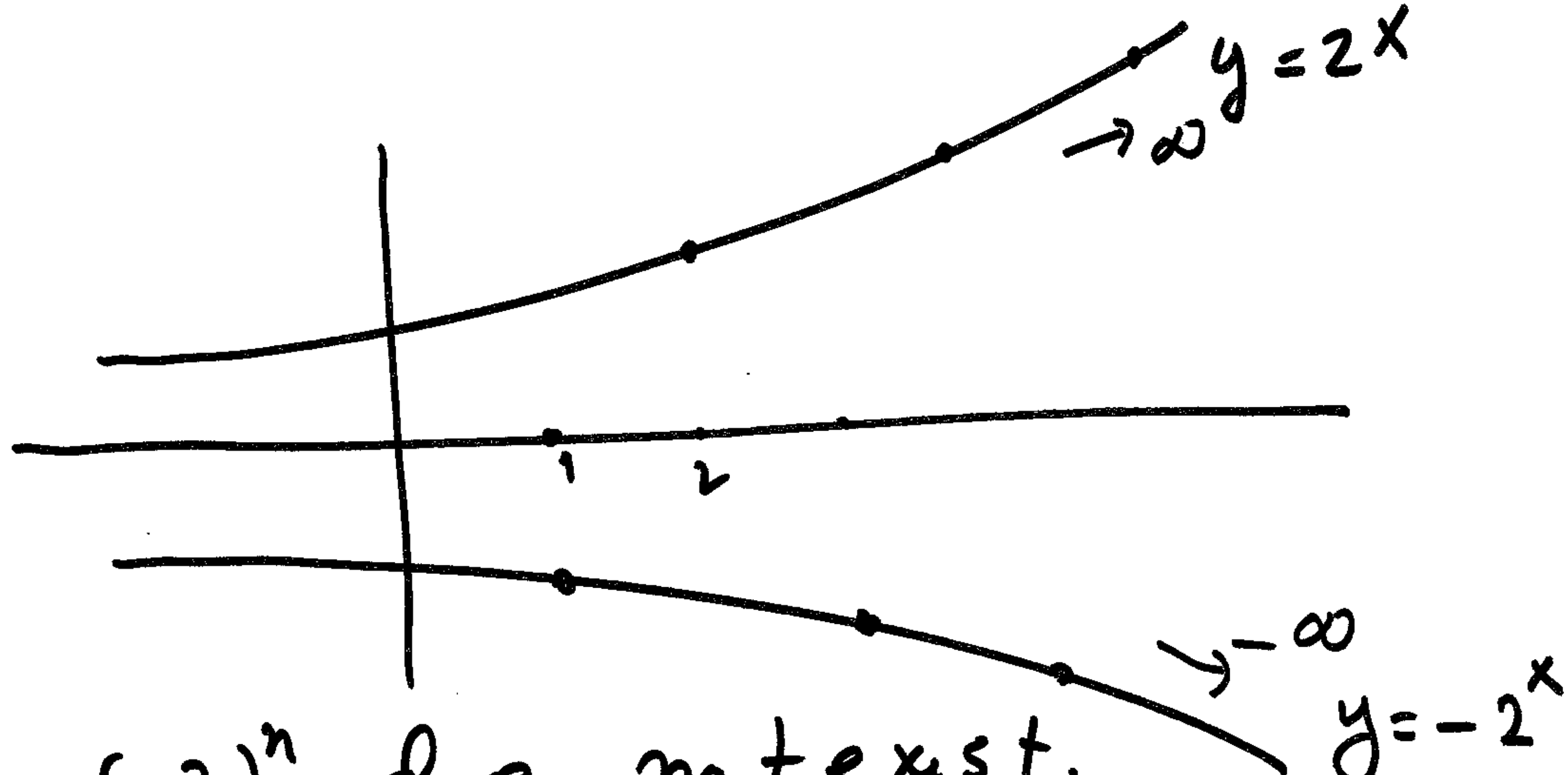


$\lim_{n \to \infty} 2^n = \infty$.

$a_n = (-2)^n$

n even: $a_n = (-2)^n = \underbrace{(-1)^n}\, 2^n = 2^n$, n even

n odd: $a_n = (-1)^n\, 2^n = -\,2^n$, n odd.



$\lim_{n\to\infty} (-2)^n$ does not exist.

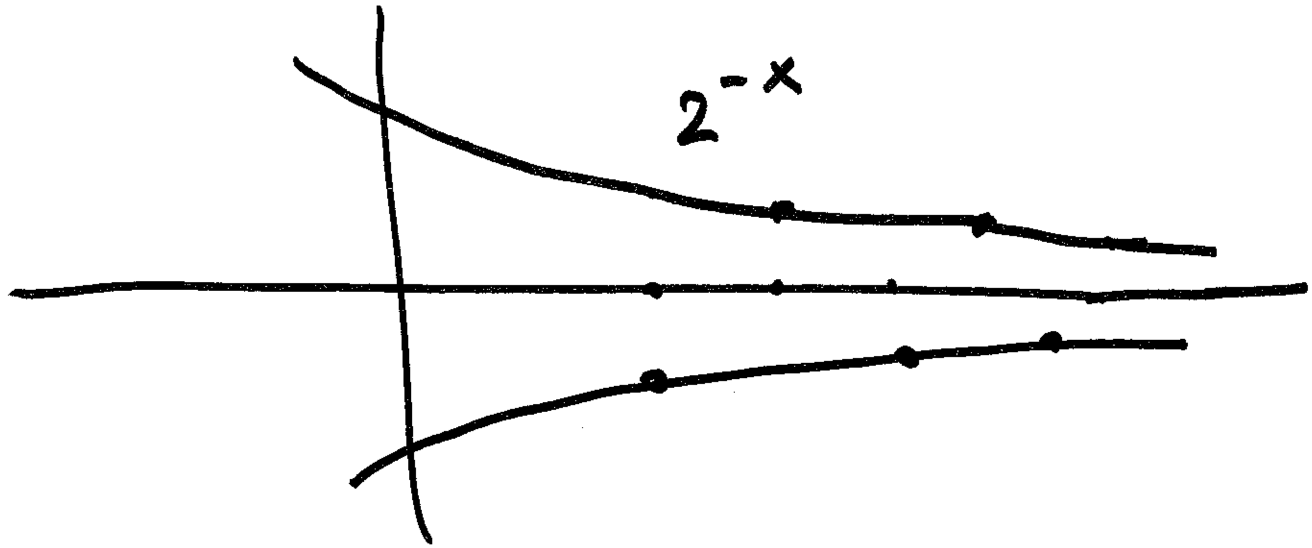
$\lim_{n \to \infty} (r)^n$ does not exist if $r \le -1$

$\lim_{n \to \infty} (r)^n = \infty$ if $r > 1$.

when $r = 1$ $\quad r^n = 1$.

when $|r| < 1$.

$$\lim_{n\to\infty}\left(\frac{1}{2}\right)^n = \lim_{n\to\infty} 2^{-n} = 0$$



$$\lim_{n\to\infty}\left(-\frac{1}{2}\right)^n = 0$$