Lesson 9 Section 7.1

Integration By Parts

Exam 1: Thursday Feb 2, 8:00 - 9:00
Elliott.
Lessons 1 to 8.

Review Problems - Course Web page.

Lesson 9:

Integration By Parts =

Converse of the product Rule

Product Rule:

$$\big(u(x)\,v(x)\big)' = u'(x)\,v(x) + u(x)\,v'(x)$$

$$\int \left(u(x)\, v(x)\right)' \, dx =$$

$$= \int u'(x)\, v(x)\, dx + \int u(x)\, v'(x)\, dx + C$$

$$\boxed{u(x)\, v(x) = \int \underbrace{u'(x)}\, v(x)\, \underbrace{dx} + \int \underline{\underline{u(x)}}\, \underbrace{v'(x)\, dx}_{dv} + C}$$

$$u'(x)\, dx = du$$

$$v'(x)\, dx = dv$$

$$u(x)v(x) = \int u\,dv + \int v\,du$$

$$\boxed{\int u\,dv = \underbrace{uv}_{} - \int v\,du}$$

<u>Examples:</u> Compute $\int x \sin x \, dx$.

<u>Two choices:</u> (1) $u = x$, $dV = \sin x \, dx$

X (2) $u = \sin x$; $dV = x \, dx$.

Start with

$$\underline{\underline{u(x)}} = \sin x, \quad \underline{\underline{dV}} = x\,dx$$

$$\int x \sin x \, dx = \int \underbrace{\sin x}_{u} \; \underbrace{x\,dx}_{dv}$$

$$du = \cos x \, dx; \quad V = \int x\,dx = \frac{x^2}{2}.$$

$$\int \underline{\underline{x}} \sin x \, dx = \frac{x^2}{2} \sin x - \int \underline{\underline{\frac{x^2}{2}}} \cdot \cos x \, dx$$

$$\int \underbrace{x}_{u} \underbrace{\sin x \, dx}_{dv} = uv - \int v \, du$$

$u = \underline{\underline{x}}, \quad dv = \sin x \, dx.$

$du = dx \quad v = \int \sin x \, dx = -\cos x$

$$\int x \sin x \, dx = -x \cos x - \int -\cos x \, dx$$

$$= -x \cos x + \int \cos x \, dx$$

$$= -x \cos x + \sin x + C.$$

$$\int \underline{\underline{x}}\,\sin x \, dx = \underline{-x\cos x + \sin x +} C.$$

<u>Check my work:</u>

$$(-x\cos x + \sin x + C)' =$$

$$= -\cos x + x\sin x + \cos x = x\sin x$$

$$\int x \ln x \, dx = \int \underbrace{\ln x}_{u} \, \underbrace{x\,dx}_{dv}$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

If I pick $dv = \ln x \, dx$, $v = \int \ln x \, dx$ Not good

Pick $u = \ln x$, $dv = x\,dx$.

$du = \frac{1}{x}\,dx$; $v = x^2/2$.

$$\int \underline{\ln x} \; x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^{\cancel{2}}}{2} \cdot \frac{dx}{\cancel{x}}$$

$$= \frac{x^2}{2} \ln x - \int \frac{x}{2} \, dx$$

$$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + c.$$

<u>Check my work.</u>

$$\left(\frac{x^2}{2} \ln x - \frac{x^2}{4} + c \right)' = x \ln x + \cancel{\frac{x}{2}} - \cancel{\frac{x}{2}}$$

$$= x \ln x$$

$$\int \underline{\underline{x}}^2 \cos x \, dx$$

$$\int u \, dv = uv - \int v \, du$$

$$u = x^2, \quad dv = \cos x \, dx$$

$$du = 2x \, dx, \quad v = \sin x$$

$$\int \underbrace{x^2}_{u} \underbrace{\cos x \, dx}_{dv} = x^2 \sin x - 2\int x \sin x \, dx$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + C$$

$$\int \underbrace{\ln x}_{u}\, \underbrace{dx}_{dv} = x \ln x - \int x \cdot \frac{dx}{x}$$

$$= x \ln x - \int dx$$

$u = \ln x, \quad dv = dx$

$$= x \ln x - x + C.$$

$du = \frac{1}{x}\, dx; \quad v = x$

$$\boxed{\int \ln x \, dx = x \ln x - x + C}$$

$$\tan^{-1} x = \arctan x.$$

$$\boxed{\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}.}$$

$$\int \underbrace{\tan^{-1} x}_{u} \, \underbrace{dx}_{dv} = ? = uv - \int v \, du$$

$$u = \tan^{-1} x \; ; \quad dv = dx.$$

$$du = \frac{1}{1+x^2} dx \; ; \quad v = x.$$

$$\int \tan^{-1} x \, dx = x \tan^{-1} x - \int \frac{x}{1+x^2} \, dx$$

$$\int \frac{x}{1+x^2} \, dx = \frac{1}{2} \int \frac{2x \, dx}{1+x^2}$$

$$w = 1 + x^2 \, ; \quad dw = 2x \, dx$$

$$\int \frac{2x}{1+x^2} \, dx = \int \frac{dw}{w} = \ln w + c.$$

So $$\int \frac{x}{1+x^2} \, dx = \frac{1}{2} \ln (1+x^2) + c.$$

$$\int \tan^{-1} x \, dx = x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + C.$$