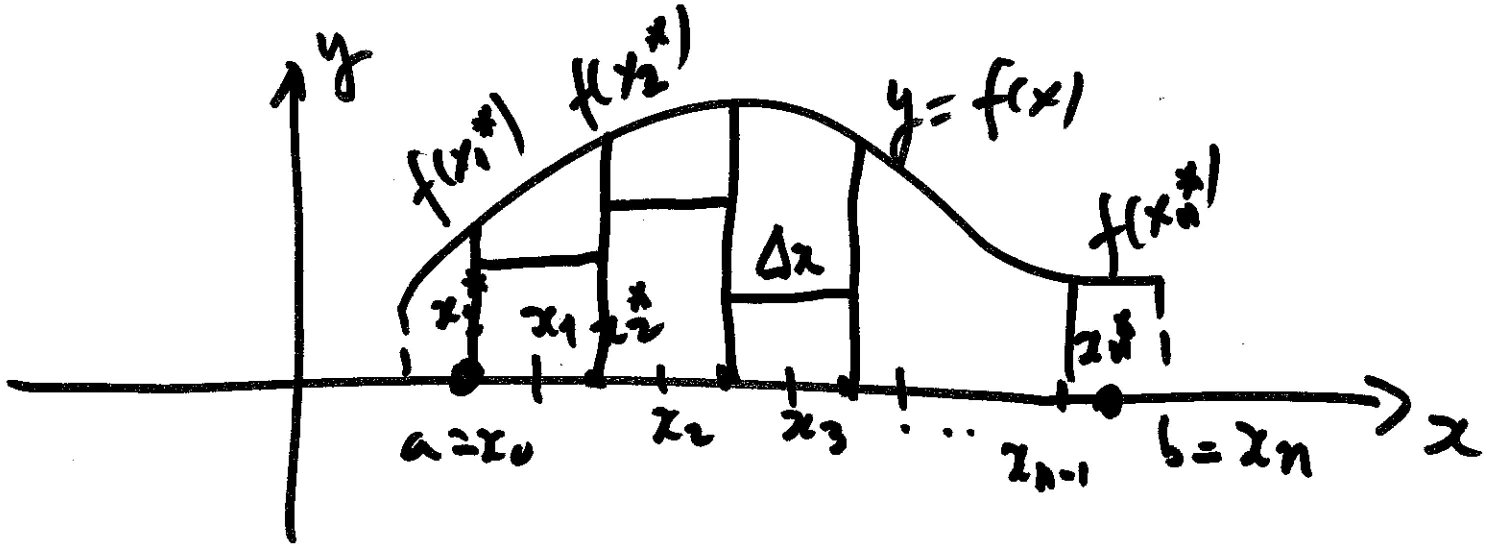
<u>LESSON 8</u> Sections 6.4 and 6.5.

<u>Exam 1</u>; Thursday Feb 2.
Elliott 8:00 P.M.
Covers: Lessons 1 to 8.

<u>Section 6.5</u> Average of a Function.

Given $\{y_1, y_2, \ldots, y_n\}$

$$\text{Average} = \bar{y} = \frac{y_1 + y_2 + \cdots + y_n}{n}$$



find the average of $f(x)$ on $[a, b]$

<u>Step 1</u>: Divide $[a, b]$ into n parts

Form the average of the values

$$f(x_1^*),\ f(x_2^*) \ldots,\ f(x_n^*)$$

$$\text{Average} = \frac{f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)}{n}$$

$$\Delta x = \frac{b-a}{n}$$

$$\cancel{n = \frac{\Delta x}{b-a}} \qquad n = \frac{b-a}{\Delta x}$$

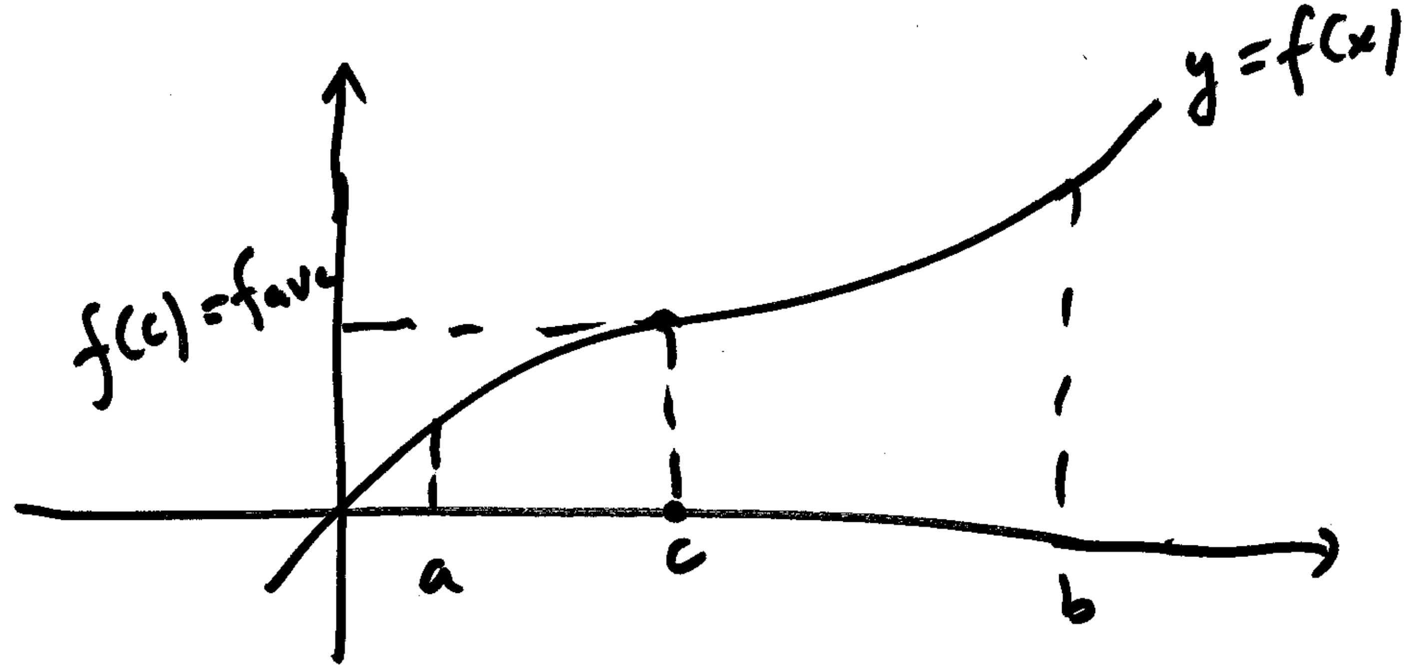
$$\text{Average} = \left(\frac{f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)}{(b-a)} \right) \Delta x$$

$$f_{ave} = \lim_{n\to\infty} \frac{1}{b-a} \underbrace{\left(f(x_1^*) + f(x_2^*) + \dots + f(x_n^*) \right) \Delta x}_{= \int_a^b f(x)\,dx}$$

$$= \frac{1}{b-a} \int_a^b f(x)\,dx.$$

$$\boxed{f_{ave} = \frac{1}{b-a} \int_a^b f(x)\,dx}$$

<u>The mean value theorem</u> If $f(x)$ is continuous on $[a,b]$



$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

There exists c in $[a,b]$ such that $f(c) = f_{ave}$.

Example: Find the average of

$$f(x) = x^2 - x + 1 \quad \text{on } [-1, 1].$$

$b = 1, \; a = -1$

$$f_{ave} = \frac{1}{2} \int_{-1}^{1} (x^2 - x + 1)\,dx$$

$$= \frac{1}{2} \left(\frac{x^3}{3} - \frac{x^2}{2} + x \right) \Big|_{-1}^{1}$$

$$= \frac{1}{2} \left\{ \left(\frac{1}{3} - \frac{1}{2} + 1 \right) - \left(-\frac{1}{3} - \frac{1}{2} - 1 \right) \right\}$$

$$= \tfrac{1}{2} \left(\tfrac{2}{3} + 2 \right) = 1 + \tfrac{1}{3} = \tfrac{4}{3}.$$

$\{1, \underline{\underline{2}}, 3\}$, $\quad \text{ave} = \dfrac{1+2+3}{3} = \underline{\underline{2}}.$

$\{1, 2, 3, 4\}$, $\quad \text{ave} = \dfrac{10}{4} = \underline{\underline{5/2}}$

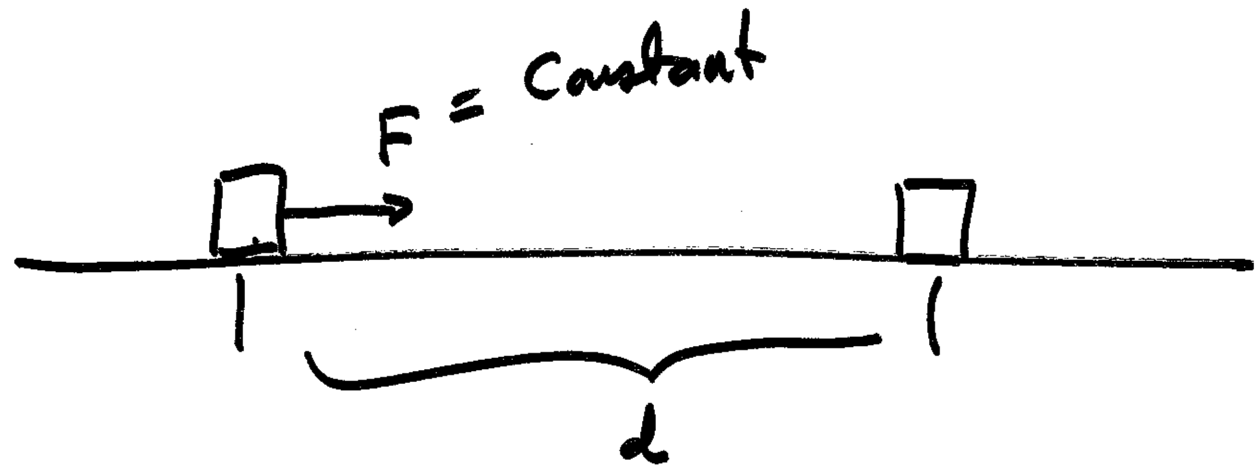
<u>Ex:</u> $f(x) = x^2 \quad x \text{ in } [0,1]$

$$f_{ave} = \frac{1}{1}\int_0^1 x^2\,dx = \int_0^1 x^2\,dx = \frac{1}{3}$$

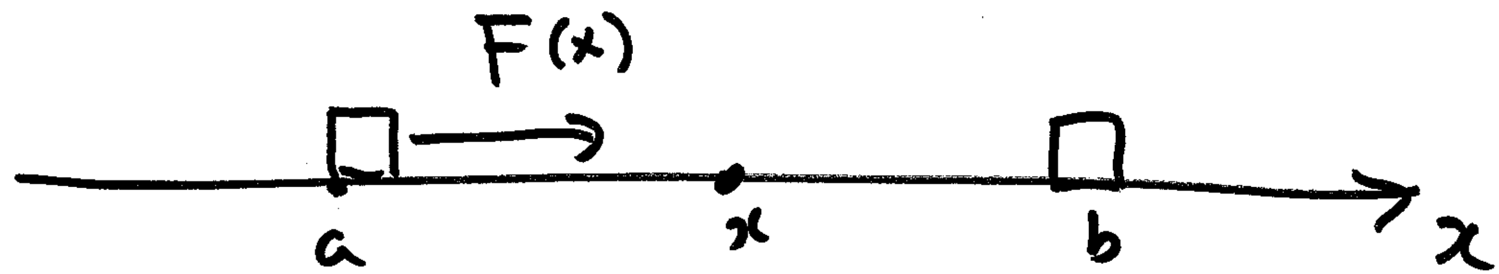
Find c such that $f(c) = 1/3$

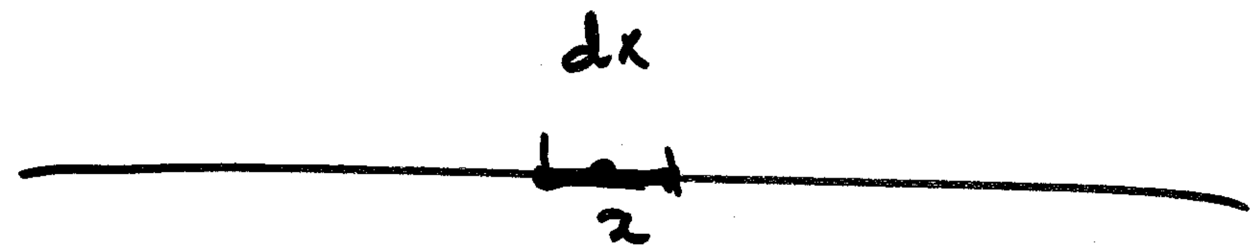
$f(c) = c^2 = 1/3 \; ; \quad \boxed{c = \dfrac{1}{\sqrt{3}}}.$

Section 6.4 Work

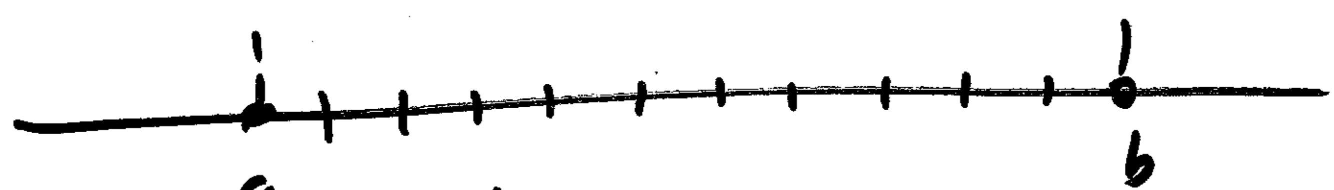


$$\text{Work} = W = F \cdot d.$$





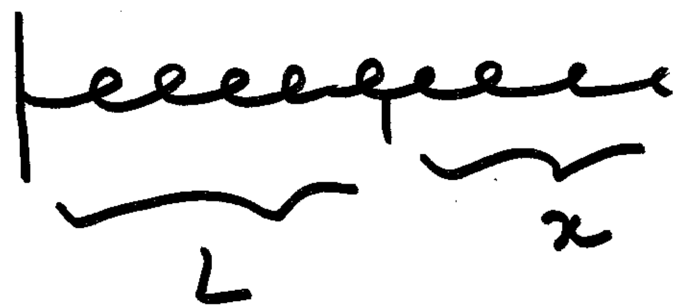
$$dW = F(x)\,dx$$



$$W = \int_a^b F(x)\,dx\,.$$

Example: Hooke's Law

$L =$ Natural length



$$\underline{F(x) = k\,x}$$ $k =$ Spring constant.

Example: A force of 10 lbs is required to hold a spring stretched 4 in beyond its natural length.

How much work is done stretching it from its natural length to 6 in beyond its natural length?

Solution: $F = kx$. $F = 10$ lbs, $x = 4$ in.

$x = \frac{1}{3}$ ft.

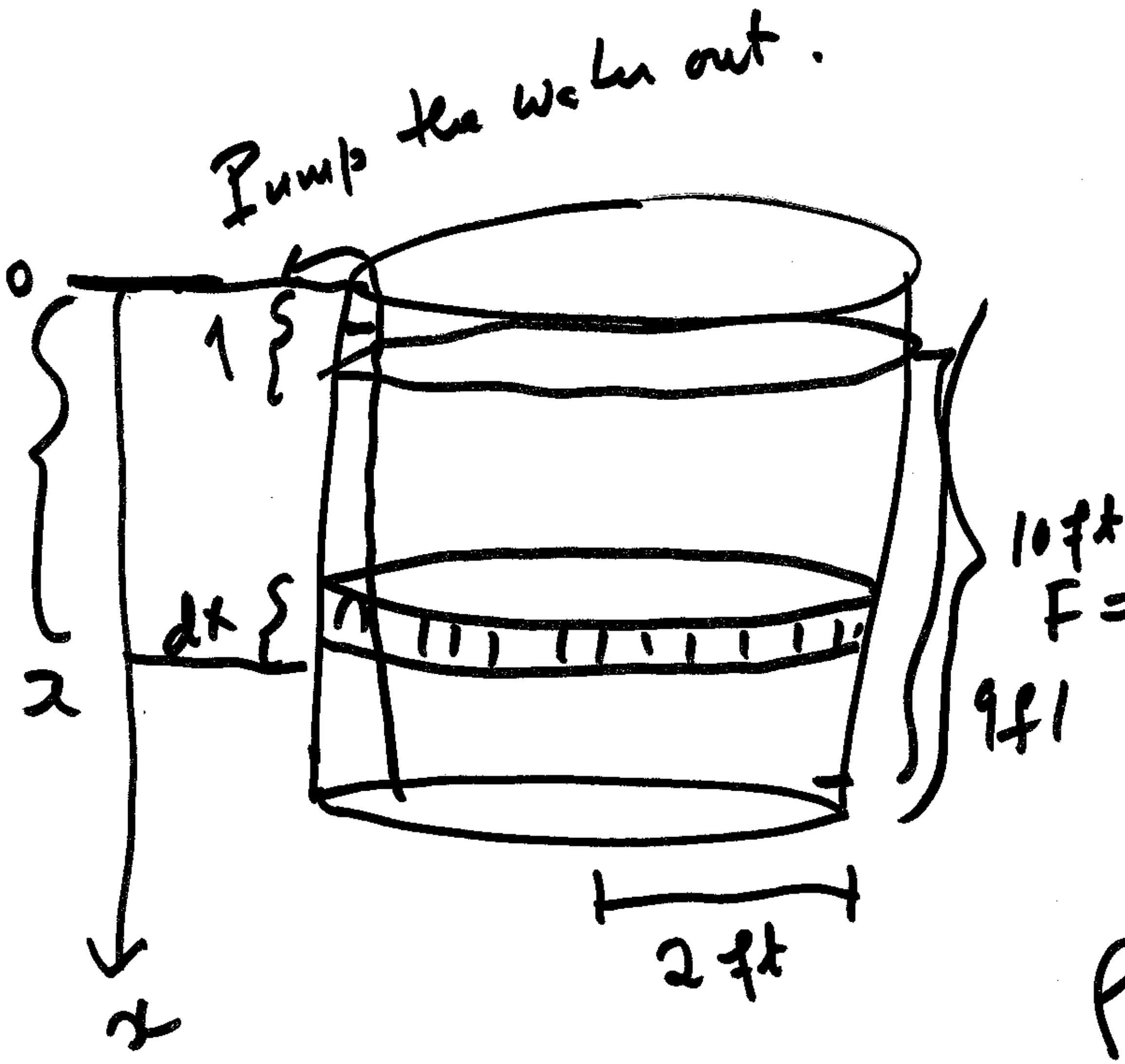
$10 = k\,\frac{1}{3}$, $\boxed{k = 30}$.

$$\text{Work} = \int_a^b F(x)\,dx$$

$$W = \int_0^{1/2} 30x\,dx = 30\,\frac{x^2}{2}\Big|_0^{1/2}$$

$$= \frac{15}{4}\ \text{ft-lb.}$$

Work to empty a tank



The tank is full of water.

F = Weight of the slab $= \underbrace{\rho}_{\text{density}} \cdot$ Volume

$\rho = 62.5\ lb/ft^3$.

Volume $= \pi \cdot R^2 \cdot dx = 4\pi\, dx$.

$F = 4\pi\rho \, dx$

dW = work to move the slab

$= 4\pi\rho \, x \, dx$.

Total work $= 4\pi\rho \int_0^{10} x \, dx$