Lesson 19 Section 11.1

Sequences

Function: A function

f where domain is

$\{1, 2, 3, 4, 5 \ldots\}$

$f(1),\ f(2),\ f(3), \ldots$

Example: $f(n) = \dfrac{1}{n}$

$n = 1, 2, 3, \ldots$

$$a_n = f(n)$$

<u>Ex:</u> $a_1 = 1,\ a_2 = 1,$

$$a_{n+1} = a_n + a_{n-1}$$

$\underline{n=2}$ $a_3 = a_2 + a_1 = 2$

$n=3$ $a_4 = a_3 + a_2 = 4$

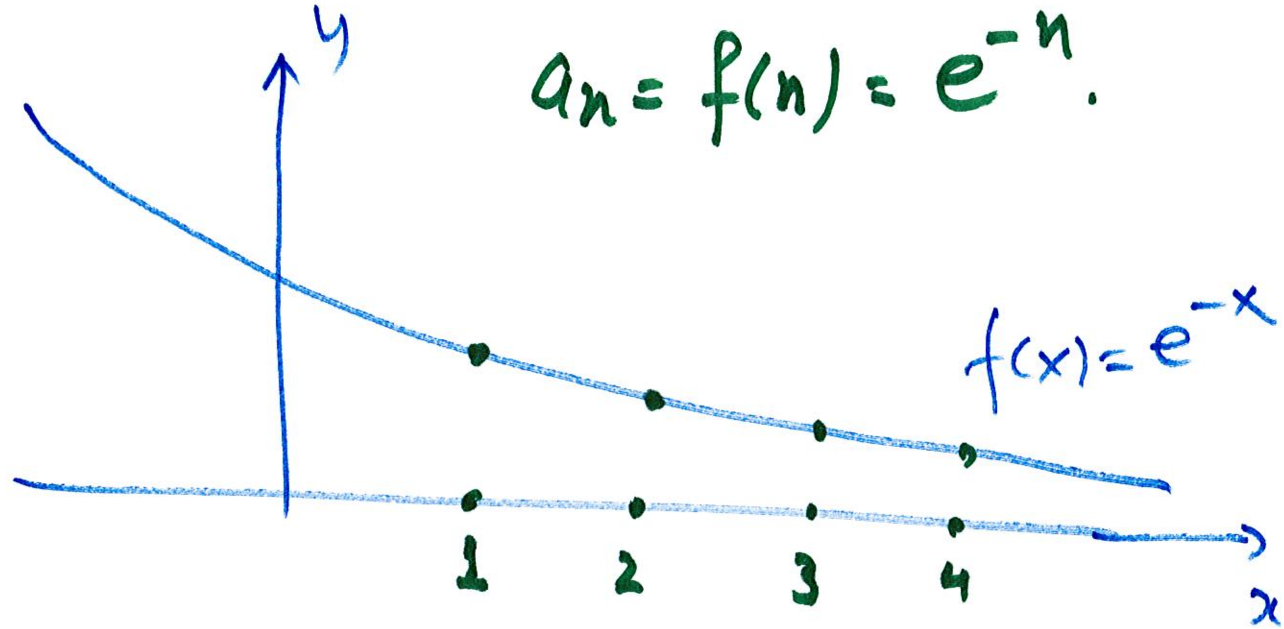
$n=4$ $a_5 = a_4 + a_3 = 6$

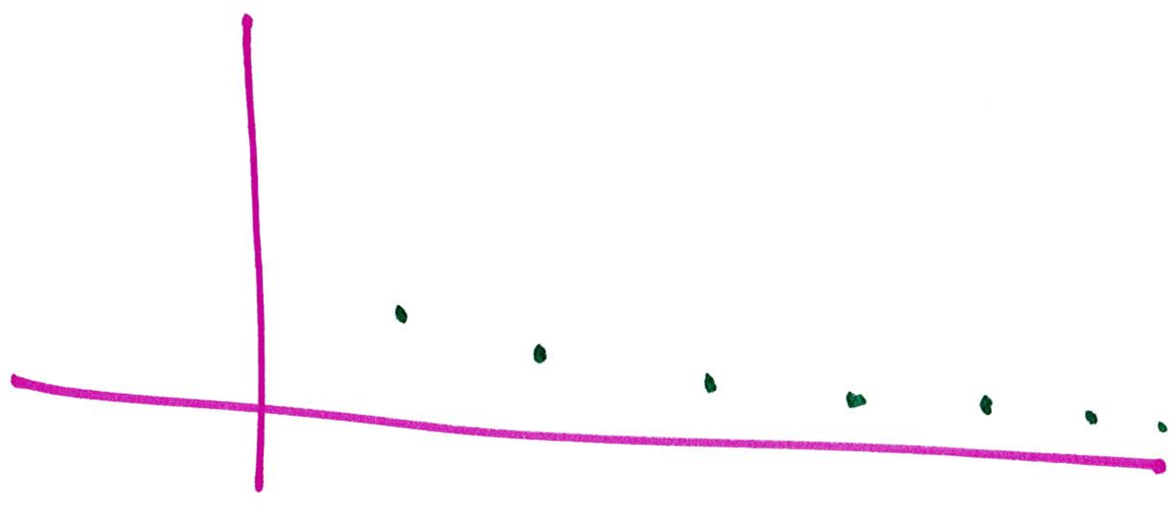
<u>Examples:</u> $a_n = \sin(1/n)$ $n=1,2,\ldots$

<u>Graph of a sequence</u>:

Ex: $a_n = e^{-n}$

$f(x) = e^{-x}$





Limits of Sequences

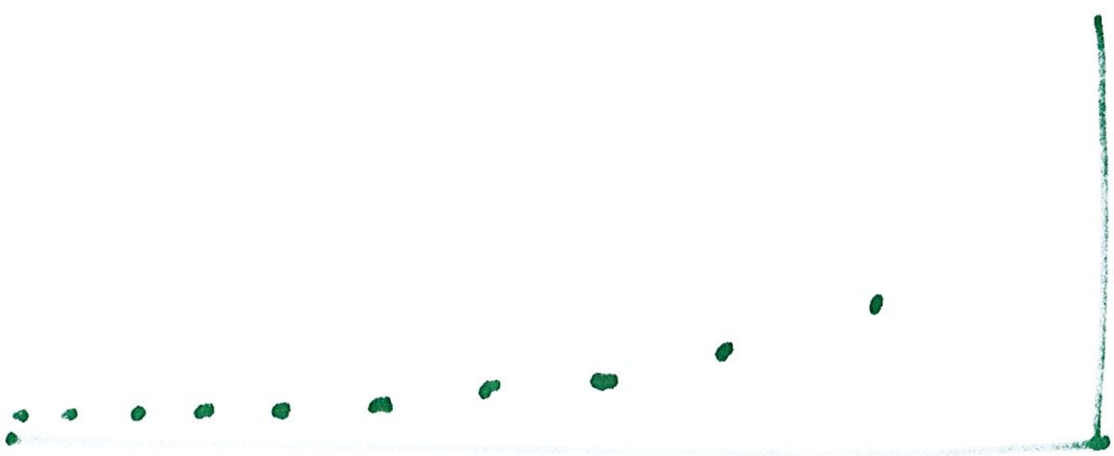
Question: Given a sequence

a_n, $n = 1, 2, 3, \ldots$

Describe the behavior of a_n

as $n \to \infty$.

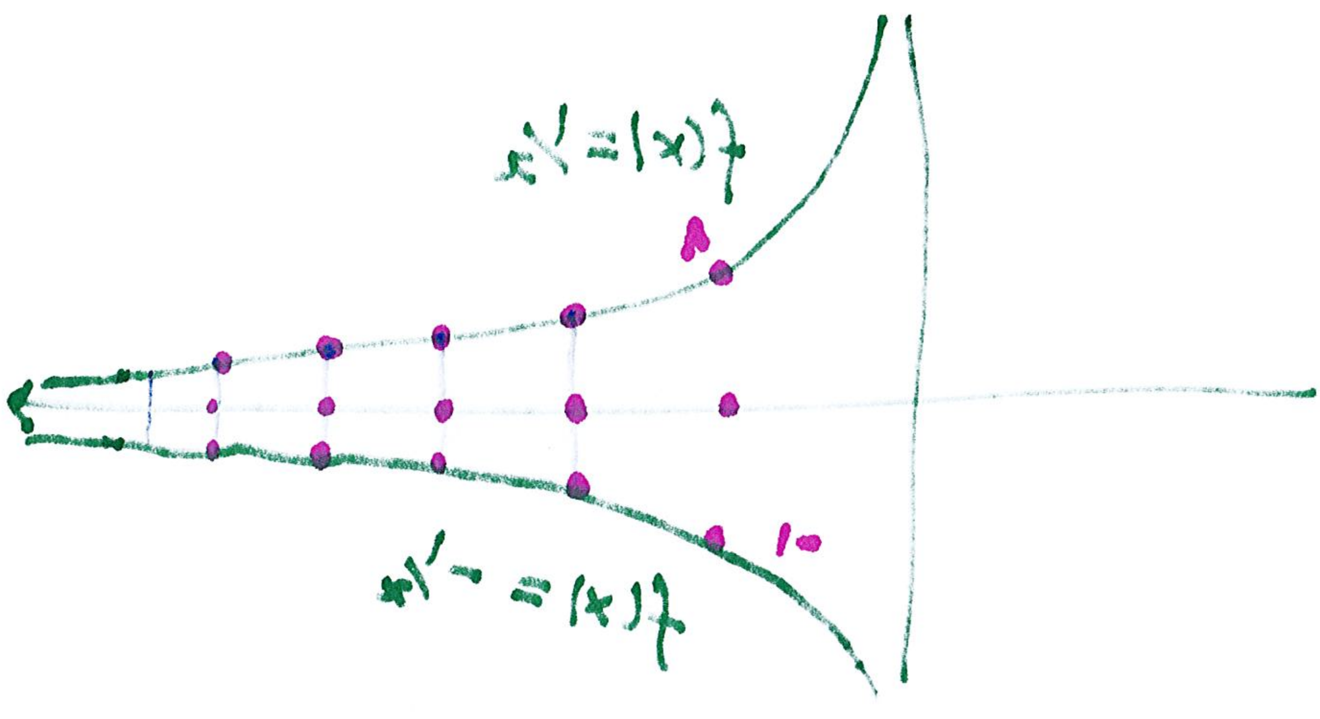
Ex: $f(n) = e^{-n} = a_n$



$a_n \to 0$ as $n \to \infty$.

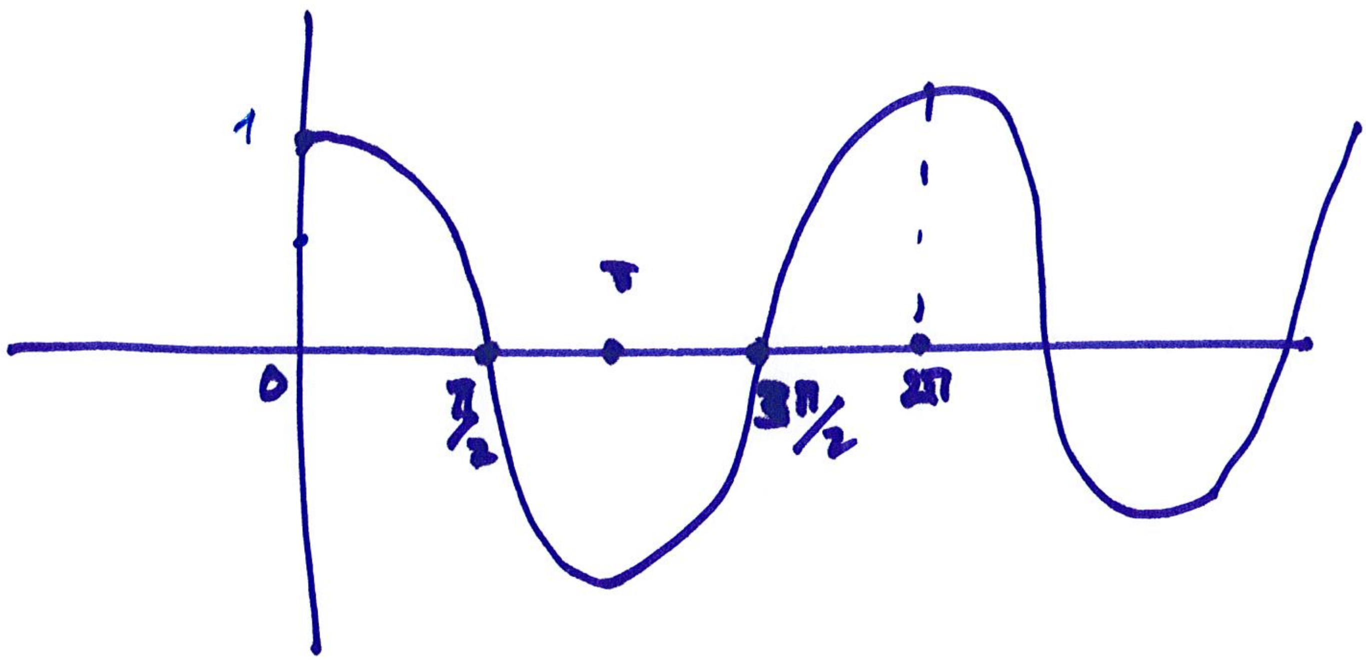
$$y = \frac{\sin x}{x}.$$

$$-\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$$



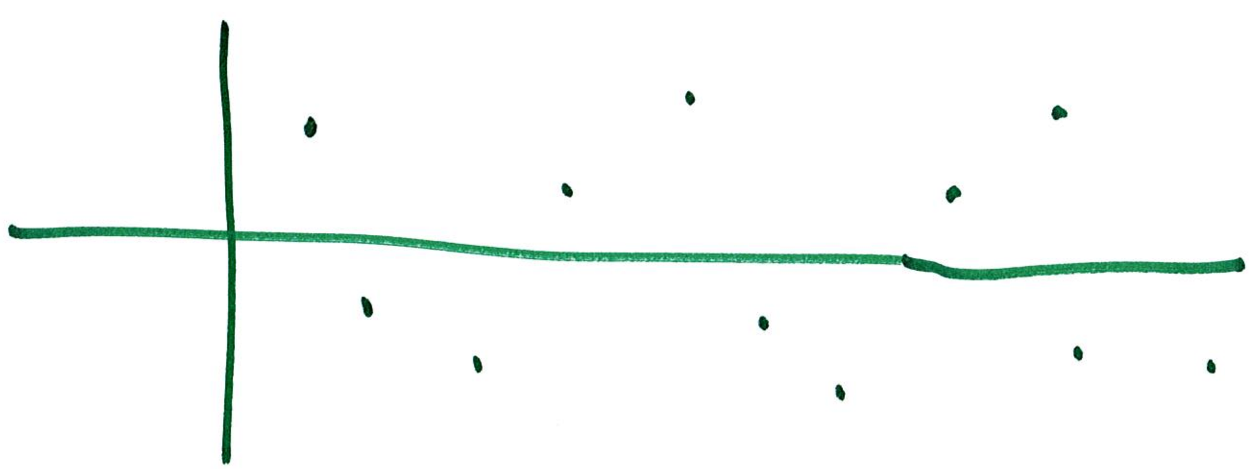
$$\lim_{x\to\infty} \frac{\sin x}{x} = 0.$$

$$b(n) = \cos n \quad , n = 1, 2, 3, \ldots$$



$\pi \sim 3.14$

$\underline{n=1} \sim \pi/3$ $\qquad n = 2 \sim 2\pi/3 \quad , n = 3 \sim \pi$



$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \cos n$$

does not <u>exist</u>

There is no number L

$$\cos n \to L \text{ as } n \to \infty$$

<u>Recall</u>:

$\lim_{x\to\infty} \cos x$ does not exist

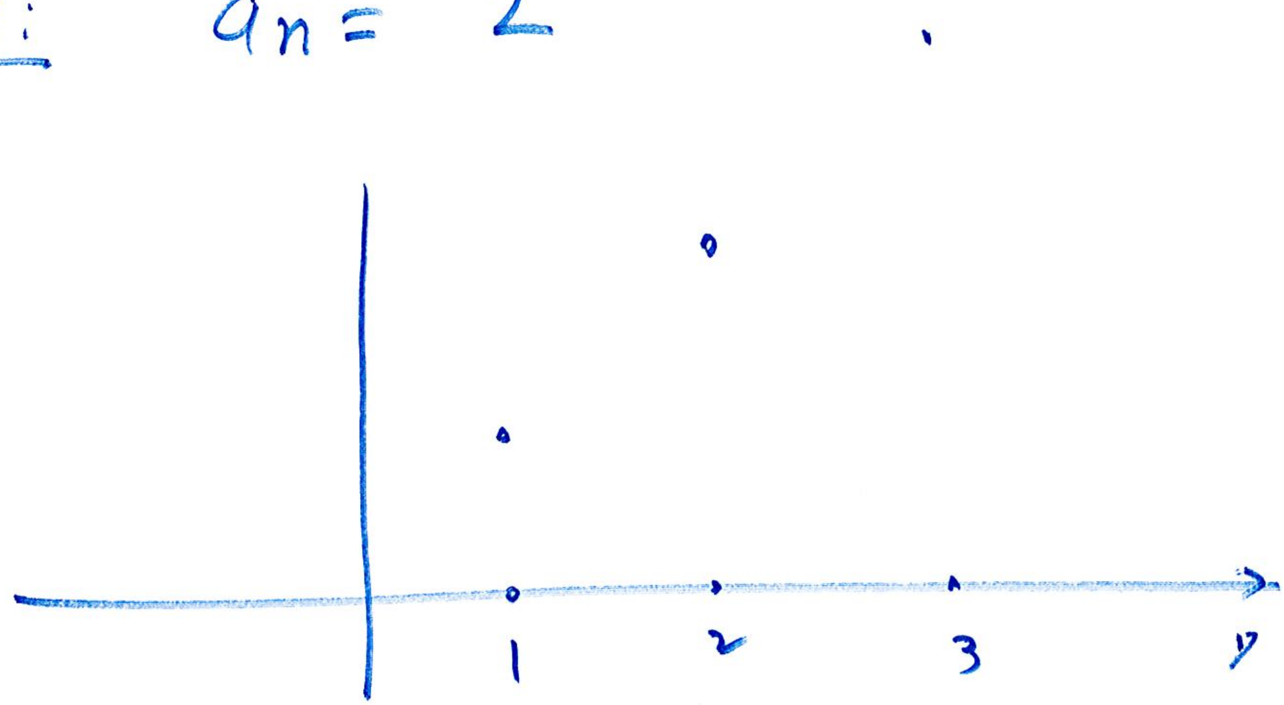
for the same reason.

We say that

$$\lim_{n \to \infty} a_n = \infty$$

if a_n gets arbitrarily large as $n \to \infty$

<u>Ex:</u> $a_n = 2^n$



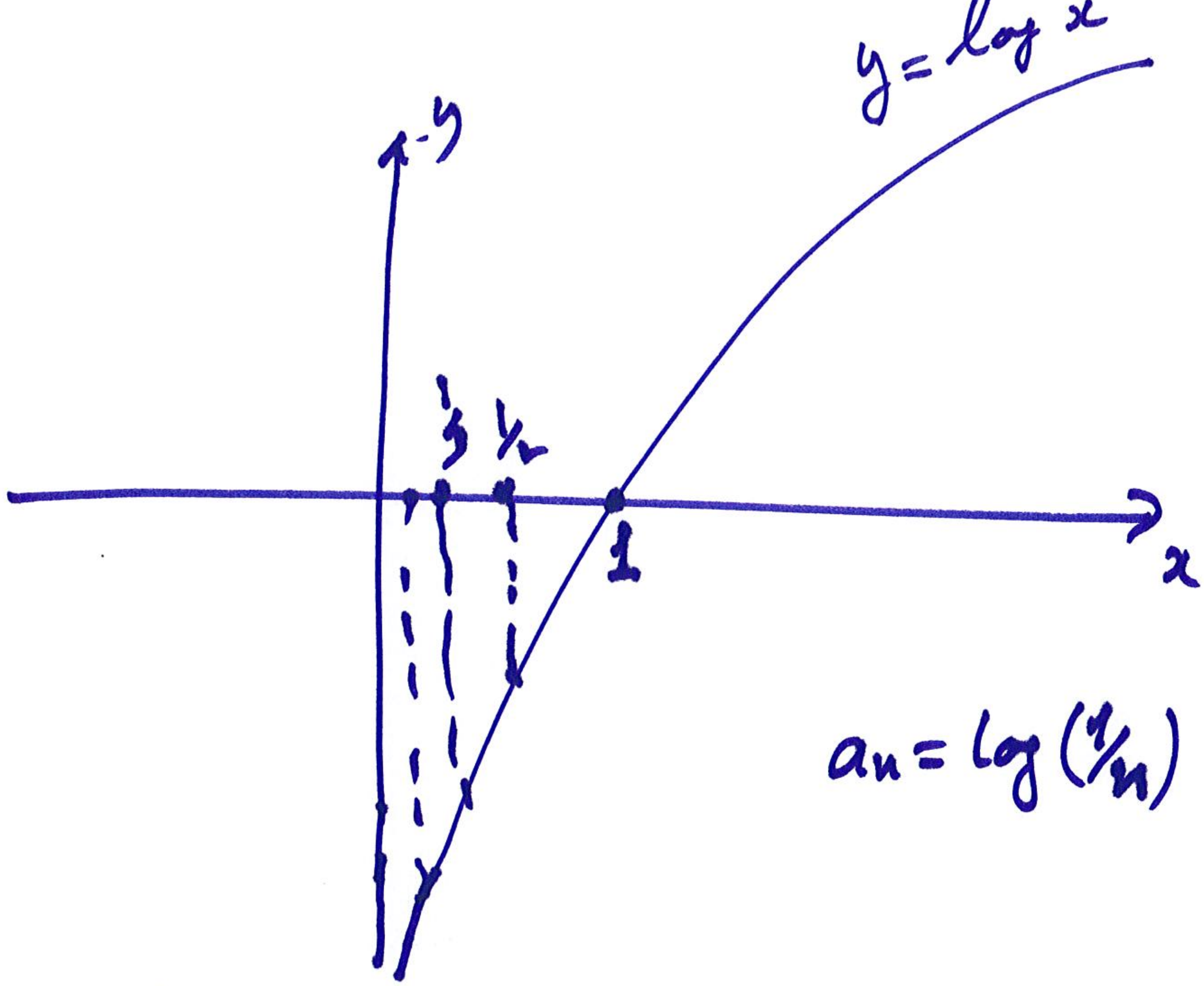
$$\lim_{n\to\infty} 2^n = \infty$$

We say that

$$\lim_{n\to\infty} a_n = -\infty$$

if a_n is arbitrarily large but negative as $n \to \infty$

$$a_n = \log 1/n.$$



$a_n = \log(1/n)$

$a_1 = \log 1 = 0$

$a_2 = \log 1/2 = -\log 2$

$$\lim_{n\to\infty} \log(1/n) = -\infty$$

General Concept.

$$a_n = f(n) \quad \text{or} \quad a_n = f(1/n)$$

When $f(x)$ is defined for x real.

<u>Ex:</u> $a_n = \log 1/n$, $a_n = f(1/n)$

$$f(x) = \log x$$

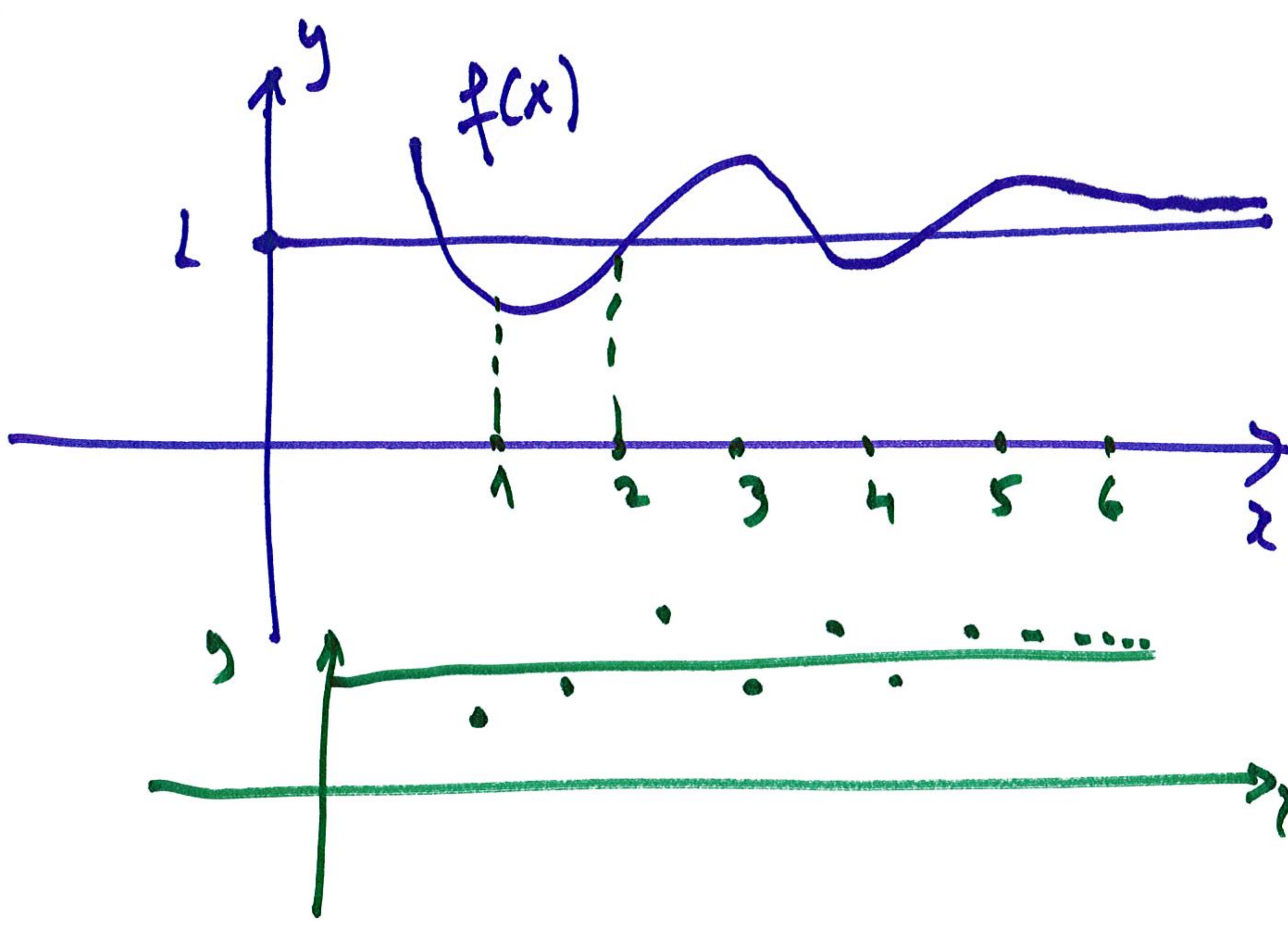
$$a_n = 2^n , \quad f(x) = 2^x$$

If

$a_n = f(n)$

and $\lim_{x \to \infty} f(x) = L$

then $\lim_{n \to \infty} a_n = L.$

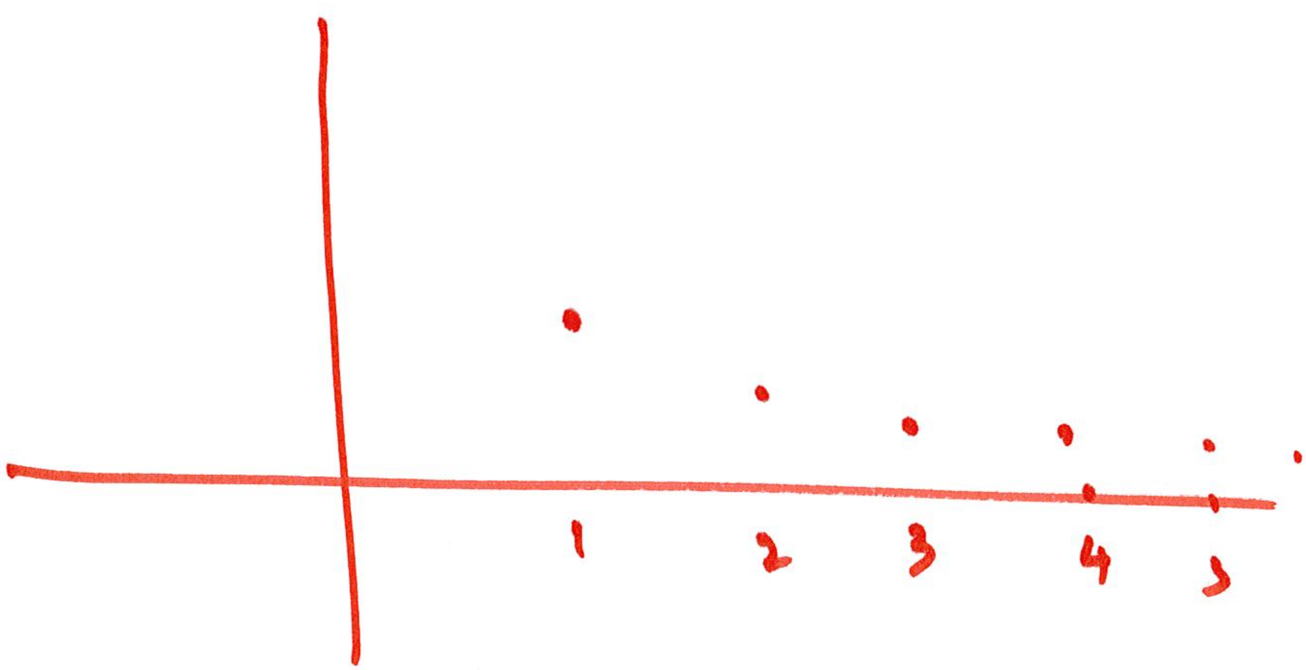


Increasing and Decreasing Sequences.

We say that

a_n is decreasing if

$a_m < a_n$ if $m > n$.

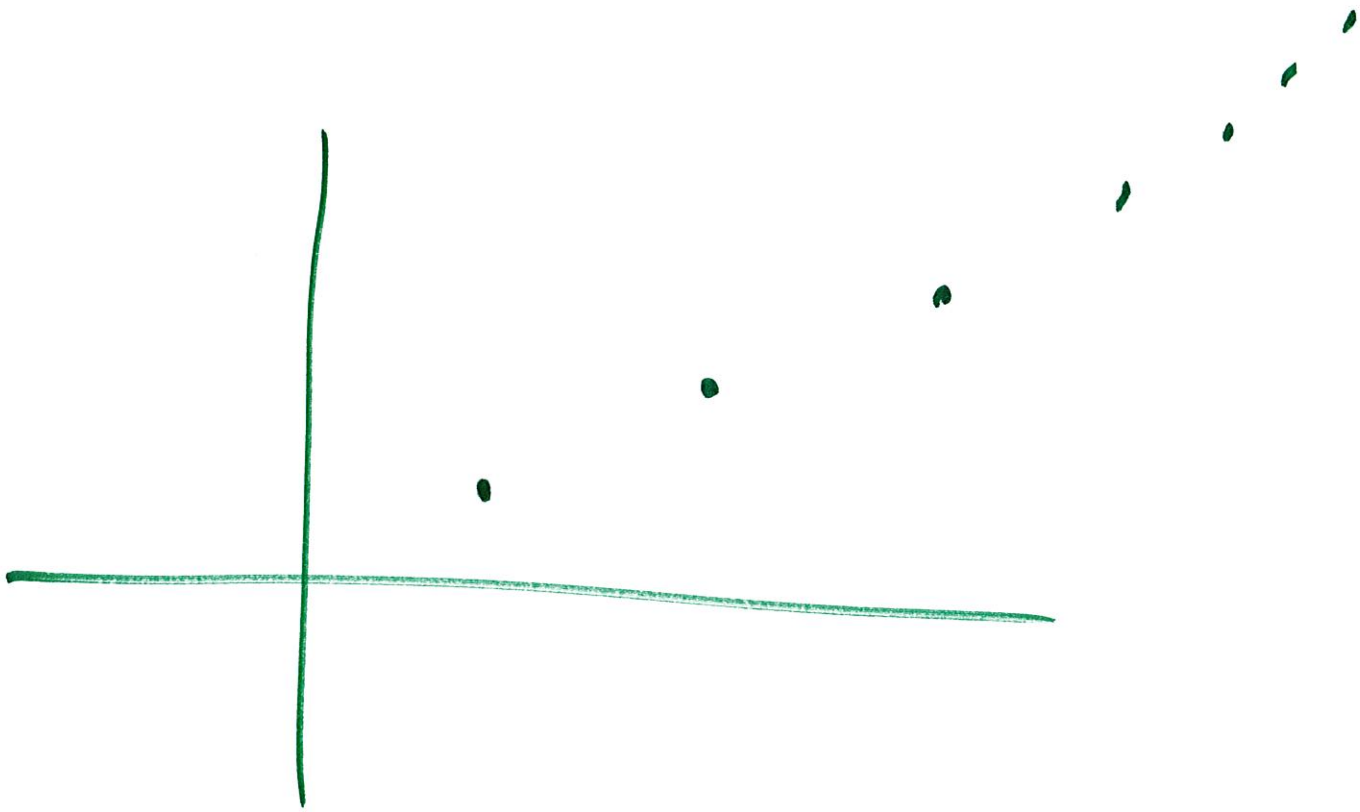


$a_n = -n$

a_n is ~~decr~~ increasing

if

$a_n < a_m$ if $m > n$



$a_n = \log n$

Compute limits

$$a_n = \frac{n^2+2}{n^2+3n+1}$$

Compute $\lim_{n\to\infty} a_n$.

Solution 1.

$$a_n = \frac{n^2+2}{n^2+3n+1}$$

$$= \frac{\cancel{n^2}(1+2/n^2)}{\cancel{n^2}(1+3/n+1/n^2)}.$$

$$a_n = \frac{1 + 2/n^2}{1 + 3/n + 1/n^2}$$

$$\lim_{n \to \infty} a_n = \frac{1}{1} = 1$$