Lesson 15: Sections 7.6 and 7.7.

Median 80

Average 74.5

Grade Curve.

A - 88 (Missed 1 question)

B - 77 (2)

C - 66 (3)

D - 55 (4)

Use of a Table of Integrals

$$\int \frac{\sec^2 x}{1 - \tan^2 x}\, dx$$

Set $u = \tan x$, $du = \sec^2 x\, dx$

$$\int \frac{\sec^2 x\, dx}{1 - \tan^2 x} = \int \frac{du}{1-u^2}$$

$$= \frac{1}{2} \ln \left| \frac{u+1}{u-1} \right| + C$$

$$\int \frac{\sec^2 x}{1 - \tan^2 x}\, dx = \frac{1}{2} \ln \left| \frac{1+\tan x}{-1+\tan x} \right| + C$$

$$\int \frac{\ln(10+\sqrt{x})}{\sqrt{x}}\,dx$$

$$u = 10+\sqrt{x} = 10 + x^{1/2}$$

$$du = \tfrac{1}{2} x^{-1/2}\,dx$$

$$= \frac{1}{2}\,\frac{dx}{\sqrt{x}}$$

$$\int \frac{\ln(10+\sqrt{x})\,dx}{\sqrt{x}} = 2\int \ln u\,du$$

$$= 2(u\ln u - u) + c$$

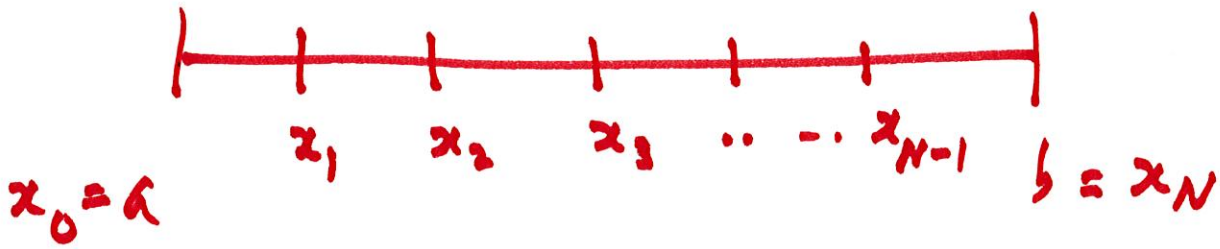
$$\int \frac{\ln(10+\sqrt{x})}{\sqrt{x}}\,dx = 2\Big((10+\sqrt{x})\ln(10+\sqrt{x}) - (10+\sqrt{x})\Big) + C.$$

Approximate Integration

<u>Task:</u> Write a computer program to compute

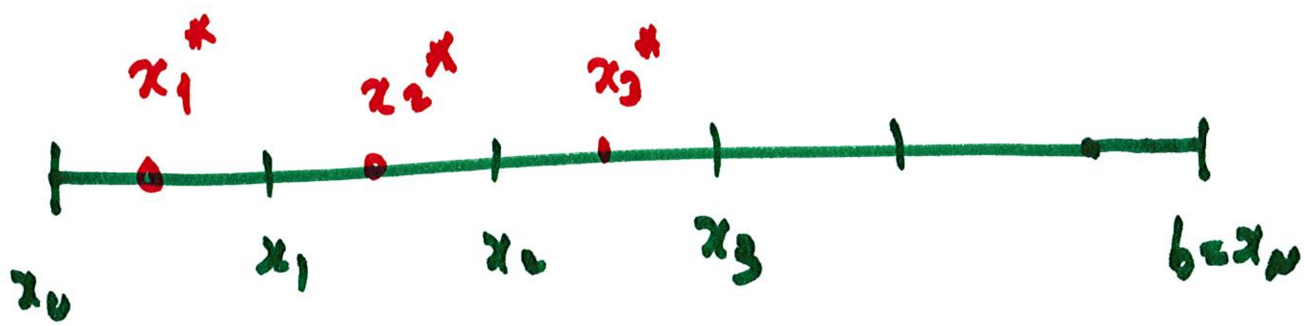
$$\int_a^b f(x)\,dx.$$

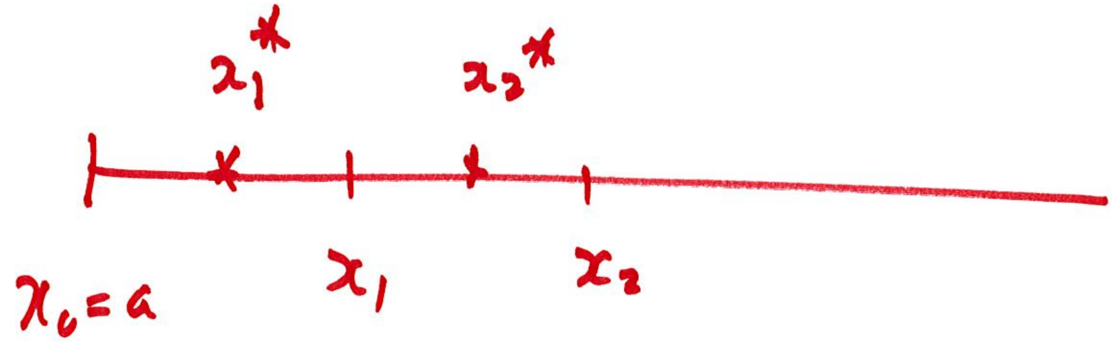
<u>Step 1</u>: Divide $[a, b]$ into N equal parts.



$$\Delta x = \frac{b-a}{N}$$

<u>Step 2</u>: Inside each interval pick x_j^* in $[x_{j-1}, x_j]$



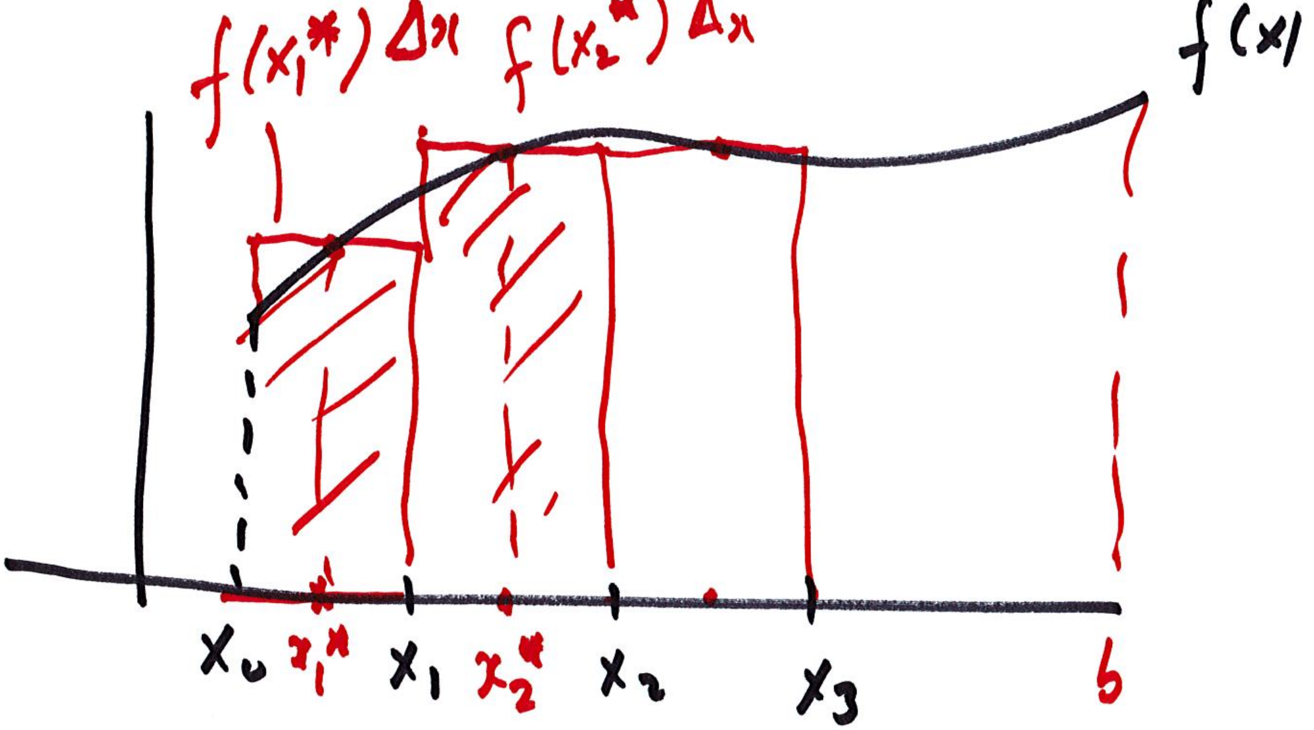


When x_j^* is the mid point of the interval.

Thus approximation is called the mid point approximation.

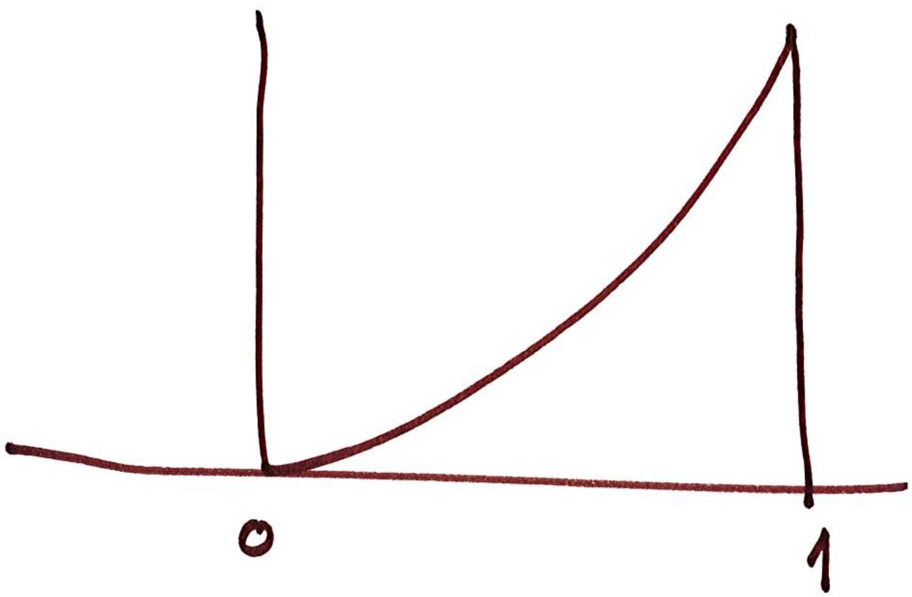
Step 3 :

$$S_N = f(x_1^*)\Delta x + f(x_1^*)\Delta x + f(x_3^*)\Delta x + \dots + f(x_{N-1}^*)\Delta x.$$

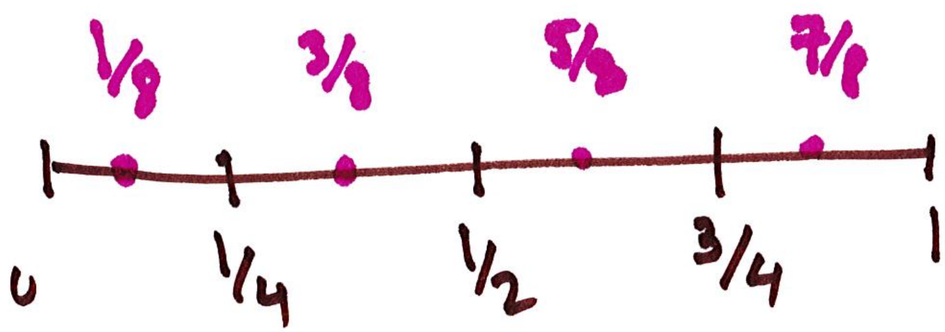


$$\int_a^b f(x)\,dx$$

Example: Use the midpoint approximation with $N = 4$ to estimate $\int_0^1 x^2\,dx = \frac{1}{3}$.



Step 1:



$$\Delta x = \frac{1-0}{4} = 1/4.$$

Step 3 :

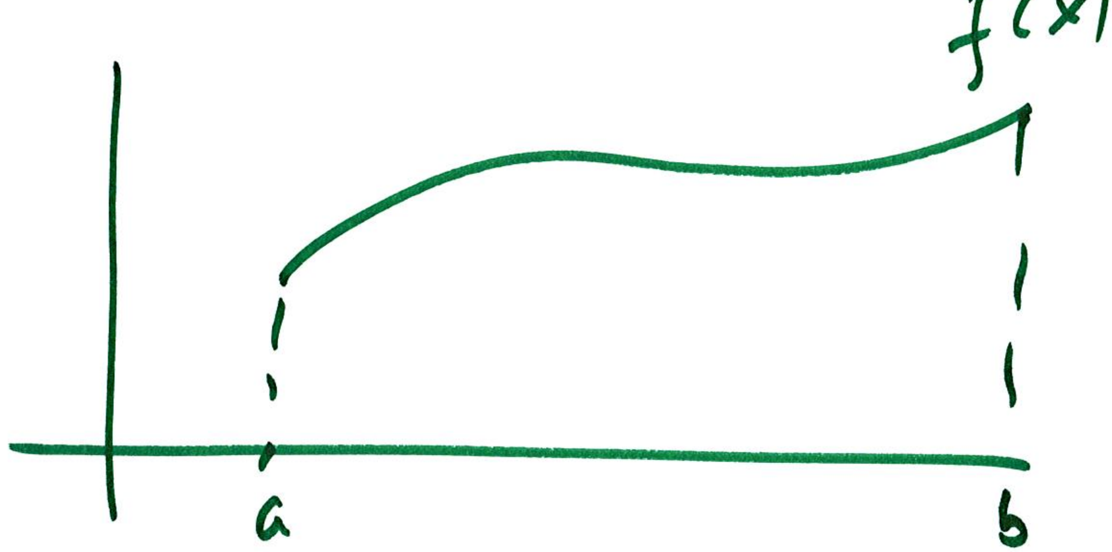
$$S_4 = f(x_1^*)\,\tfrac{1}{4} + f(x_2^*)\,\tfrac{1}{4} + f(x_3^*)\,\tfrac{1}{4} + f(x_4^*)\,\tfrac{1}{4}$$

$$= \tfrac{1}{4}\left(\left(\tfrac{1}{8}\right)^2 + \left(\tfrac{3}{8}\right)^2 + \left(\tfrac{5}{8}\right)^2 + \left(\tfrac{7}{8}\right)^2\right)$$

$$= \frac{1}{4} \cdot \frac{1}{64}\left(1 + 9 + 25 + 49\right)$$

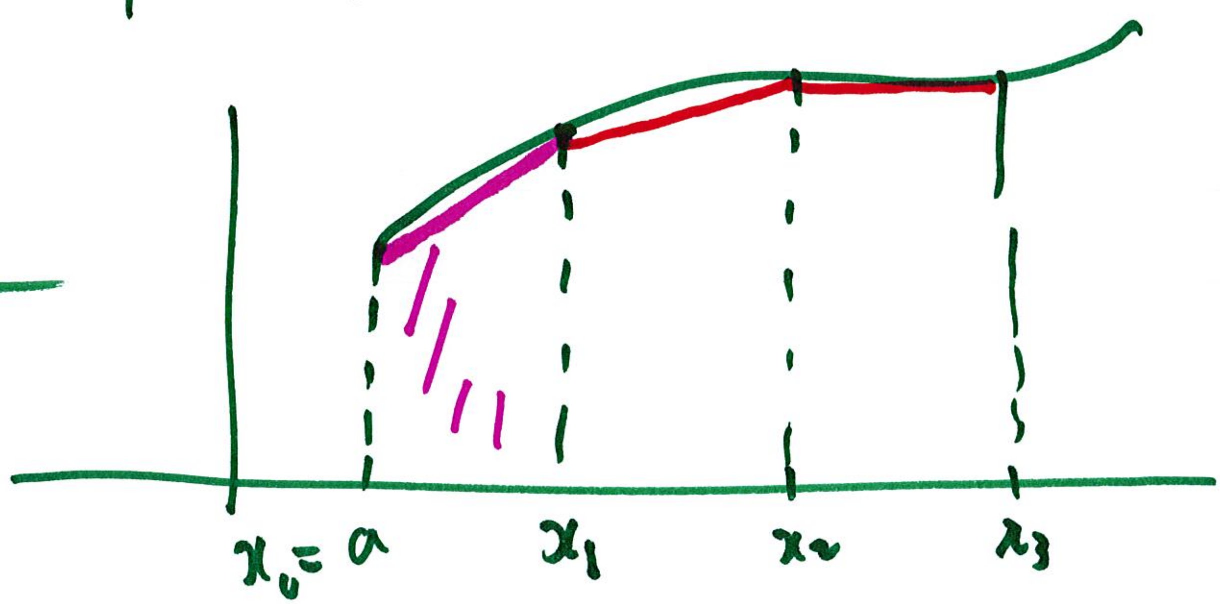
$$= \frac{1}{4} \cdot \frac{1}{64} \cdot 84 = \frac{21}{64}$$

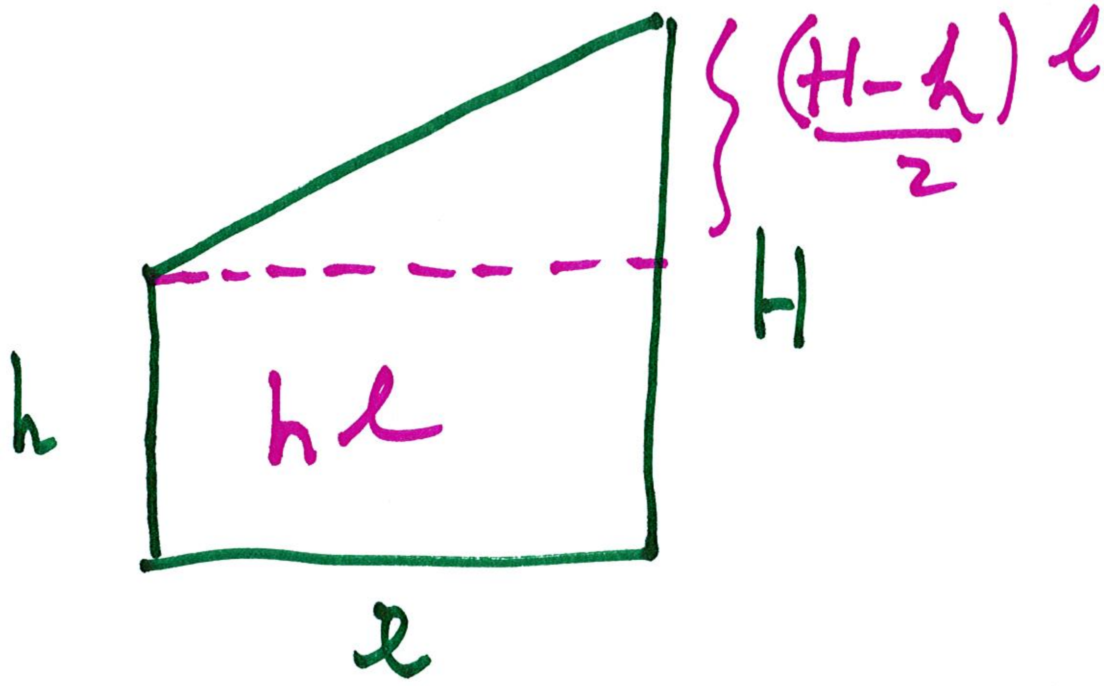
The Trapezoidal Rule



Step 1: Divide [a, b] into N equal parts

Step 2

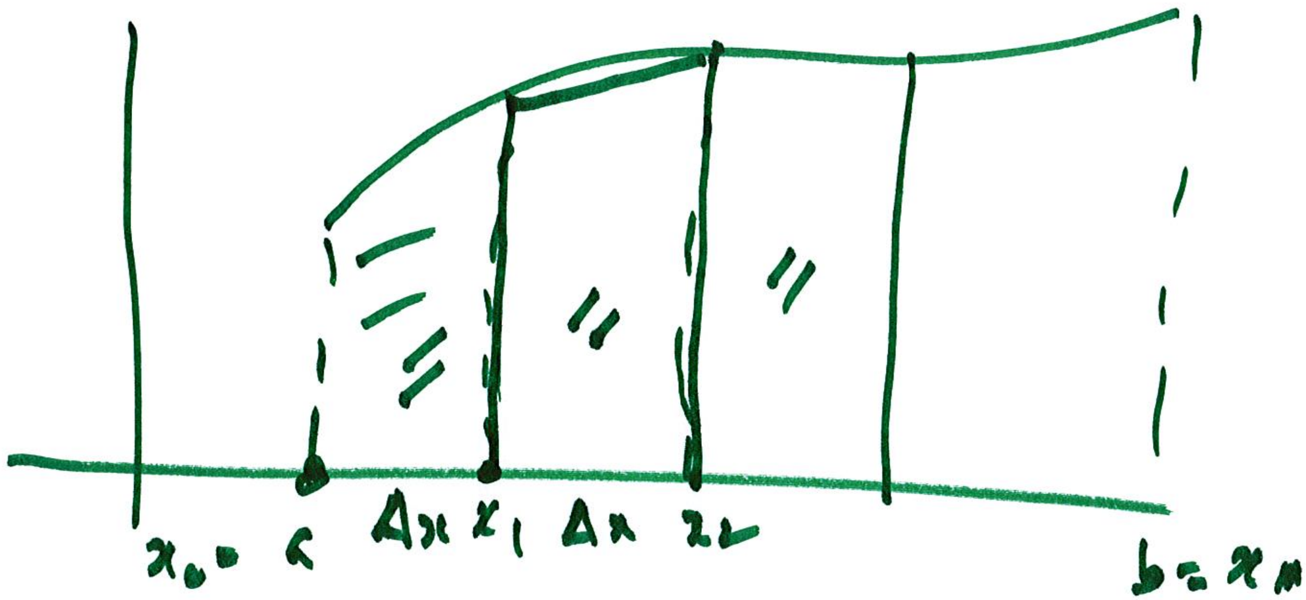




$$h\ell + \left(\frac{H-h}{2}\right)\ell$$

$$= \ell\left[h + \frac{H-h}{2}\right]$$

$$= \ell\left[\frac{h+H}{2}\right]$$



$$\left(\frac{f(x_1)+f(x_0)}{2}\right)\Delta x \;+$$

$$\left(\frac{f(x_2)+f(x_1)}{2}\right)\Delta x \;+\dots$$

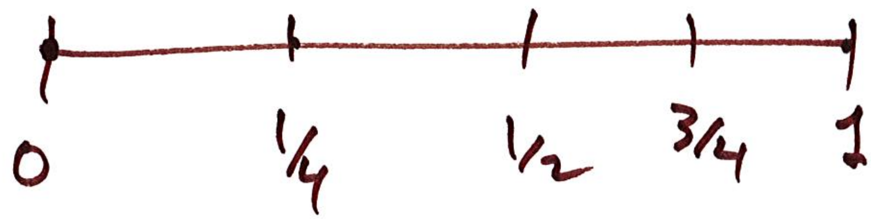
$$+\;\left(\frac{f(x_{N-1})+f(x_N)}{2}\right)\Delta x$$

$$= \Delta x\left(\frac{f(x_0)}{2} + f(x_1)+f(x_2)+\dots+f(x_{N-1}) + \frac{f(x_N)}{2}\right).$$

$$S_N = \Delta x \left(\frac{f(x_0)}{2} + f(x_1) + f(x_2) + \dots \right.$$
$$\left. + f(x_{N-1}) + \frac{f(x_N)}{2} \right)$$

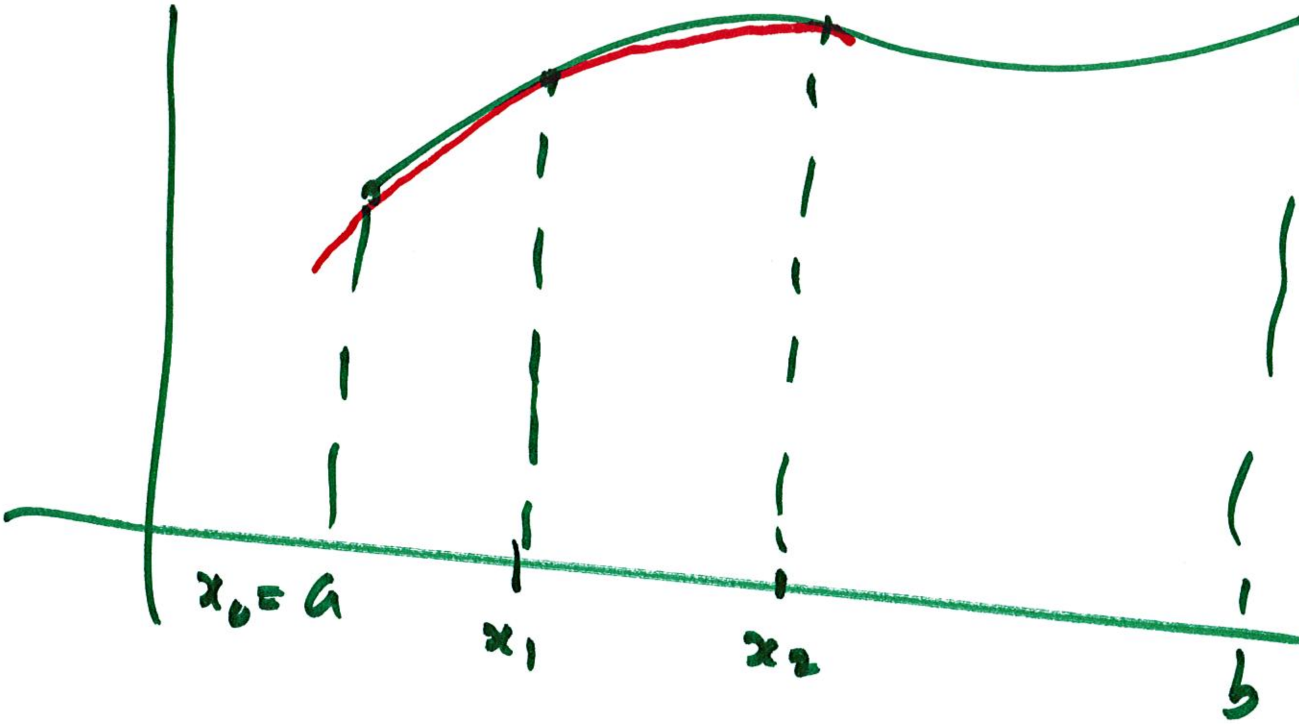
<u>Ex:</u> $\int_0^1 x^2 \, dx$

<u>$N=4$</u>:



$$S_4 = \frac{1}{4}\left(0 + \frac{1}{16} + \frac{1}{4} + \frac{9}{16} + \frac{1}{2} \right)$$

Simpson's Rule



N equal parts, <u>N even</u>

A

$$S_N = \frac{\Delta x}{3}\Big(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 4f(x_{N-1}) + f(x_N) \Big).$$