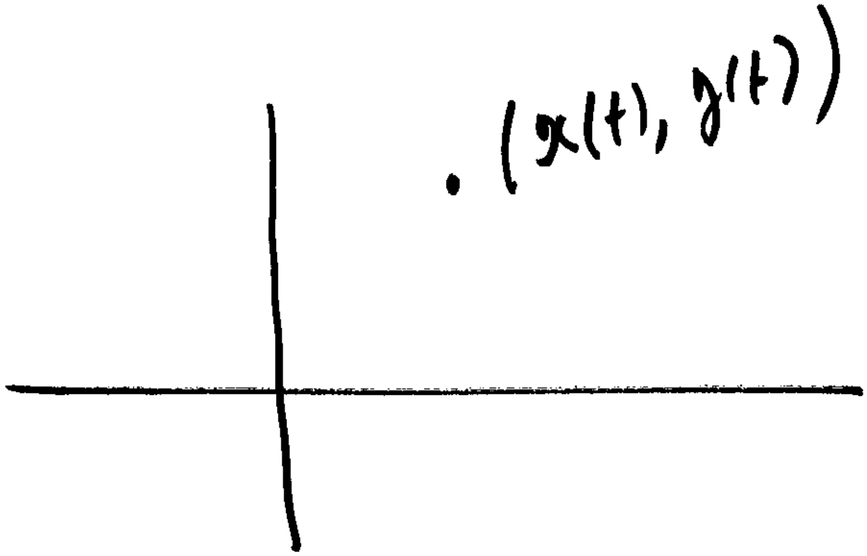
Lesson 31   Section 10.1

Curves defined by Parametric Equations.



We have an object moving on the plane. Suppose that at time t the object is located at $(x(t), y(t))$

As the point $\underbrace{\quad}_{(x(t),\, y(t))}$ moves

it describes a curve on the

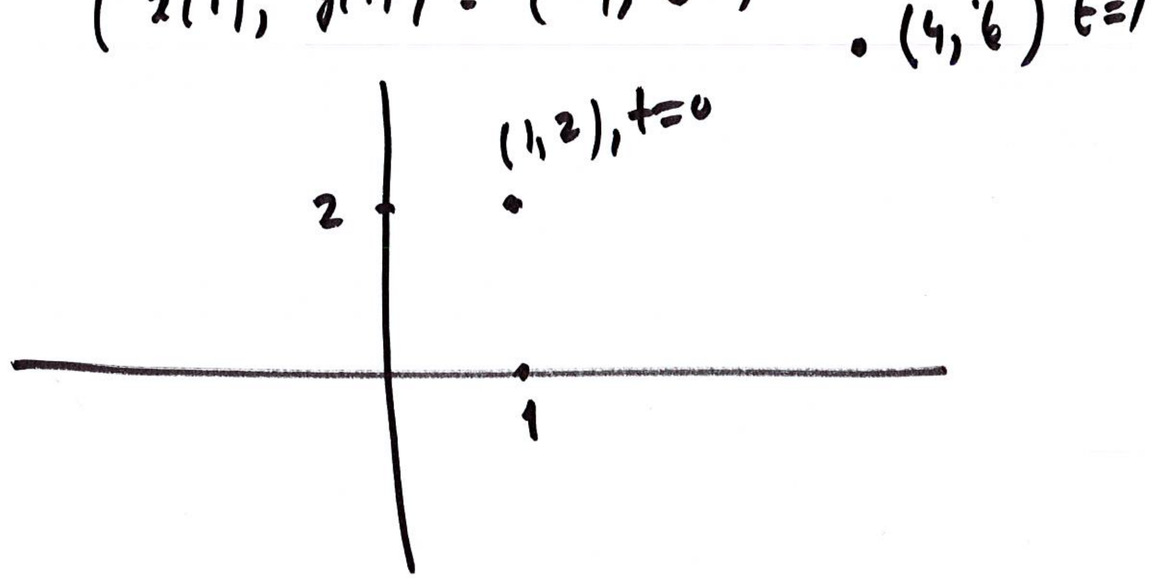
plane.

<u>Example</u>: $x(t) = 1 + 3t$, $y = 2 + 4t$.

t = time.

$\underline{t=0}$: $(x(0), y(0)) = (1, 2)$

$t=1$ $\quad (x(1), y(1)) = (4, 6)$



Questions

1) How the curve looks like?

2) Orientation of the motion along the curve.

(1) $\underline{\underline{x}} = x(t) = \underline{1+3t}$

$\underline{\underline{y}} = y(t) = \underline{2+4t}$

$x - 1 = 3t, \quad t = \frac{x-1}{3}$

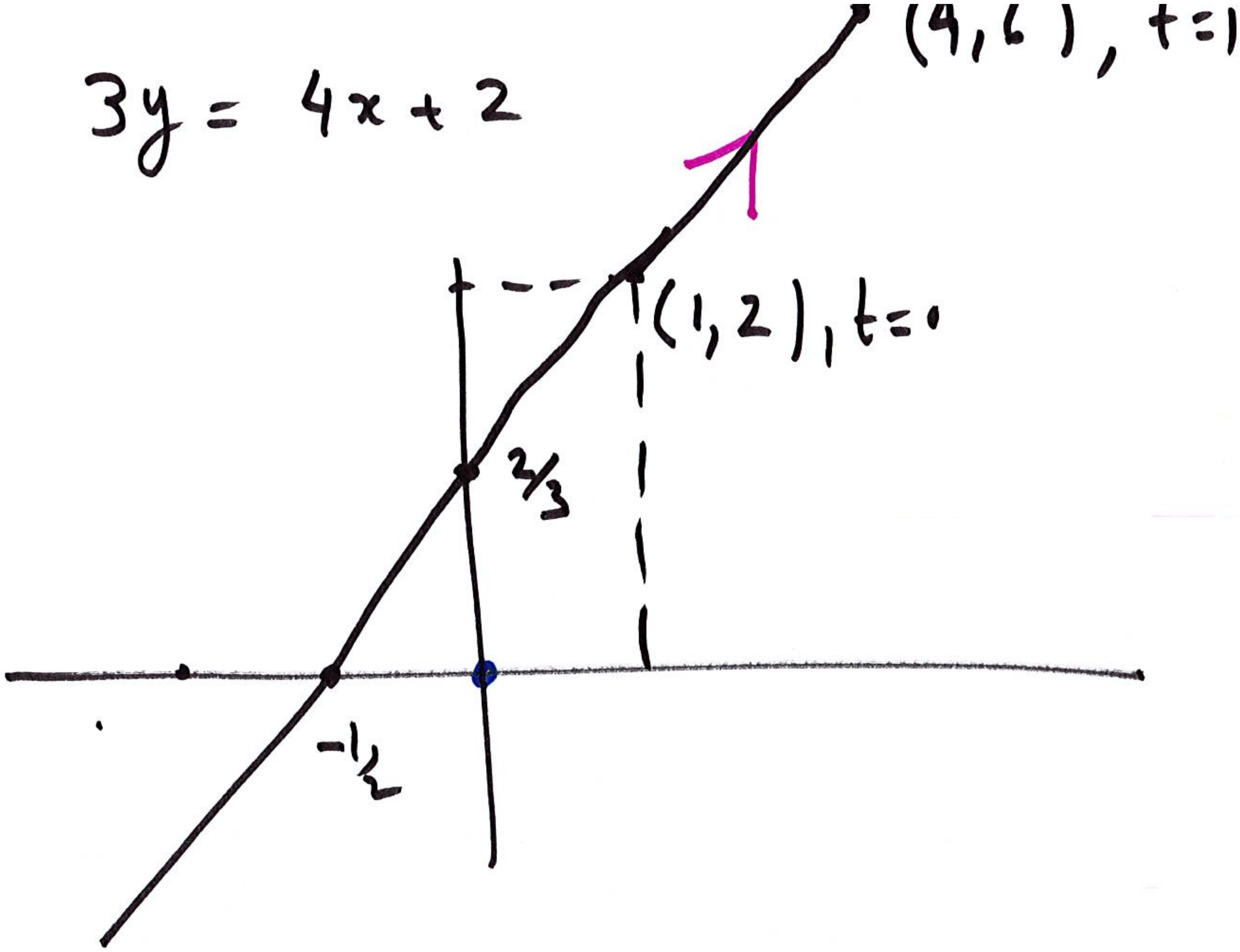
$y - 2 = 4t, \quad t = \frac{y-2}{4}.$

$$\frac{x-1}{3} = \frac{y-2}{4}$$

$4x - 4 = 3y - 6 \qquad 4x = 3y - 2$

$$\boxed{3y = 4x + 2}$$

$$3y = 4x + 2$$

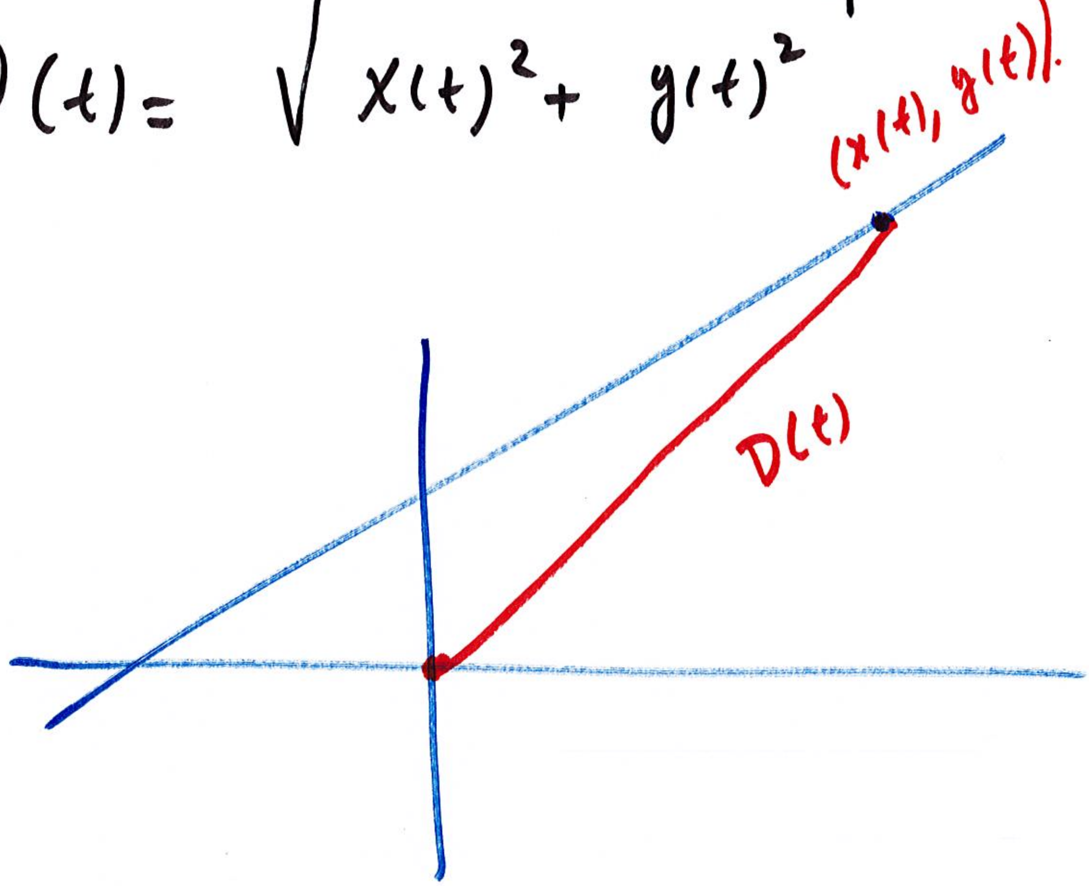


$$x = 1 + 3t, \quad y = 2 + 4t.$$

Question: How fast is the object moving? with respect to the origin?

Distance to the origin

$$D(t) = \sqrt{x(t)^2 + y(t)^2}$$



$$\text{Speed} = \frac{d\,D(t)}{dt} = D'(t)$$

$$D(t) = \sqrt{(1+3t)^2 + (2+4t)^2}$$

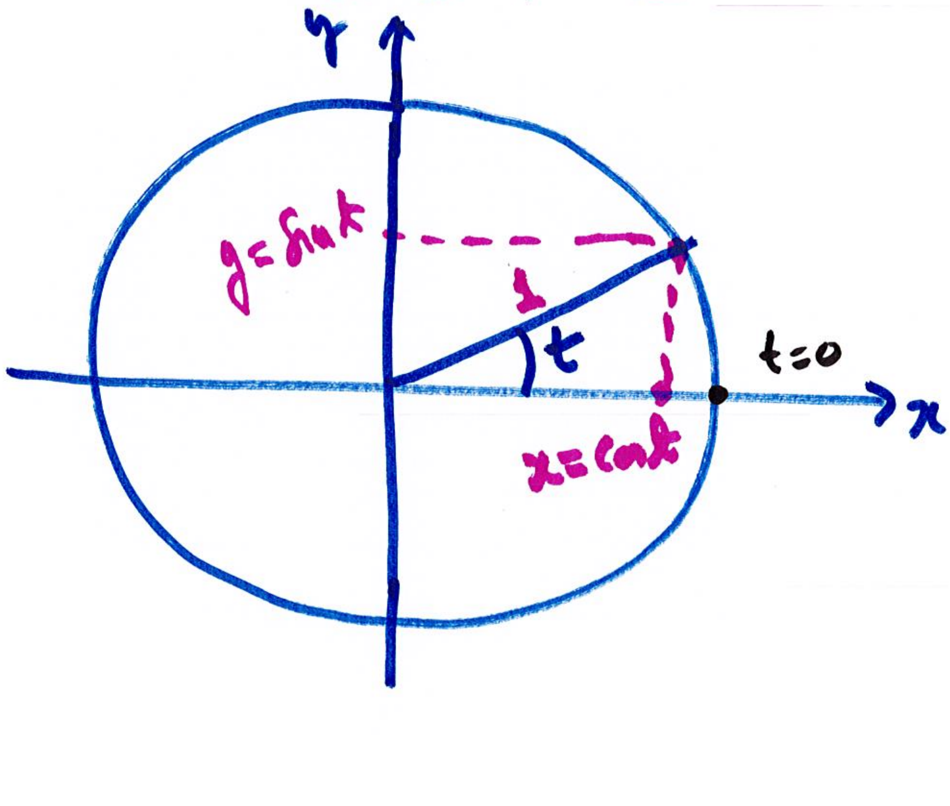
$$D'(t) = \frac{\cancel{2}(1+3t)\cdot 3 + \cancel{2}(2+4t)\cdot 4}{\cancel{2}\sqrt{(1+3t)^2 + (2+4t)^2}}$$

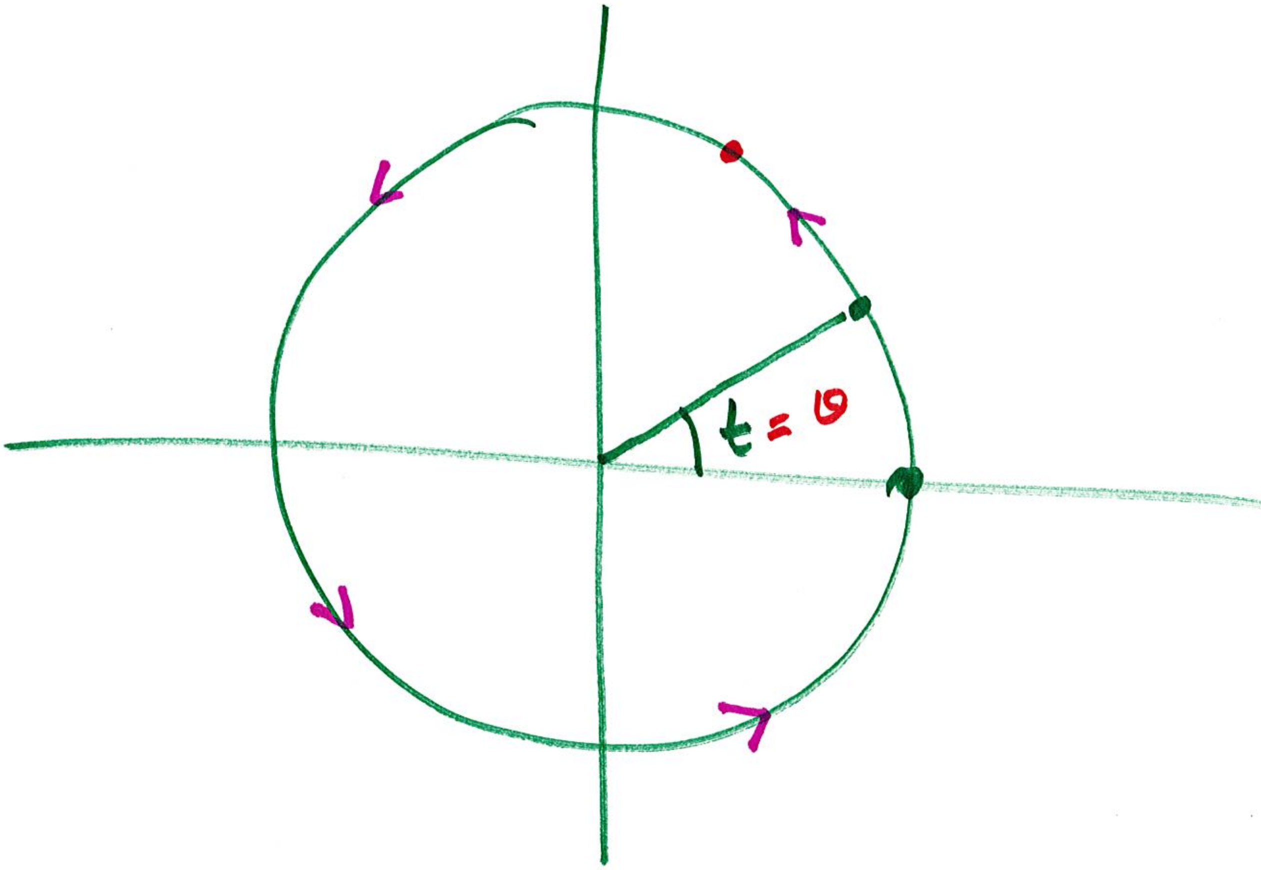
$$D'(t) = \frac{25t + 11}{\sqrt{(1+3t)^2 + (2+4t)^2}}.$$

at $\underline{t=0}$: $$D'(0) = \frac{11}{\sqrt{5}}$$

$$x(t) = \cos t, \quad y(t) = \sin t$$

$$0 \le t \le 2\pi.$$





$t = 0, \quad x = 1, \quad y = 0$

$t = 2\pi, \quad x = 1, \quad y = 0$

$\theta = \text{angle}, \quad \theta = t$

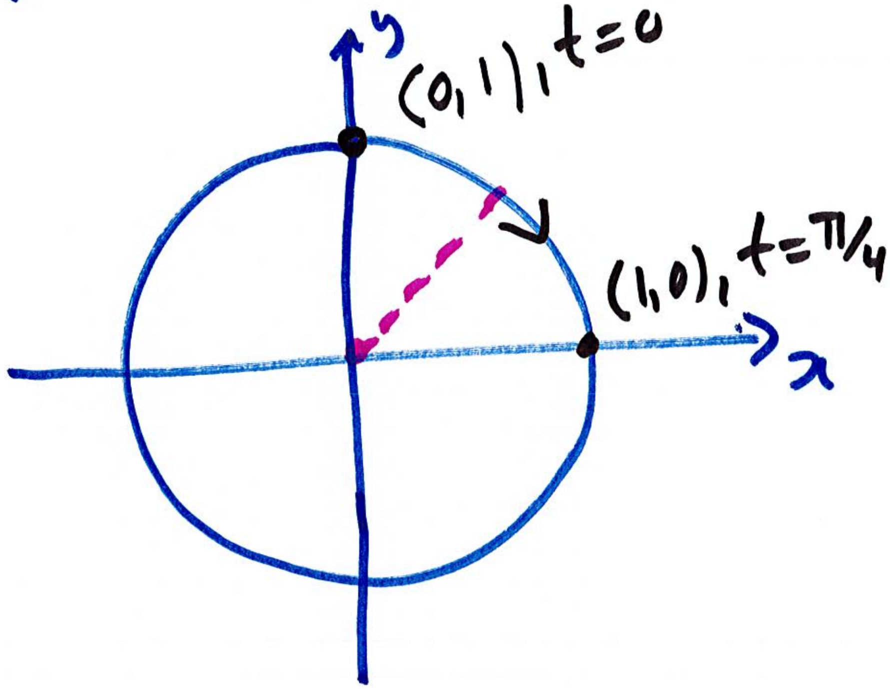
$$\frac{d\theta}{dt} = 1.$$

$x = \sin 2t, \quad y = \cos 2t$

$0 \le t \le 2\pi.$

$x^2 = (\sin 2t)^2, \quad y^2 = (\cos 2t)^2$

$x^2 + y^2 = 1.$



when $t = 0$ $\quad x = \sin 0 = 0, \quad y = \cos 0 = 1$

when $t = \pi/4$ $\quad x = \sin \pi/2 = 1, \quad y = \cos \pi/2 = 0$

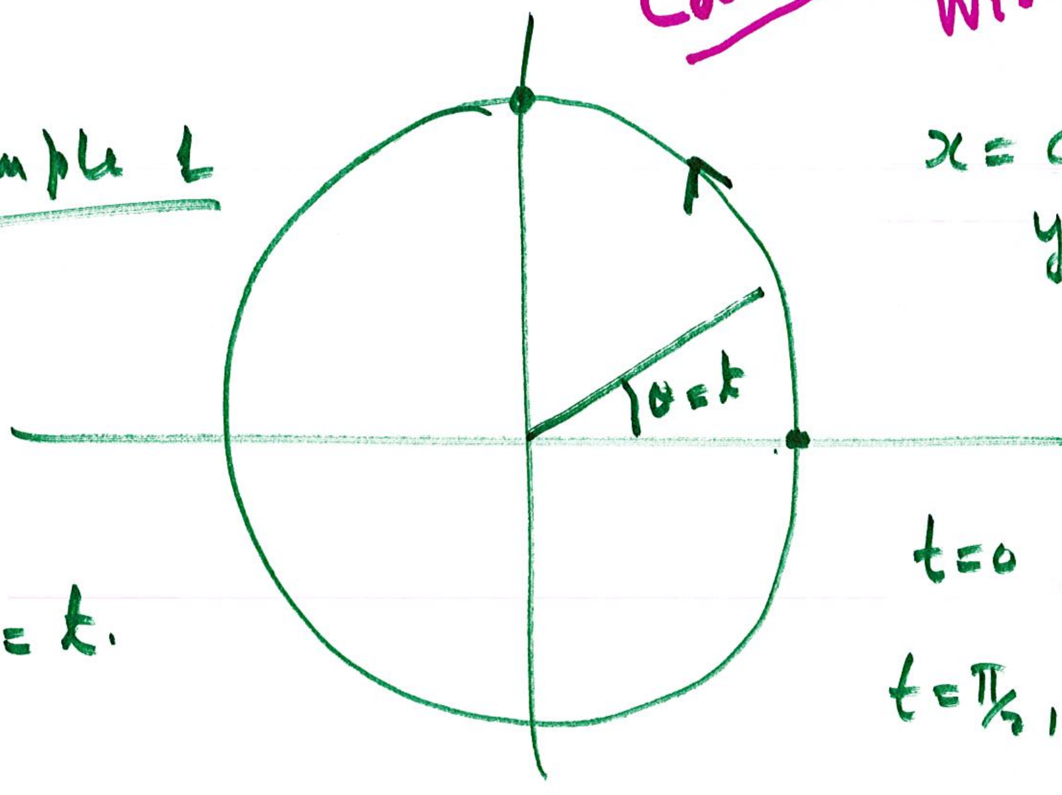
$x = \sin 2t, \quad y = \cos 2t.$

Angular speed.

$\theta =$

COUNTERCLOCK WISE

Example 1



$x = \cos t$

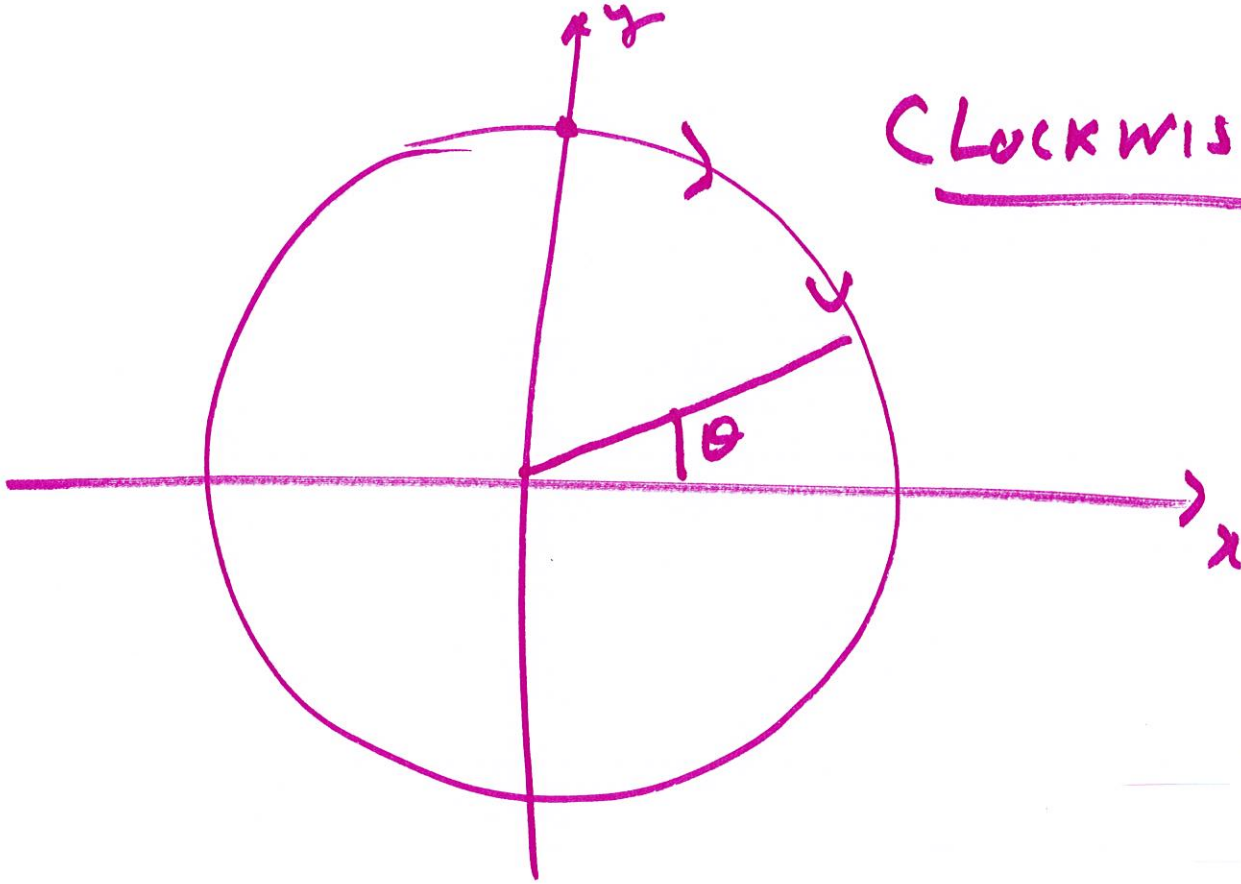
$y = \sin t$

$\theta = t.$

$t = 0, \; x = 1, \; y = 0$

$t = \pi/2, \; x = 0, \; y = 1$

CLOCKWISE



$$x = \sin 2t, \quad y = \cos 2t$$

$$t = 0, \quad x = 0, \quad y = 1$$

$t = 0, \quad \theta = \pi/2$

$$\theta = \pi/2 - 2t$$

$t = \pi/4, \quad \theta = 0$

$$\boxed{\frac{d\theta}{dt} = -2}$$

$$\cos^2 t + \sin^2 t = 1.$$

$$x^2 + y^2 =$$

$$x = \cos t, \quad y = \sin t$$

$$x = \underline{2\cos t}, \quad y = 3\sin t.$$

$$x^2 = 4\cos^2 t, \quad \frac{x^2}{4} = \cos^2 t$$

$$y^2 = 9\sin^2 t, \quad \frac{y^2}{9} = \sin^2 t$$

$$x^2/4 + y^2/9 = 1$$

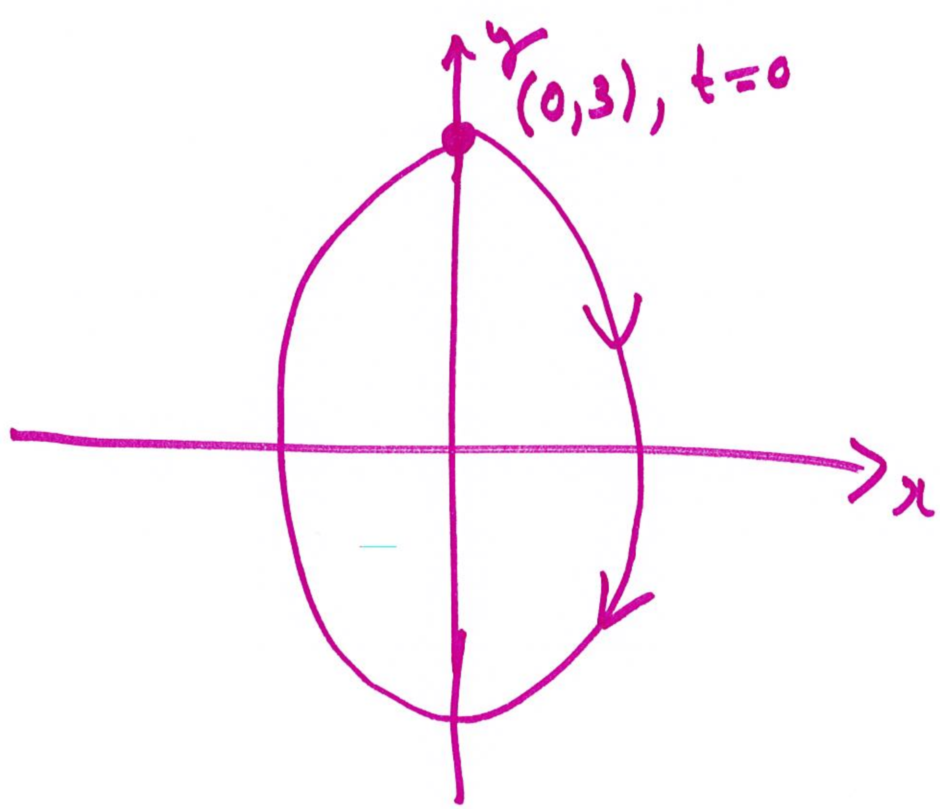
$$\boxed{x = 2\cos t, \quad y = 3\sin t}$$

$x = 2\sin 3t, \quad y = 3\cos 3t$

$x/2 = \sin 3t \qquad y/3 = \cos 3t$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\frac{x^2}{4} = \sin^2 3t, \quad \frac{y^2}{9} = \cos^2 3t$$



Find a parametric

Equation for

$$\boxed{\frac{x^2}{4} + \frac{y^2}{9} = 1.}$$

We want $x = x(t)$

$y = y(t)$

Ellipse