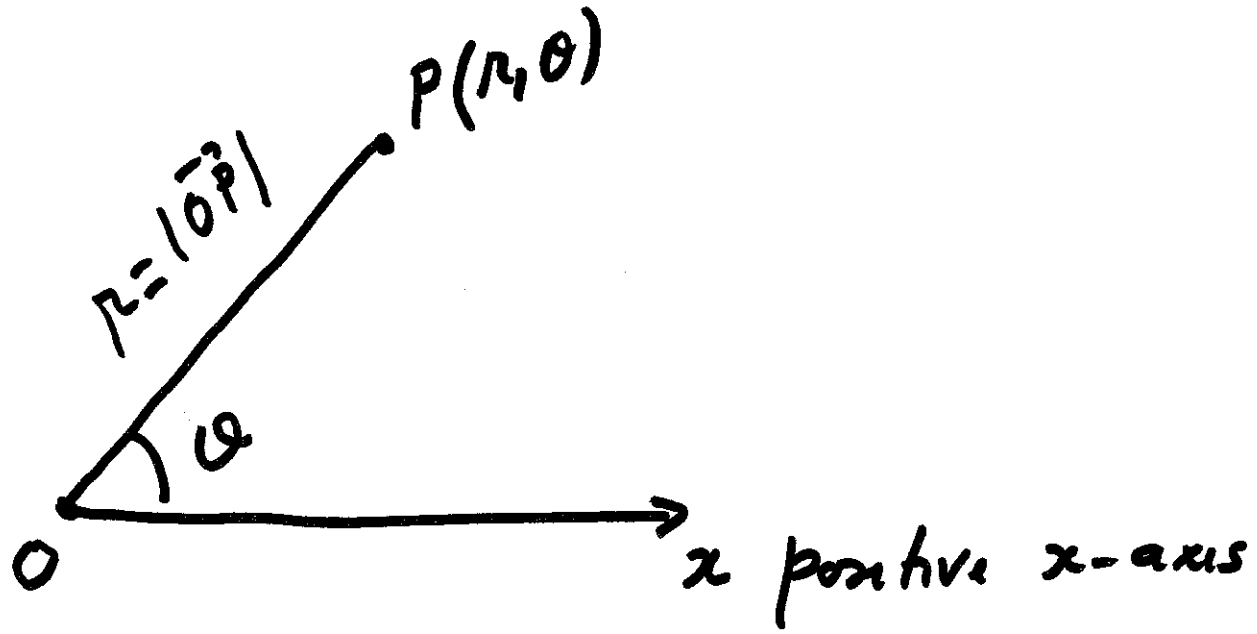
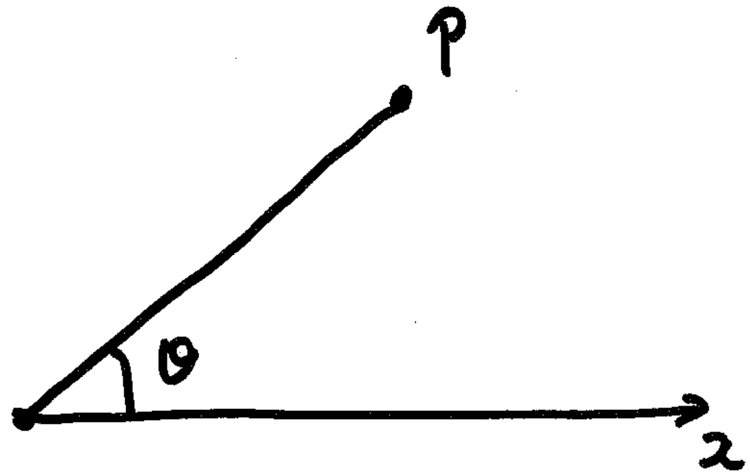
LESSON 3.3 – SECTION 10.3
Polar Coordinates



r = distance from O to P

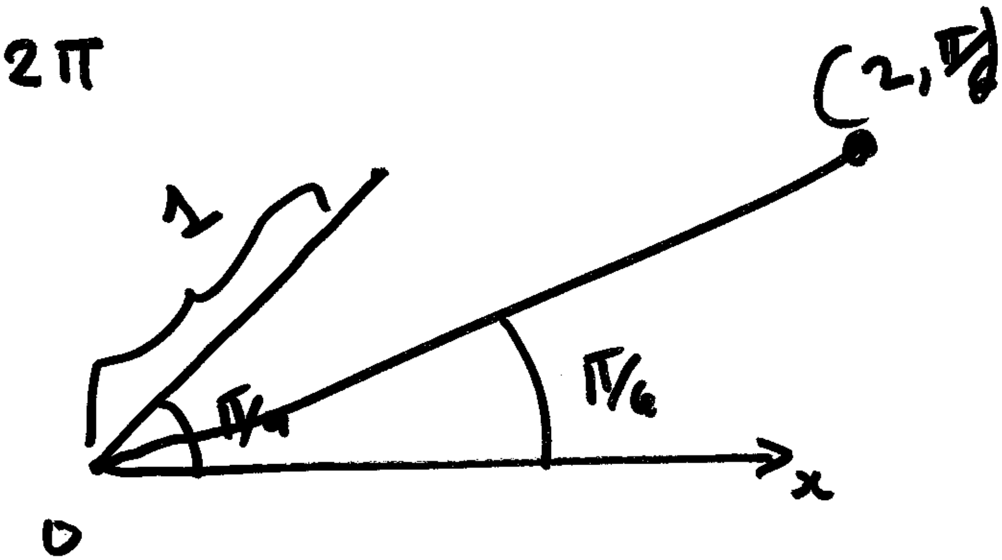
θ = Angle between $\overline{OP}$ and the positive x-axis.



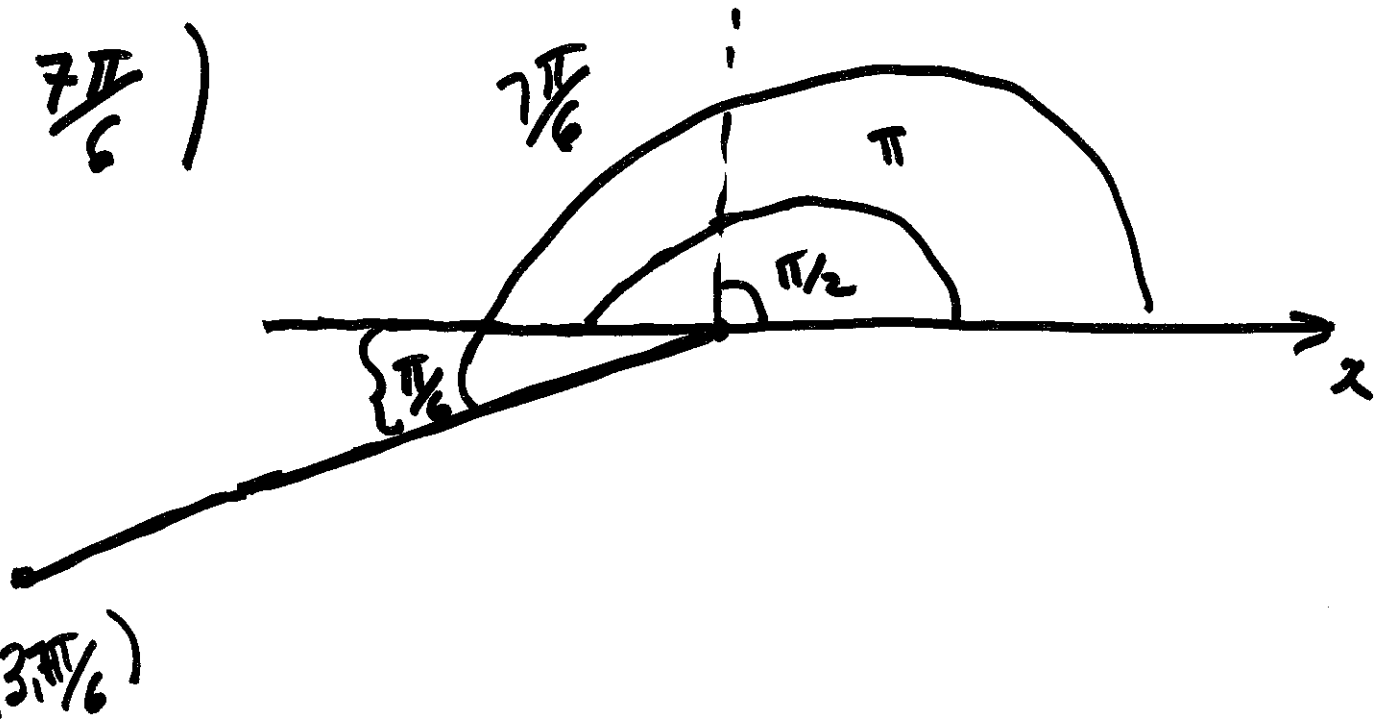
At first $0 \le \theta < 2\pi$

$P(1, \pi/4)$

$P(2, \pi/6)$

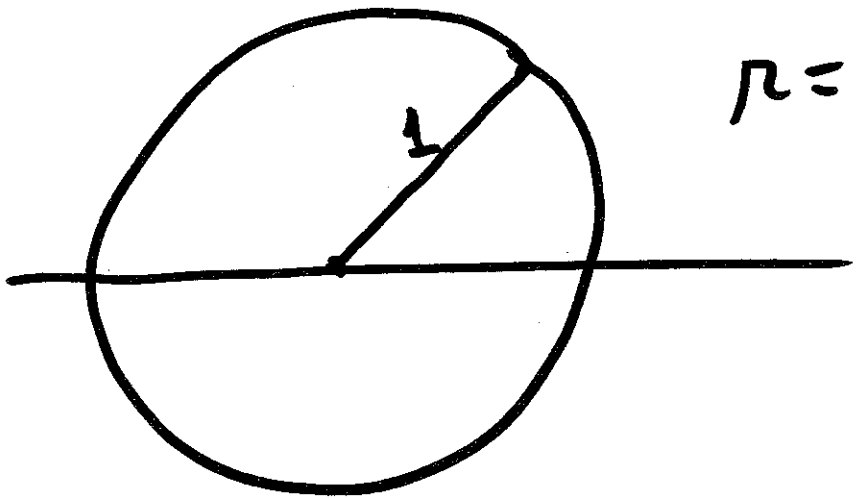


$(3, \frac{7\pi}{6})$



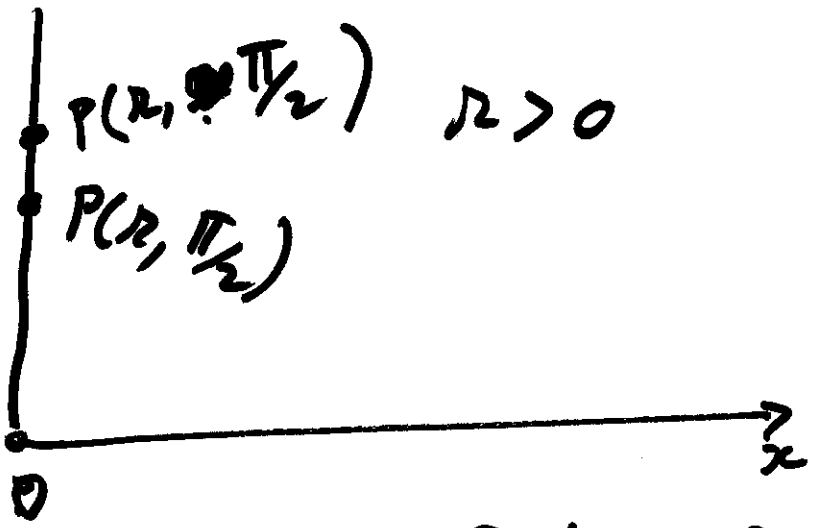
$\frac{7\pi}{6} = \pi + \frac{\pi}{6}$

Ex Describe the curve $r = 1$

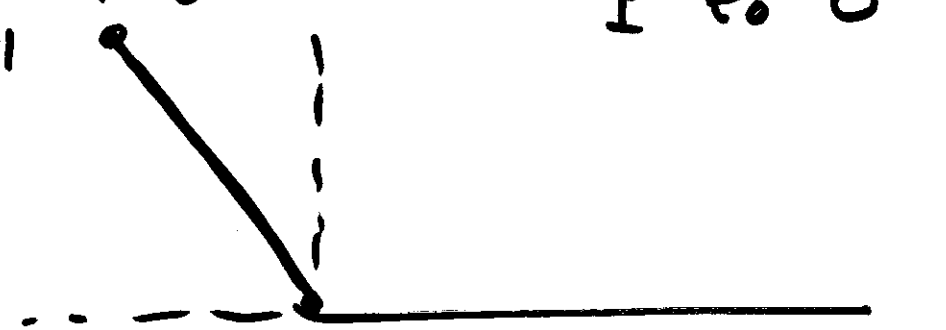


$r = 1$ = circle of radius 1

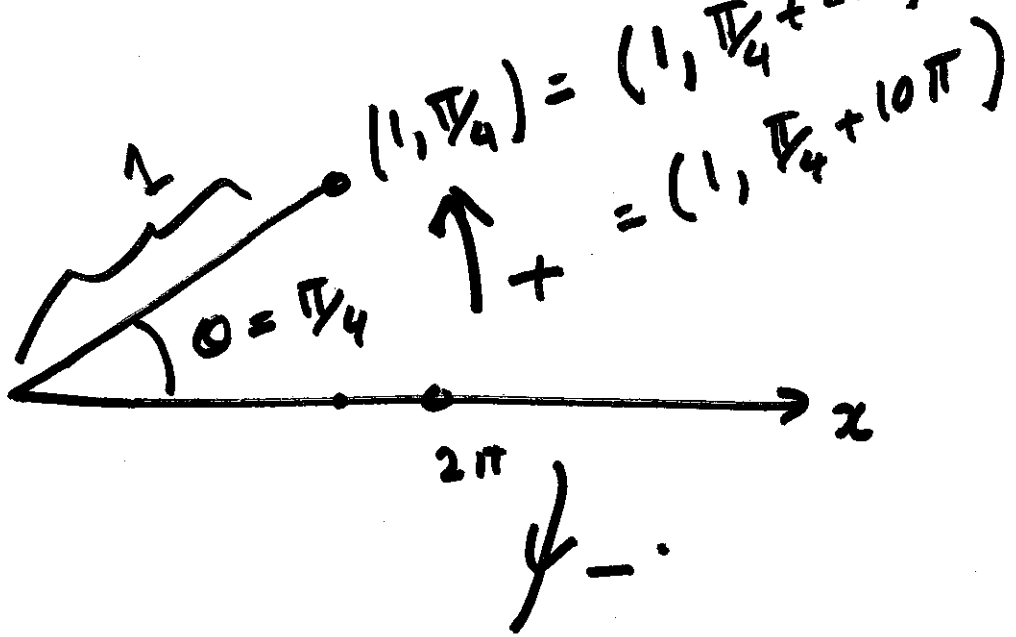
$\theta = \pi/2$.



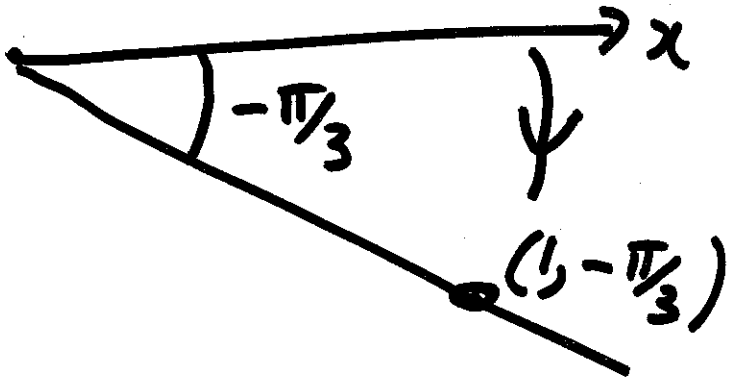
$\theta = 3\pi/4 = \pi/2 + \pi/4$



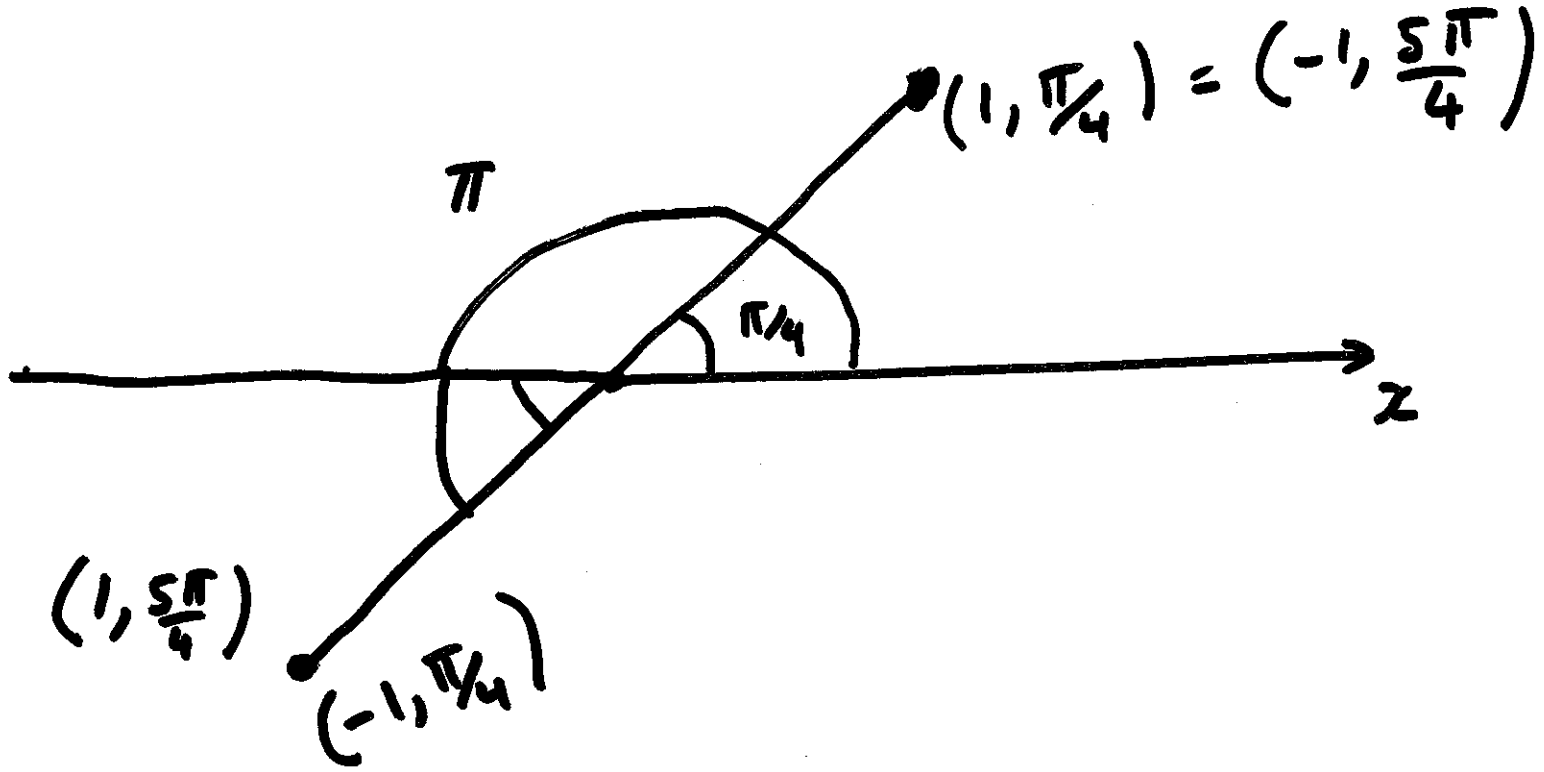
We want to extend this definition for all θ and all r



Example: $(1, -\pi/3)$

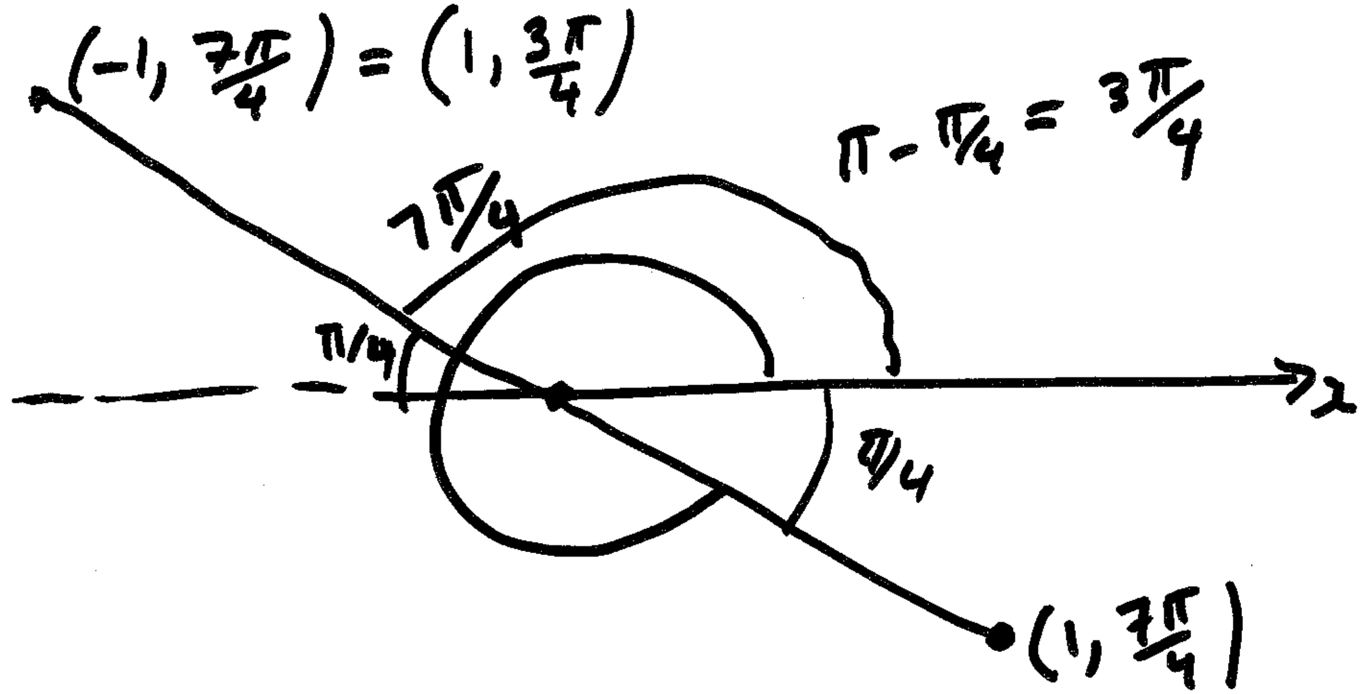


Negative r $(-2, \pi/4)$; $(-1, \pm 5\pi/4)$.



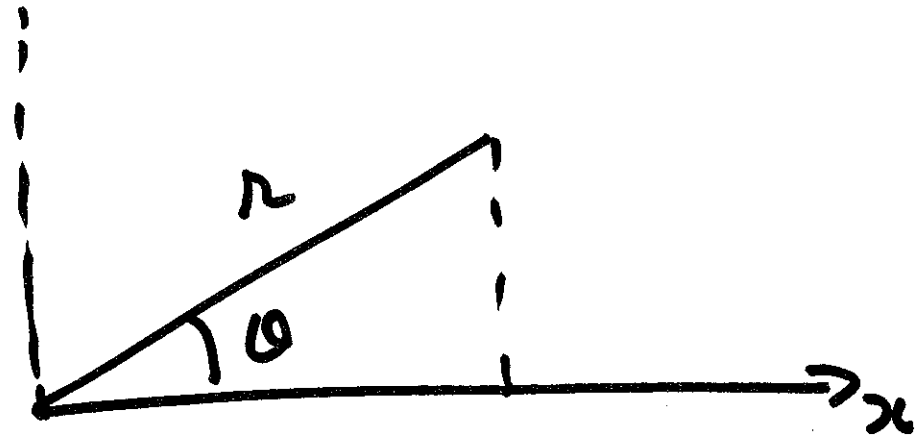
$(1, \frac{5\pi}{4}) =$

~~$(-1, \frac{3\pi}{4}$~~ $\left(-1, \frac{7\pi}{4}\right)$



$\frac{7\pi}{4} = 2\pi - \pi/4$

Polar Coordinates to Cartesian Coordinates and Vice-Versa.



$$x = r\cos\theta$$

$$y = r\sin\theta.$$

$P(r, \theta)$ to (x, y)

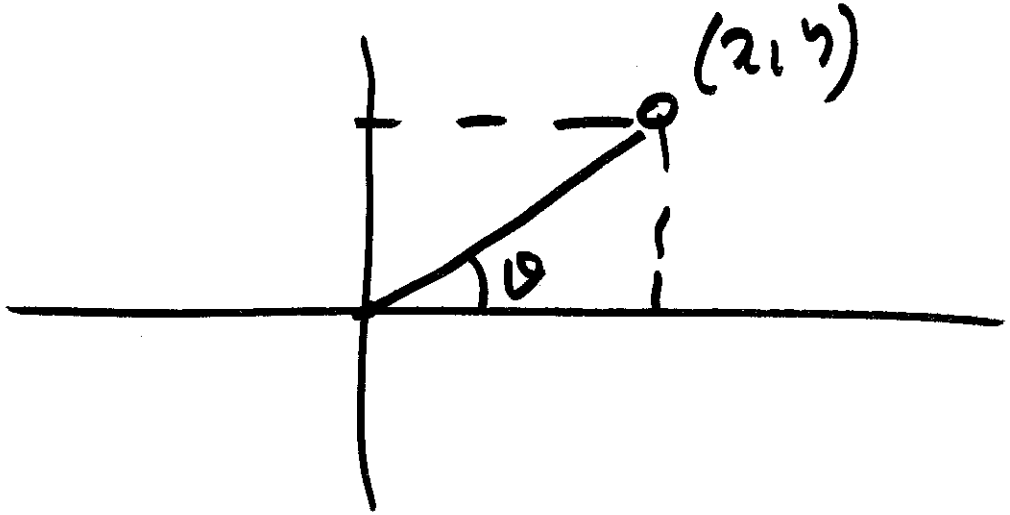
$P(1, \pi/4)$ $\quad r = 1, \ \theta = \pi/4$

$$x = \cos \pi/4 = \frac{\sqrt{2}}{2}$$

$$y = \sin \pi/4 = \frac{\sqrt{2}}{2}$$

$$P(1, \pi/4) \longrightarrow \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

<u>Going backwards</u> From Cartesian to Polar.

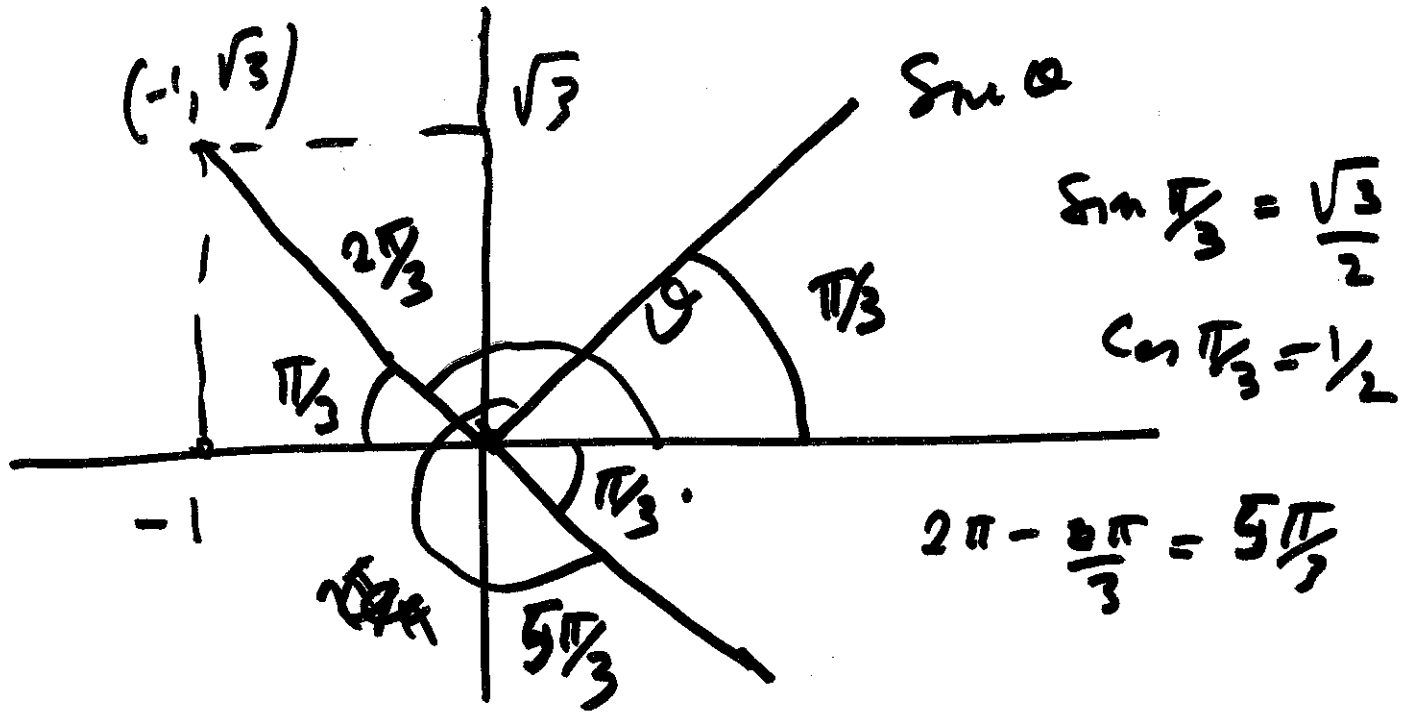


$r = \sqrt{x^2 + y^2}$

$\tan\theta = \left(\frac{y}{x}\right), \quad x \neq 0$

$\theta = \arctan\left(\frac{y}{x}\right)$

Express the point $(-1, \sqrt{3})$ in polar coordinates.



$$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2.$$

$$\tan\theta = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

$$\sin\alpha = \frac{\sqrt{3}}{2}$$

$$\cos\alpha = -\frac{1}{2}$$

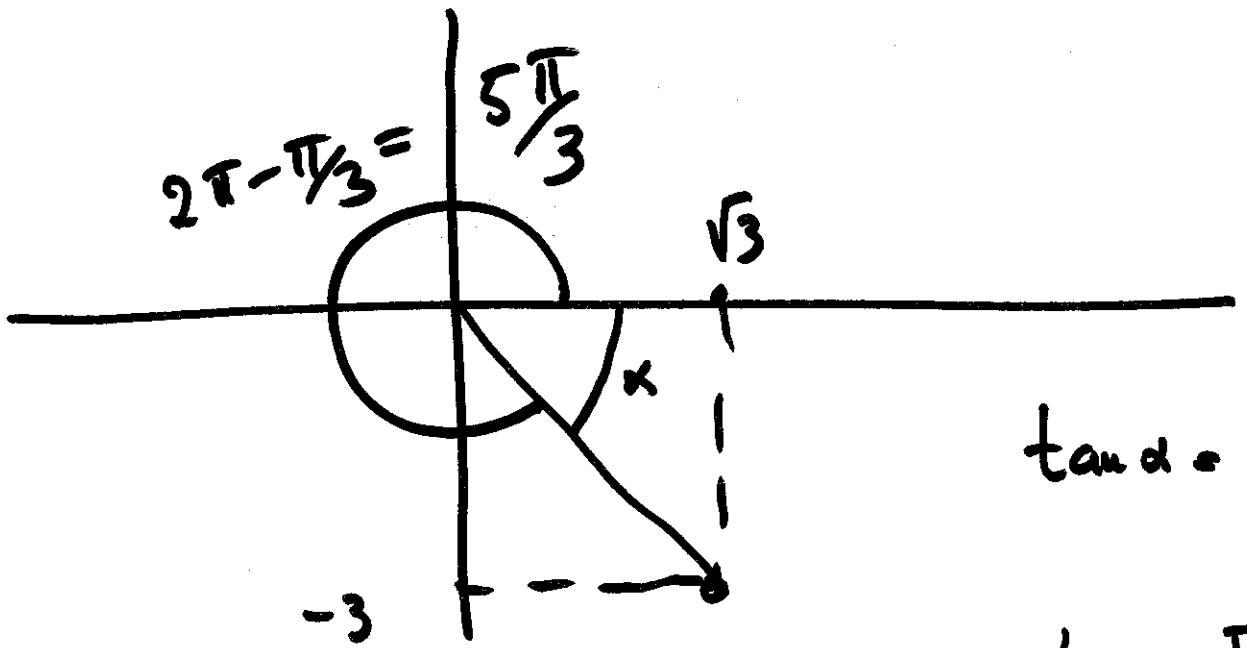
$$\boxed{\theta = \frac{2\pi}{3}}$$

$(\sqrt{3}, -3)$ in Cartesian coordinates.

$$r = \sqrt{(\sqrt{3})^2 + 3^2} = \sqrt{3+9} = \sqrt{12} = \sqrt{4 \times 3}$$
$$= 2\sqrt{3}.$$

$$\boxed{r = 2\sqrt{3}}$$

$$\tan\theta = \frac{-3}{\sqrt{3}} = -\sqrt{3}$$



$\tan\alpha = -\frac{3}{\sqrt{3}} = -\sqrt{3}$

$\alpha = -\pi/3$

$(\sqrt{3}, -3)$ is $\left(2\sqrt{3}\,;\, 5\pi/3\right)$ in Polar Coordinates

$r = \cos\theta$

Describe the curve given by

$$r = \cos\theta$$

in polar coordinates.

Multiply the equation by r:

$$r^2 = r\cos\theta.$$

$$x^2 + y^2 = x \qquad x^2 - x + y^2 = 0$$

$$\boxed{(x-1/2)^2 + y^2 = 1/4}$$