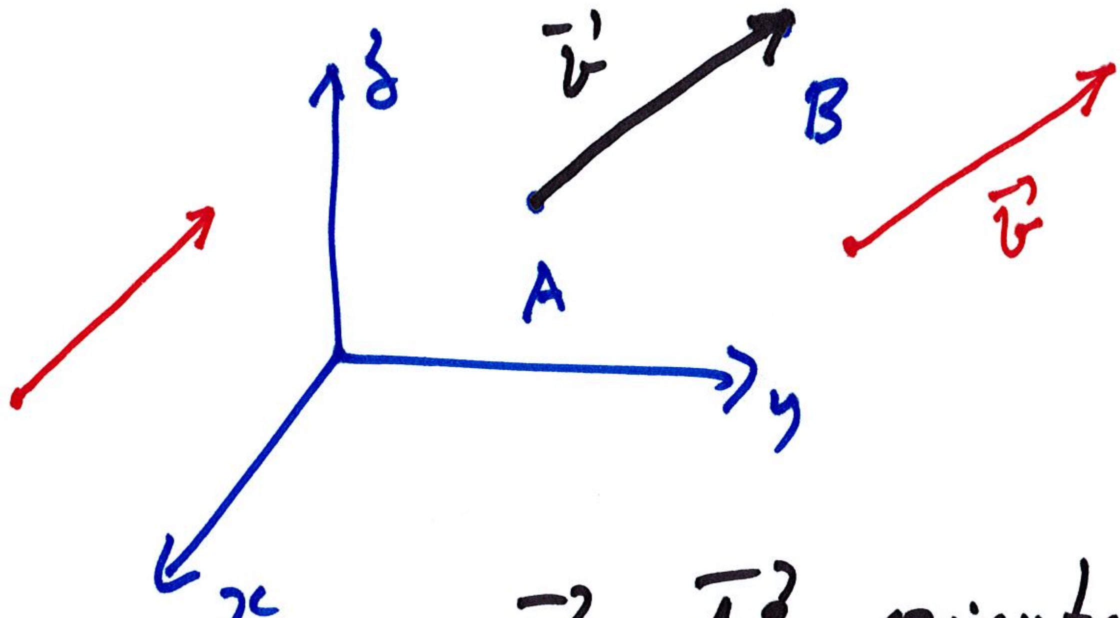
Lesson 2 Sections 12.2 and 12.3

12.2 Vectors



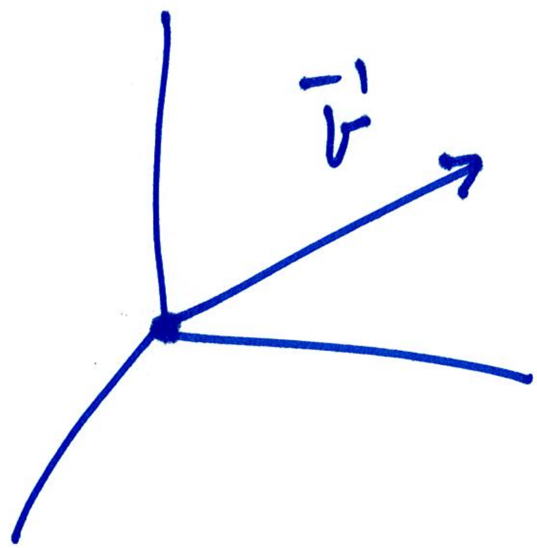
$\vec{v} = \overrightarrow{AB}$ oriented segment.

Magnitude or length of

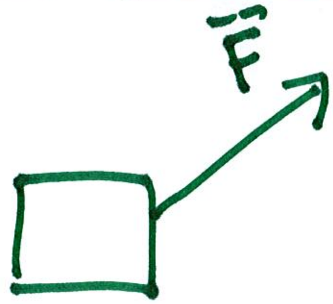
$\vec{v} = |\overrightarrow{AB}|$ = length of segment.

Equivalent vectors differ by translation only.

We always think of vectors as a segment starting at 0



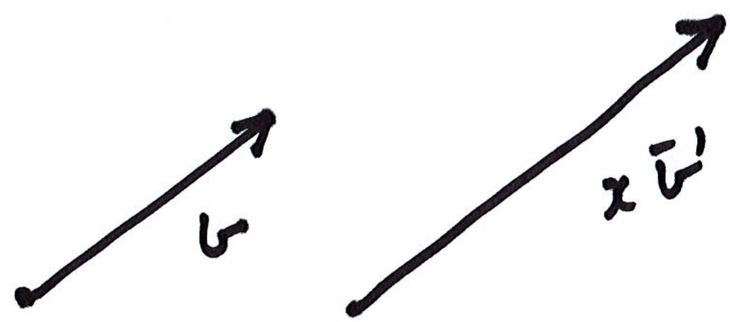
<u>Example:</u> <u>Force</u>



Operations with vectors

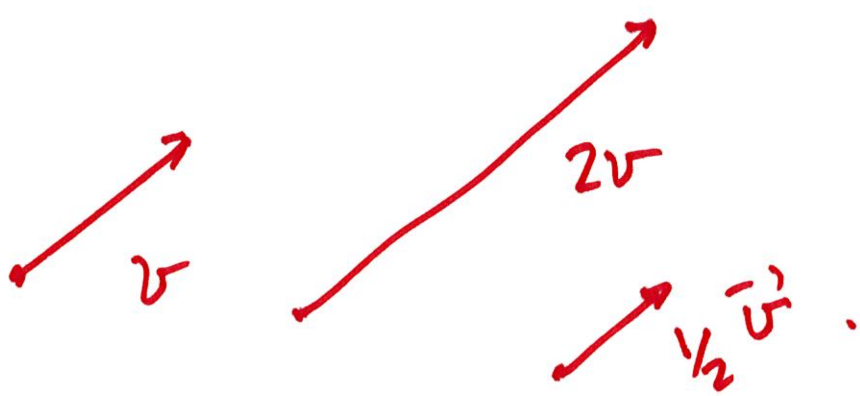
1. Multiplication by a number. x.
   (Scalar)

<u>$x > 0$</u>

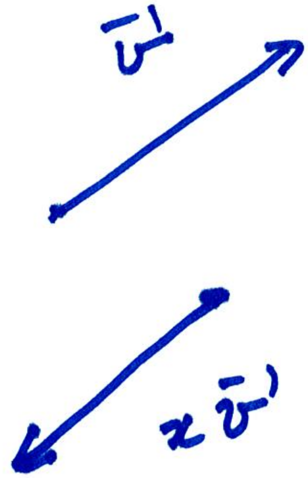


$x\vec{v}$ has same direction as $\vec{v}$

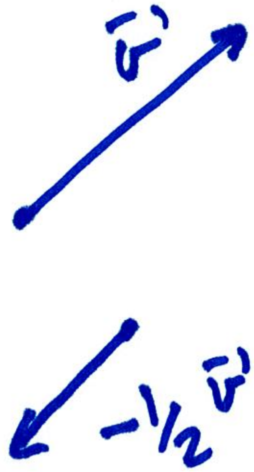
Magnitude $|x\vec{v}| = x|\vec{v}|$.



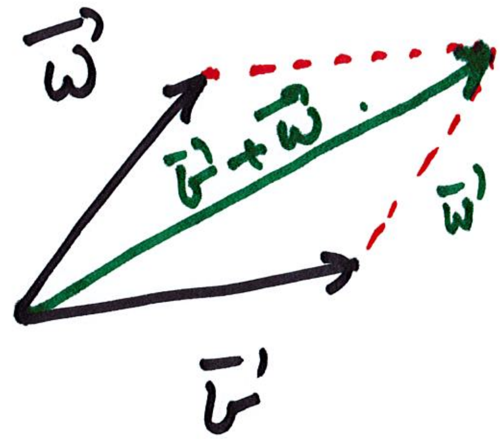
If $x < 0$



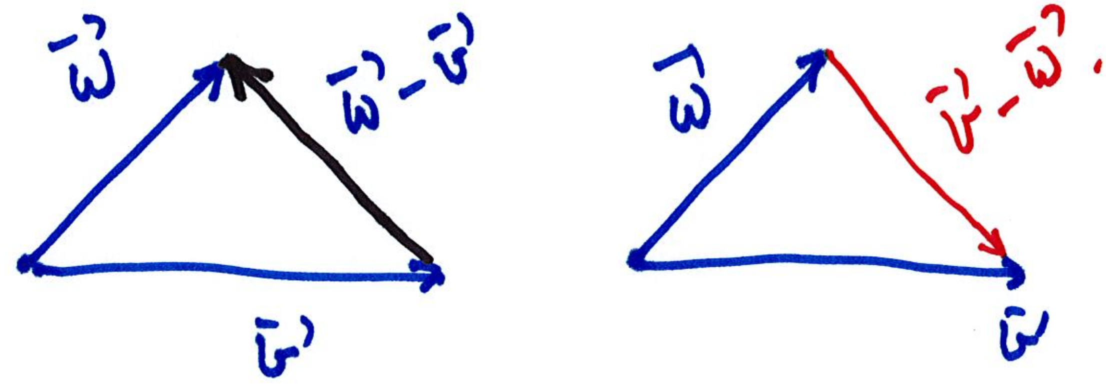
$$|x\vec{v}| = |x| . |\vec{v}|$$



Addition and Subtraction.



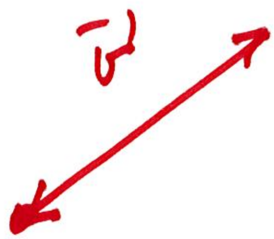
$\vec{v}+\vec{w}$ and $\vec{v}-\vec{w}$.



The zero vector. $\vec{0}$

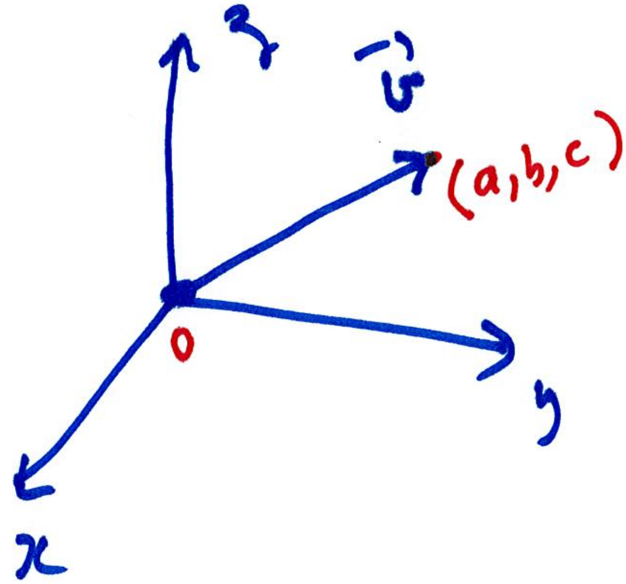
$$\vec{v} + \vec{0} = \vec{v}$$

$$\vec{v} - \vec{v} = \vec{0}.$$

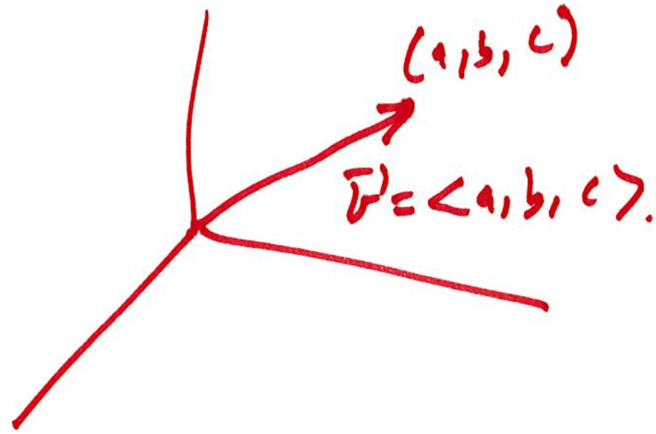


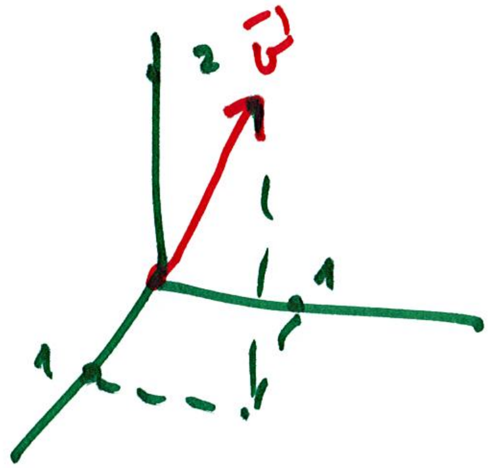
We want to write equations for vectors and vector operations.

<u>Components</u>..



$\vec{v} = \langle a, b, c \rangle$





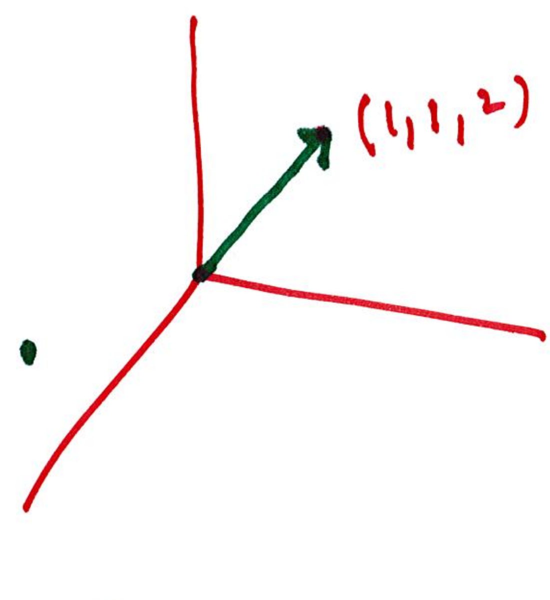
$\vec{v} = \langle 1, 1, 2 \rangle$

Operations with vectors

1) $\vec{v} = \langle a, b, c \rangle$; x

$$x\vec{v} = \langle xa, xb, xc \rangle$$

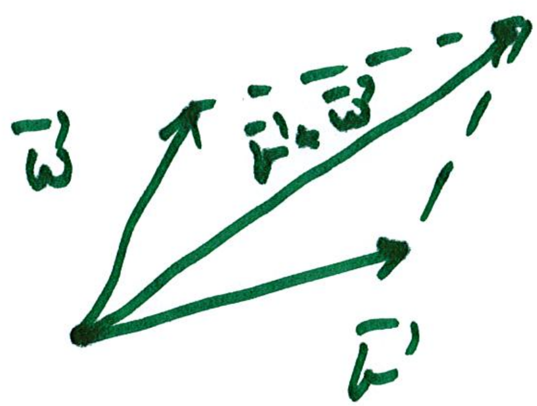
$\vec{v} = \langle 1, 1, 2 \rangle$ $\quad x = -1$

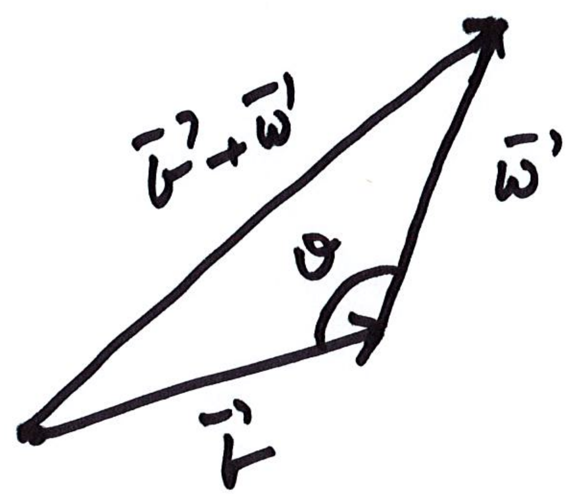


$-\vec{v} = \langle -1, -1, -2 \rangle$

$\vec{v} = \langle a_1, b_1, c_1 \rangle$, $\vec{w} = \langle a_2, b_2, c_2 \rangle$

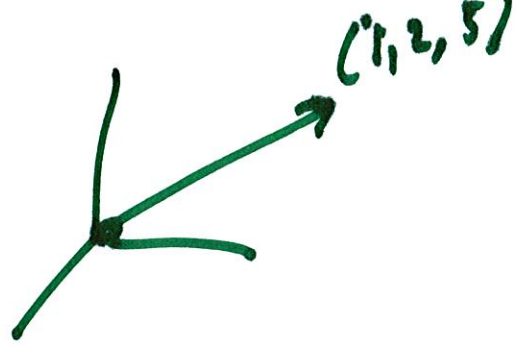
$\vec{v} + \vec{w} = \langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle$





$\vec{v} = \langle 1, 1, 2 \rangle, \quad \vec{w} = \langle 0, 1, 3 \rangle$

$$\vec{v} + \vec{w} = \langle 1+0, 1+1, 2+3 \rangle$$
$$= \langle 1, 2, 5 \rangle$$



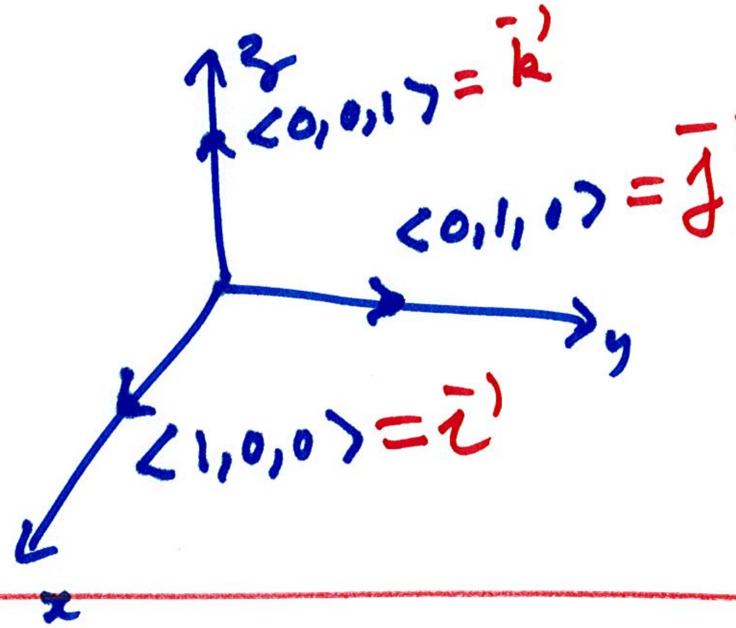
$$|\vec{v} + \vec{w}| = \sqrt{1+4+25}$$
$$= \sqrt{30}.$$

$$\vec{v} - \vec{w} = \langle 1, 1, 2 \rangle - \langle 0, 1, 3 \rangle$$
$$= \langle 1, 0, -1 \rangle$$

Special Vectors

$$\langle a_1, a_2, a_3 \rangle = a_1 \langle 1, 0, 0 \rangle$$

$$+ a_2 \langle 0, 1, 0 \rangle + a_3 \langle 0, 0, 1 \rangle.$$



$$\boxed{\langle a_1, a_2, a_3 \rangle = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}}$$

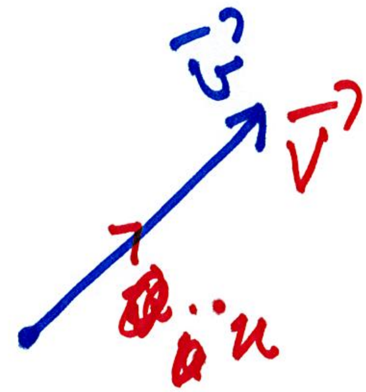
Example: $\langle 1, 2, 5 \rangle = \vec{i} + 2\vec{j} + 5\vec{k}.$

Unit Vectors.

$\vec{u}$, $|\vec{u}|=1$.

Given a vector $\vec{v}$ of length >0

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|}$$



Example: $\vec{v} = \langle 1,4,5 \rangle$

$$|\vec{v}| = \sqrt{1+16+25} = \sqrt{42}.$$

$\vec{u} = \frac{1}{\sqrt{42}} \langle 1, 4, 5 \rangle$ has length 1 and the same direction as $\vec{v}$.

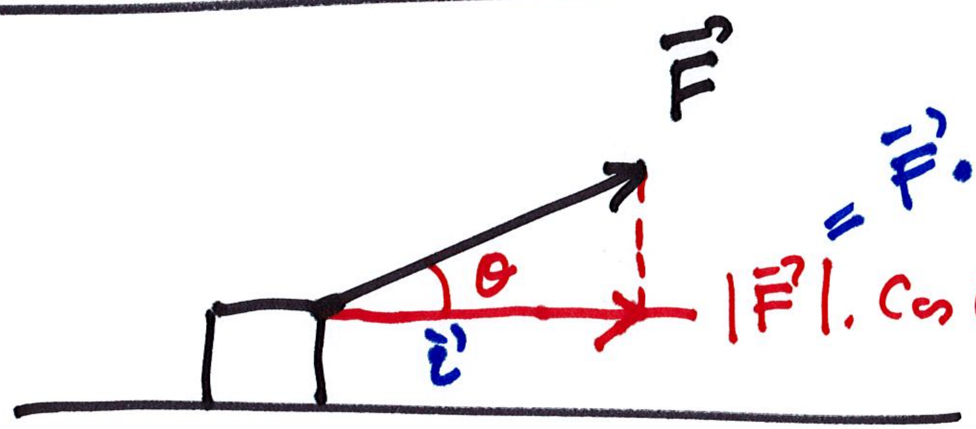
The vector

$$w = \frac{10}{\sqrt{42}} \langle 1, 4, 5 \rangle$$

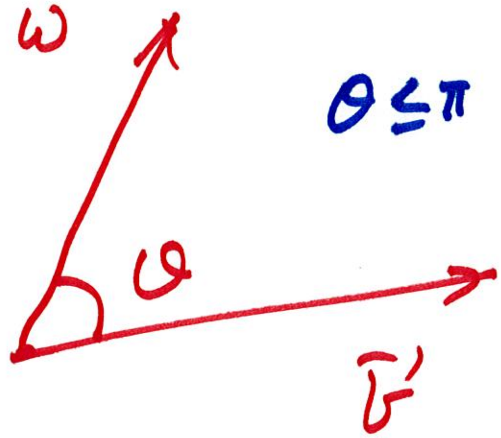
has the same direction of $\vec{v} = \langle 1, 4, 5 \rangle$

and length 10.

The Dot Product

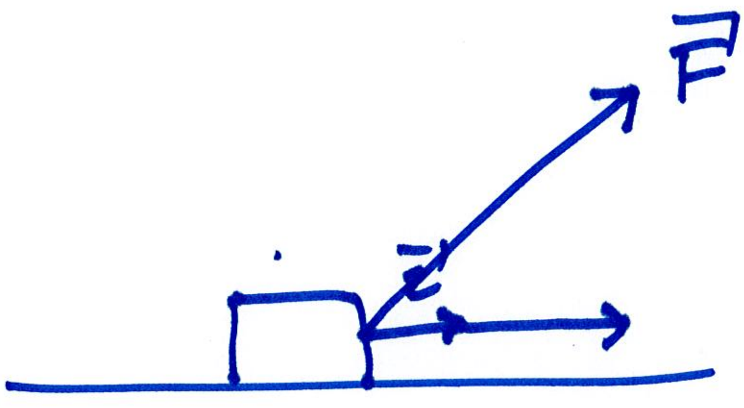


The dot product between two vectors $\vec{v}$ and $\vec{w}$



$\theta \leq \pi$

$$\vec{v} \cdot \vec{w} = |\vec{v}| \cdot |\vec{w}| \cdot \cos\theta$$



$$\vec{F} \cdot \vec{e} = |\vec{F}| \cdot |\vec{e}| \cdot \cos\theta$$

$$= |\vec{F}| \cdot \cos\theta .$$