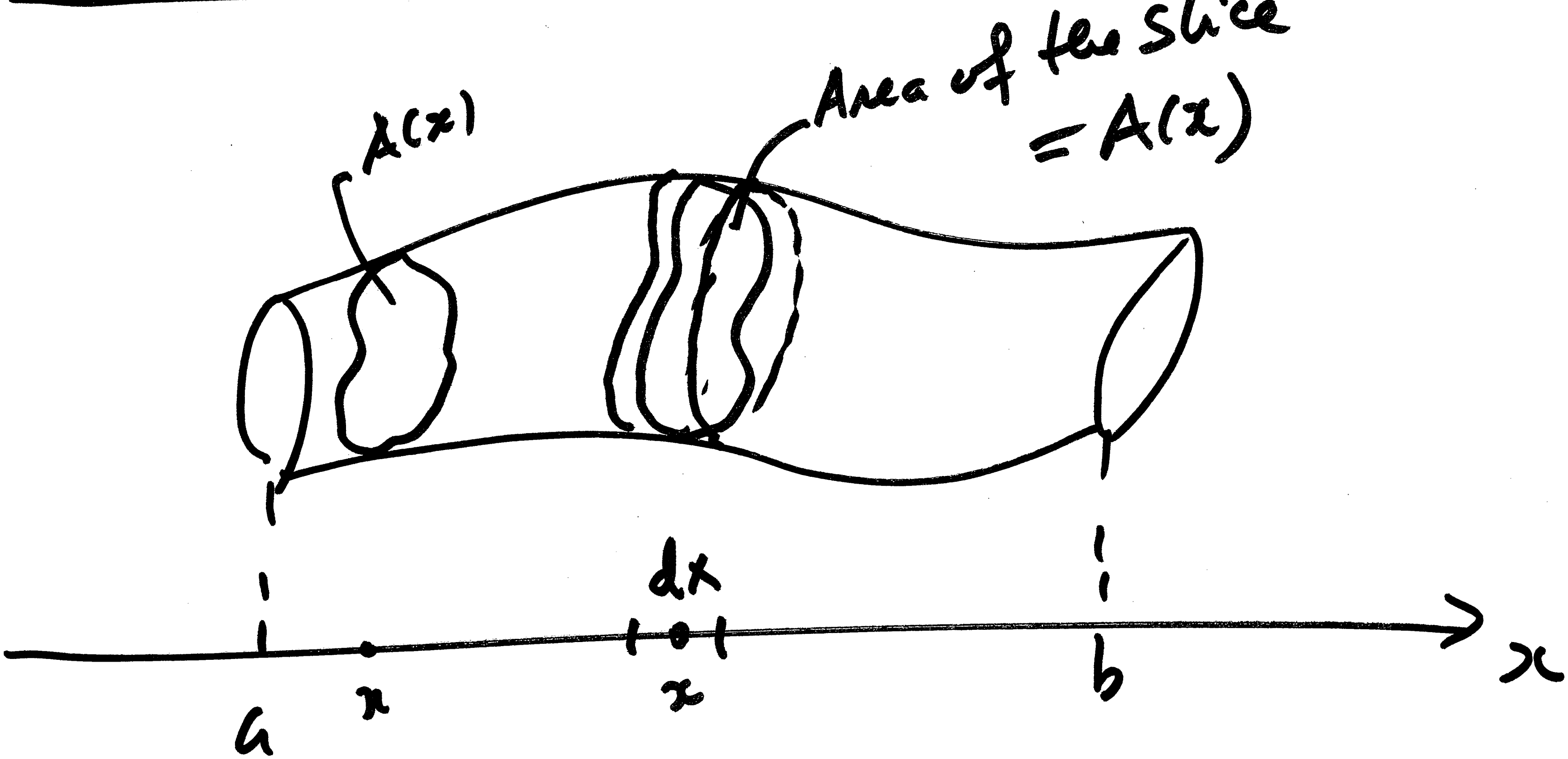
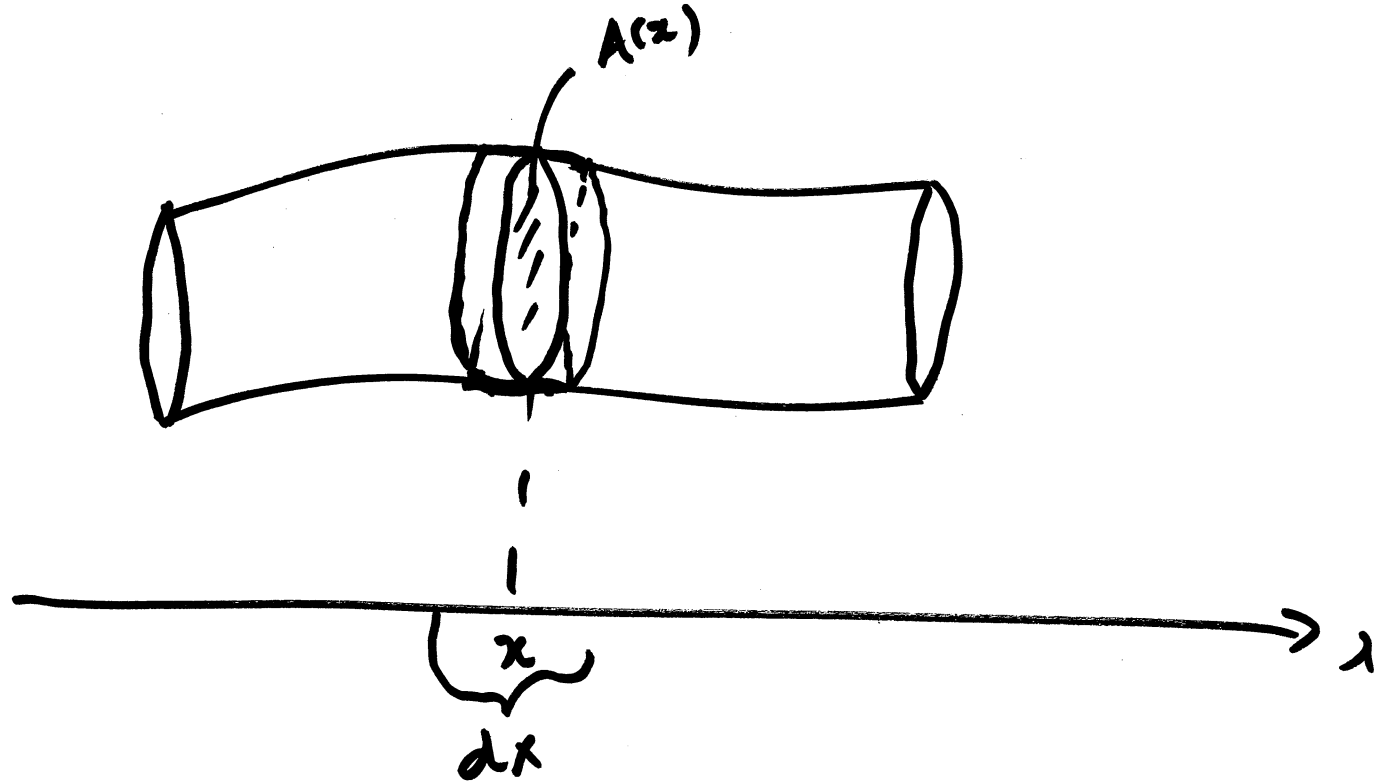
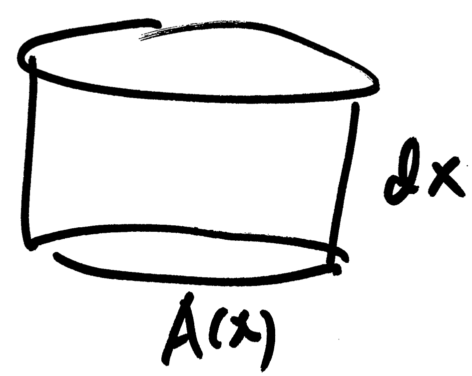
Lesson 6 Section 6.2

Volumes:



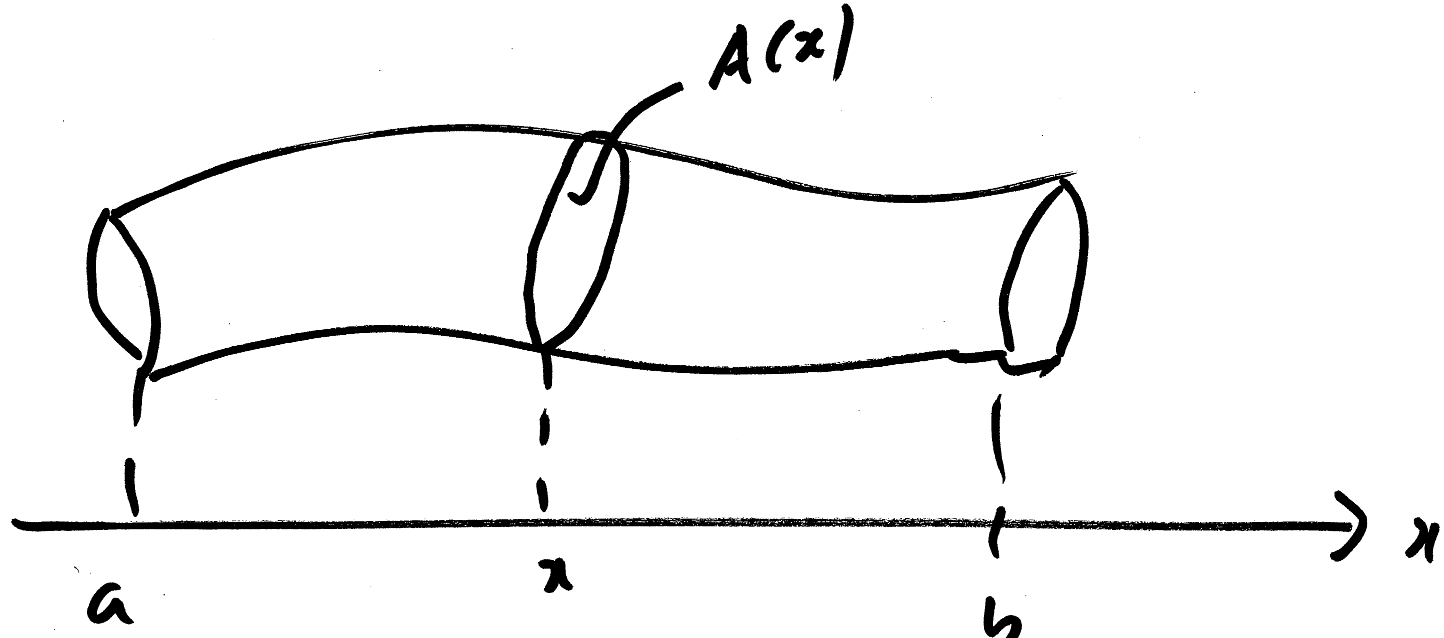
$$dV = A(x)\,dx$$





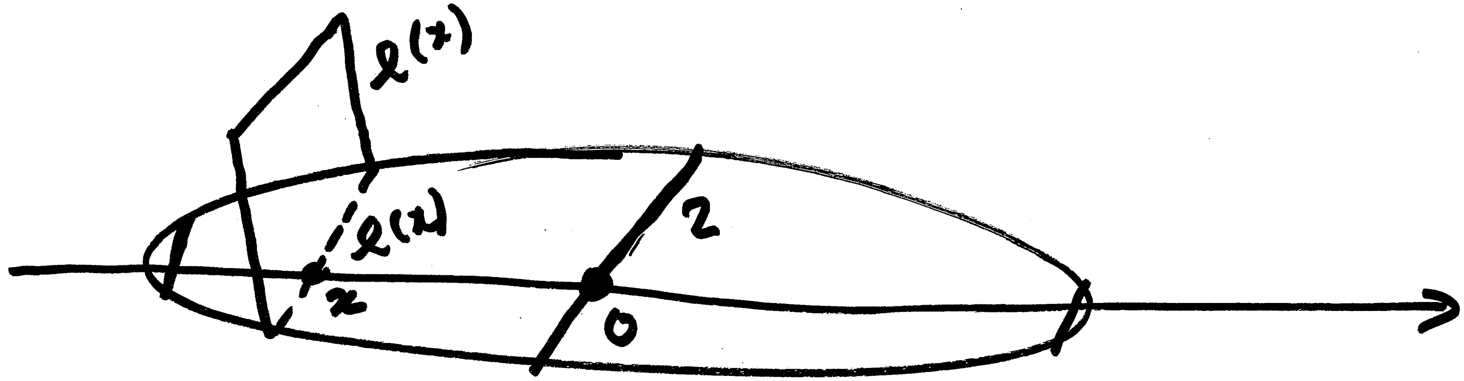
$$dV = A(x)\,dx$$

Volume of the Solid



$$V = \int_a^b A(x)\,dx.$$

Example 1: Find the volume of the solid

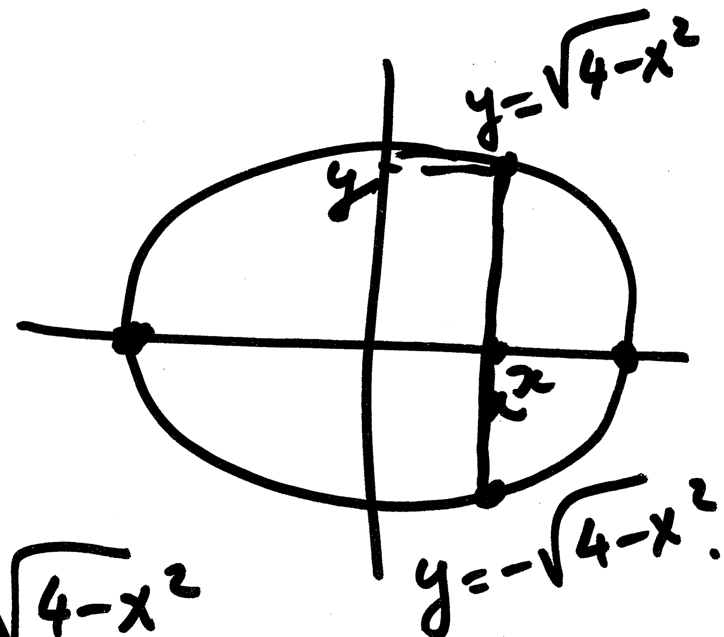


$$\boxed{A(x) = (\ell(x))^2.}$$

Circle: $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4-x^2}$, $y = -\sqrt{4-x^2}$



$$\ell(x) = 2\sqrt{4-x^2}.$$

$$\int_{-2}^{2} 4\,dx = 4x\Big|_{-2}^{2} = 4(2-(-2)) = 16$$

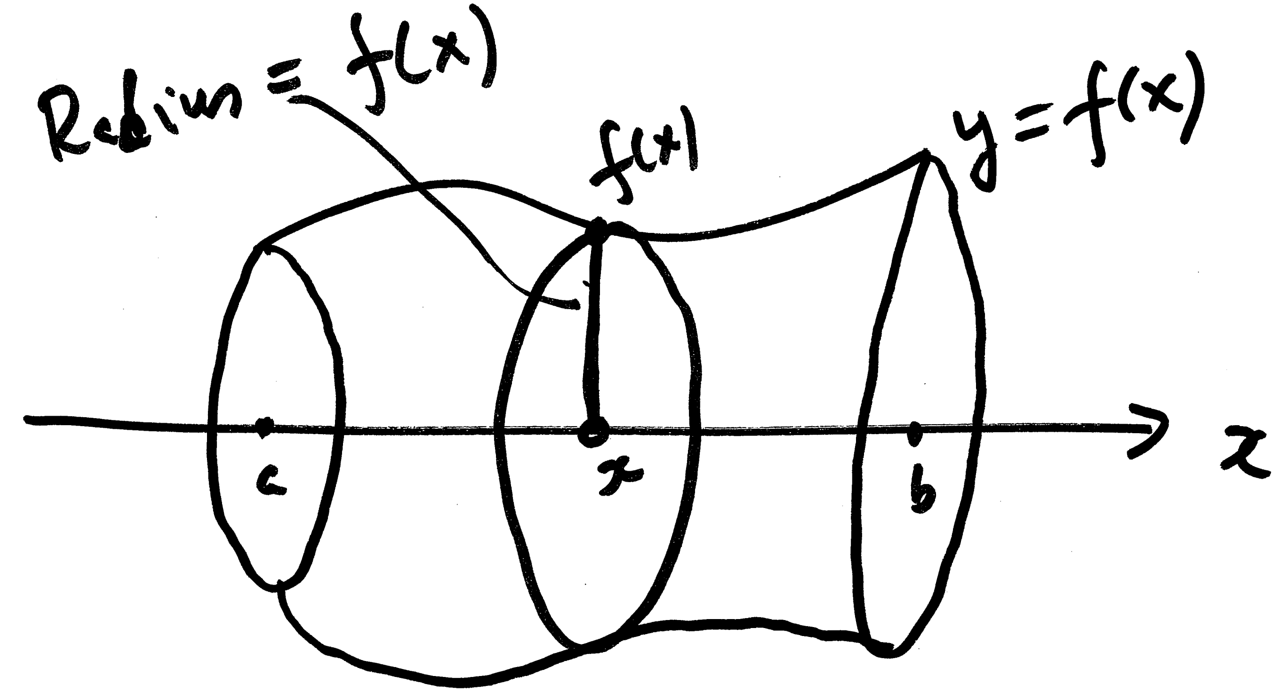
$$A(x) = 4(4-x^2)$$

$$V = \int_a^b A(x)\,dx = \int_{-2}^{2} 4(4-x^2)\,dx$$

$$= 4\int_{-2}^{2}(4-x^2)\,dx. = 4\left(16 - \frac{x^3}{3}\Big|_{-2}^{2}\right.$$

$$= 4\left(16 - \frac{16}{3}\right) = 64\left(1 - \frac{1}{3}\right) = \frac{128}{3}.$$

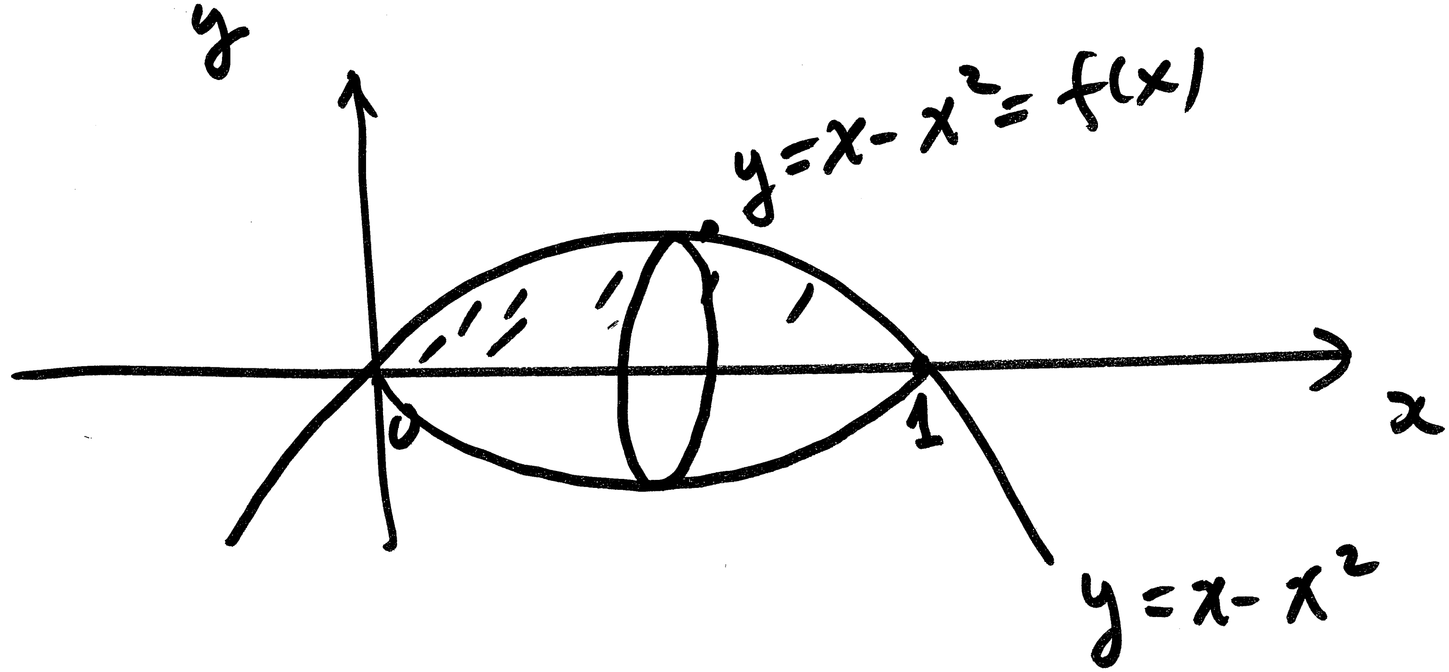
Solids of Revolution.



$$A(x) = \pi \cdot (f(x))^2$$

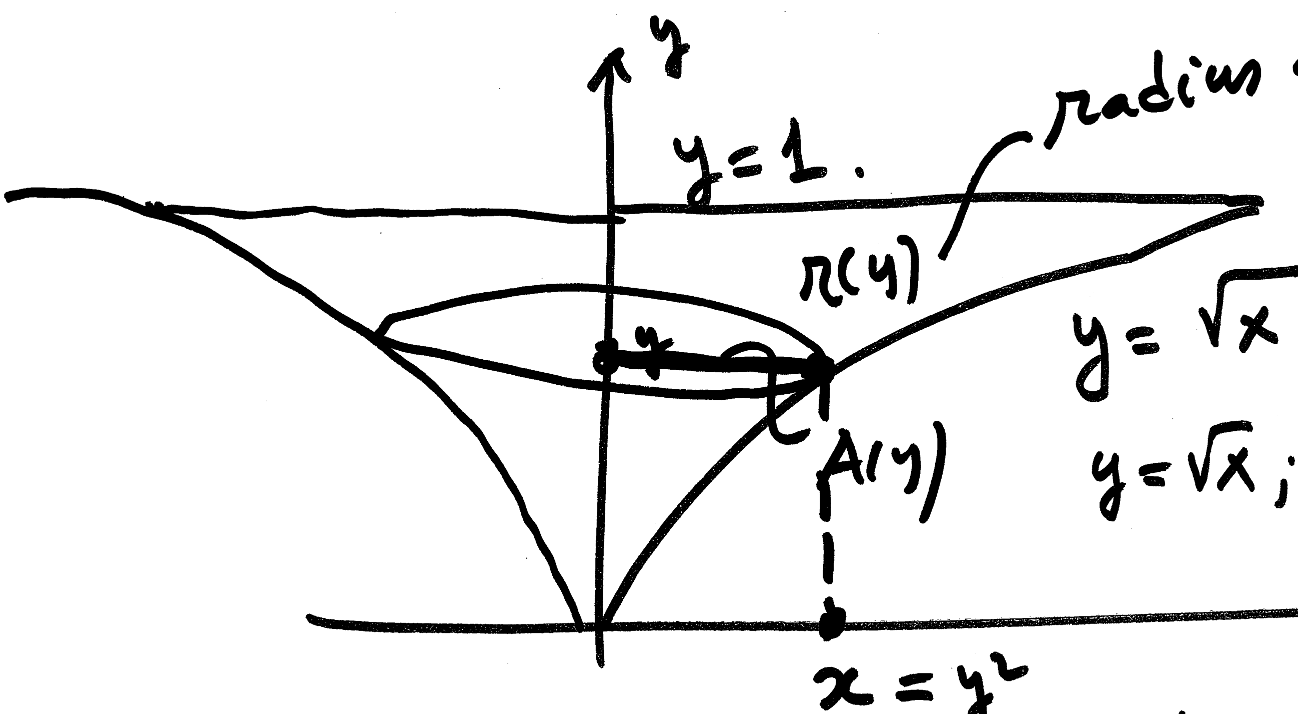
$$V = \int_a^b \pi (f(x))^2 \, dx$$

Ex: Rotate the region bounded by $y = x - x^2$ and the x-axis.



$$V = \int_0^1 \pi (x - x^2)^2 \, dx$$

$$V = \pi \int_0^1 (x^2 - 2x^3 + x^4)\, dx = \ldots$$



$A(y) = \pi (r(y))^2$

$y = \sqrt{x}$; $x = y^2$ $\qquad r(y) = y^2$

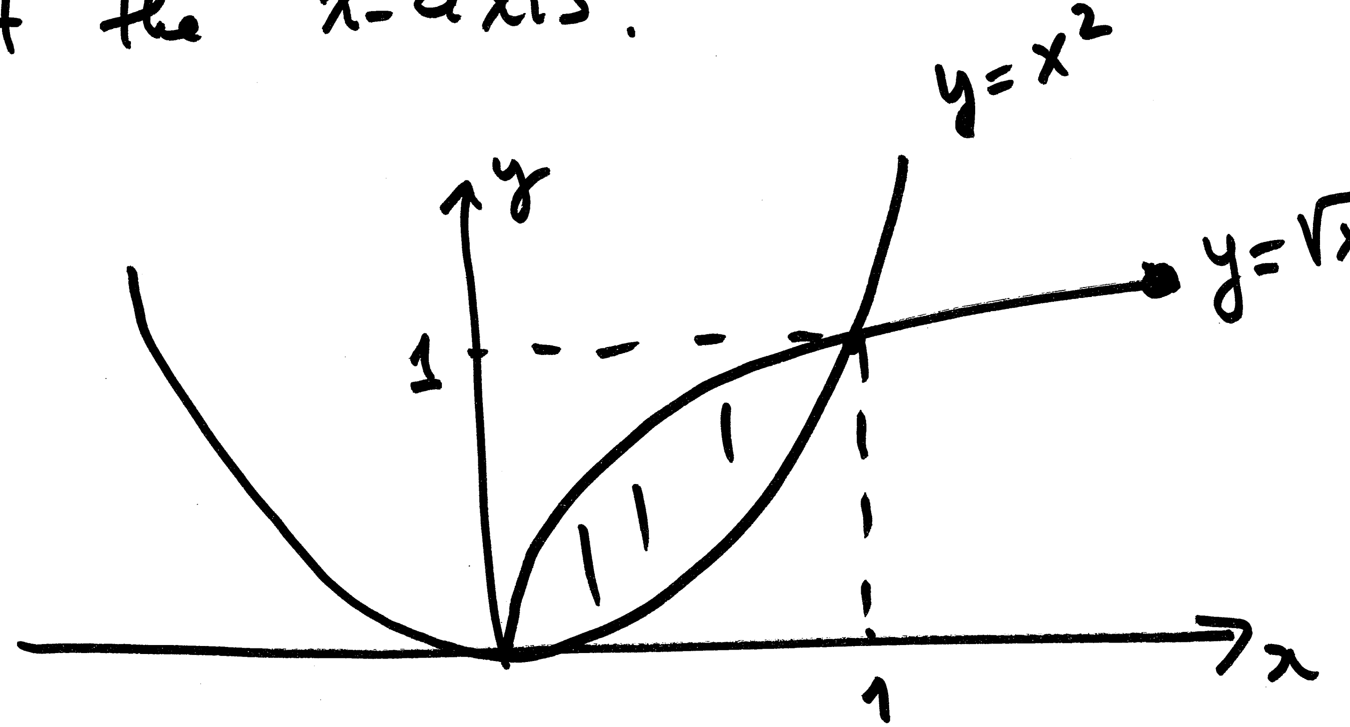
$$V = \int_0^1 A(y)\, dy = \int_0^1 \pi (y^2)^2\, dy = \int_0^1 \pi y^4\, dy$$

$$= \pi/5.$$

Rotate the region bounded by

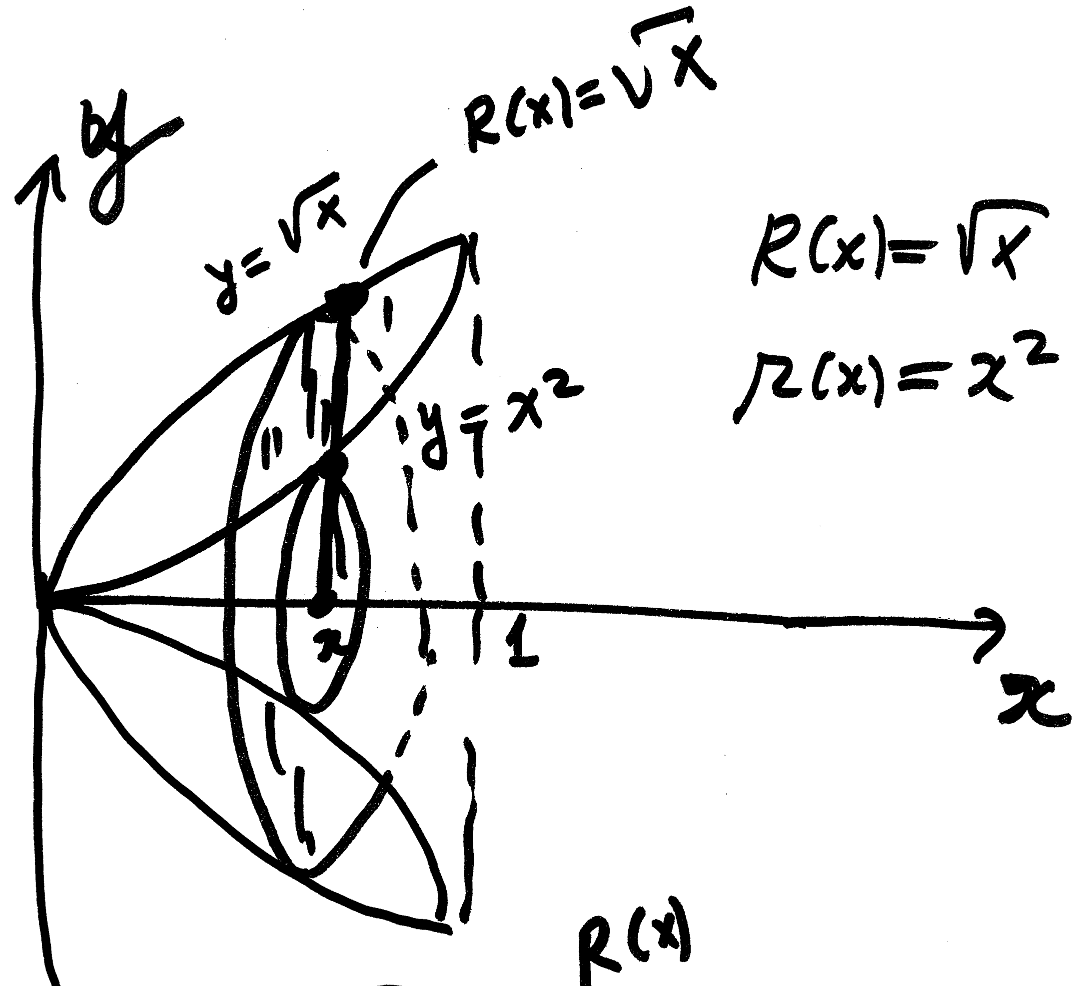
$y = x^2$ and $y = \sqrt{x}$

about the x-axis.



$x^2 = \sqrt{x}$; $x = 0$, $x = 1$

$$V = \int_0^1 A(x)\,dx$$

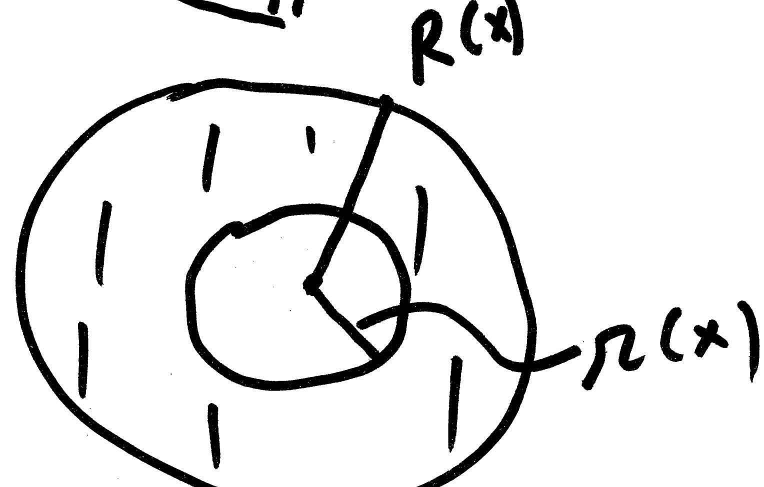


$R(x) = \sqrt{x}$

$r(x) = x^2$

<u>Cross-Section:</u>

$$A = \pi\left((R(x))^2 - (r(x))^2\right)$$

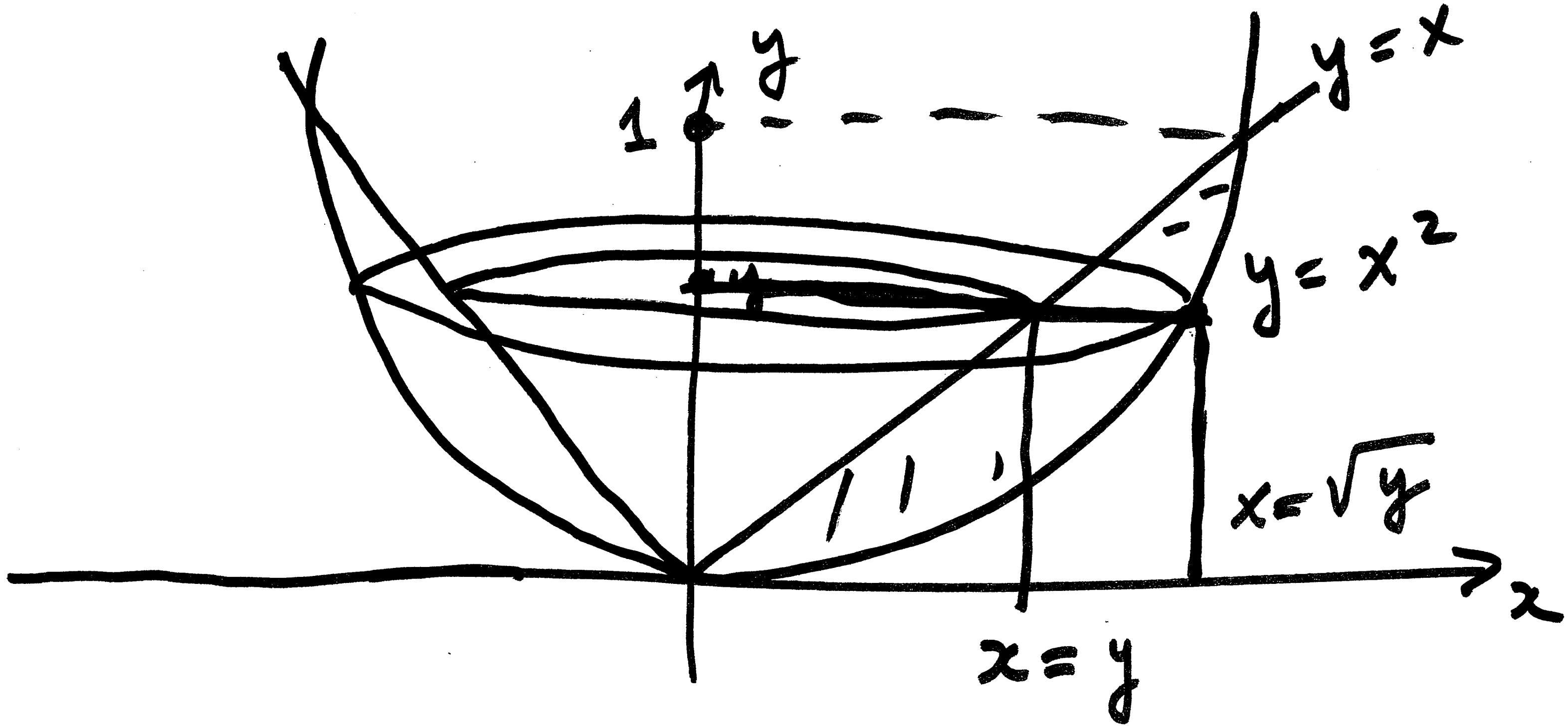


$$A(x) = \pi(x - x^4)$$

$$V = \pi \int_0^1 (x - x^4)\,dx = \pi\left(\tfrac{1}{2} - \tfrac{1}{5}\right)$$

$$= \tfrac{3\pi}{10}.$$

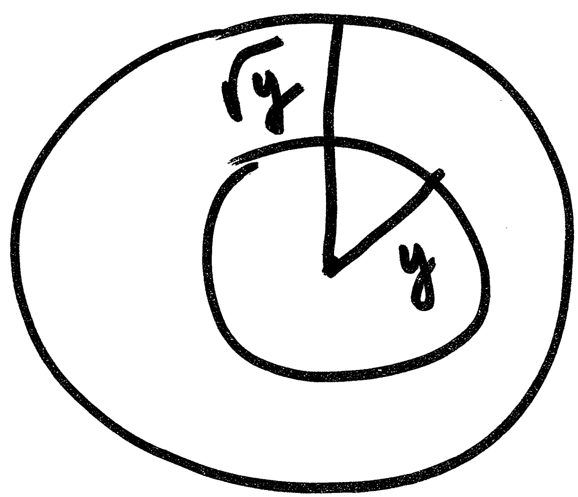
Rotate the Region bounded by $y = x$ and $y = x^2$ about the y-axis.



$y = x, \quad y = x^2$

$x = x^2 \quad x = 0$

$x = 1$



A

$$V = \int_0^1 A(y)\, dy$$

$$A(y) = \pi\left((\sqrt{y})^2 - (y)^2 \right)$$

$$= \pi\,(y - y^2)$$

$$V = \int_0^1 \pi (y - y^2)\, dy = \pi\left(\tfrac{1}{2} - \tfrac{1}{3}\right) = \tfrac{\pi}{6}.$$