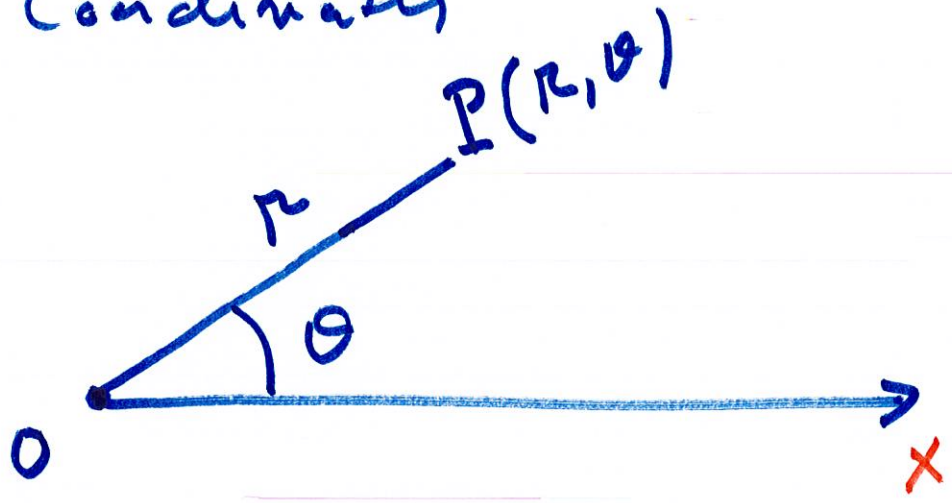
Lesson 33 Polar Coordinates

Section 10.3

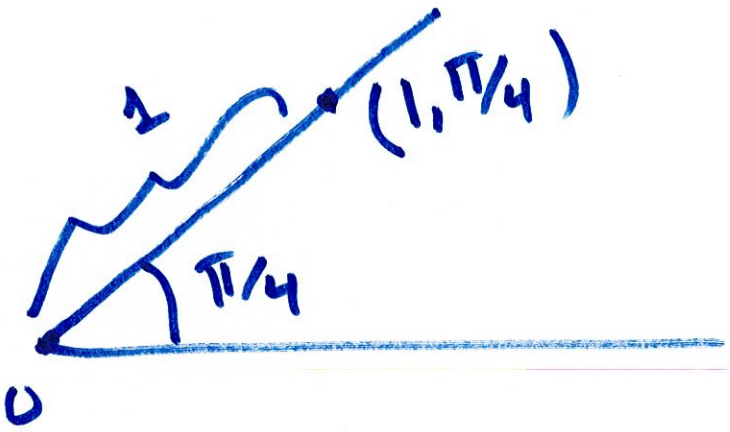
Review Exercises Course Page

Polar Coordinates

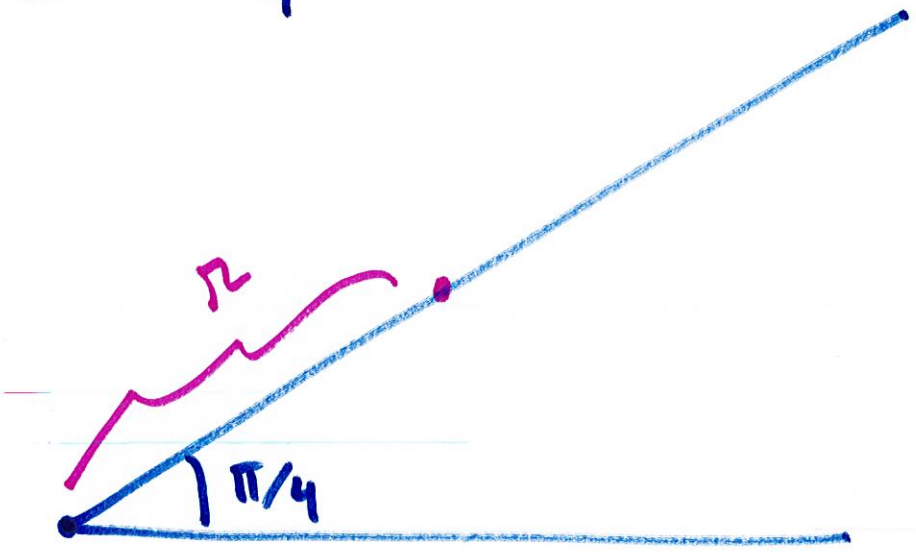


$r = |OP|$, θ = angle between $\overrightarrow{OP}$ and the axis

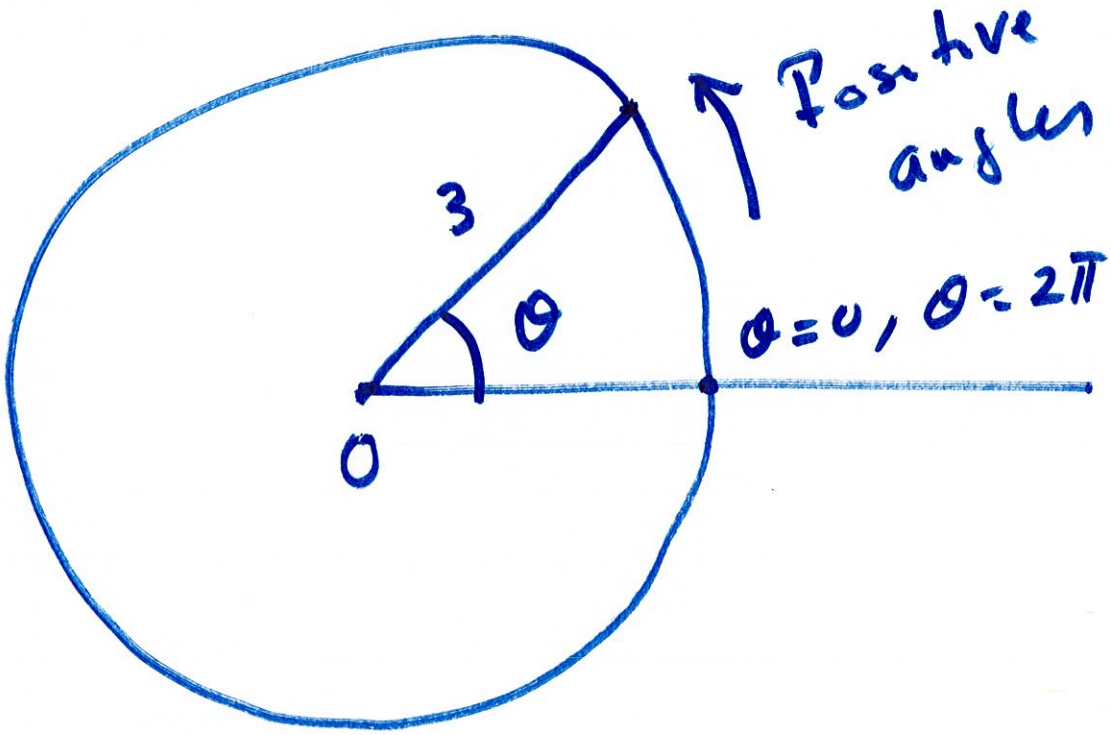
Examples : $(1, \pi/4)$



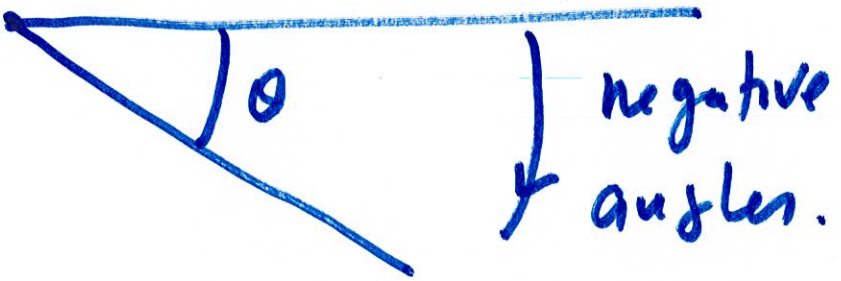
Angle : $\theta = \pi/4$



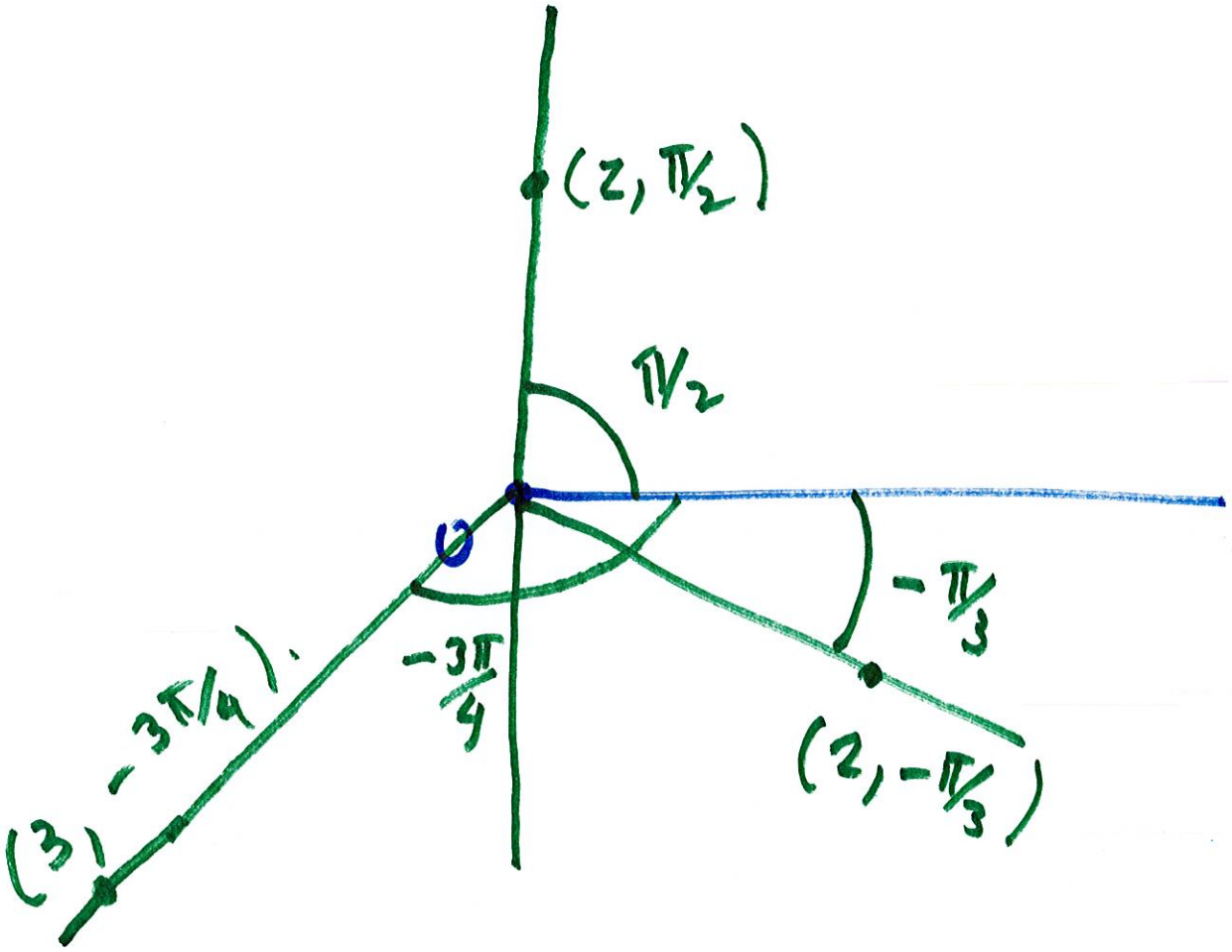
$r = 3$



Range of θ

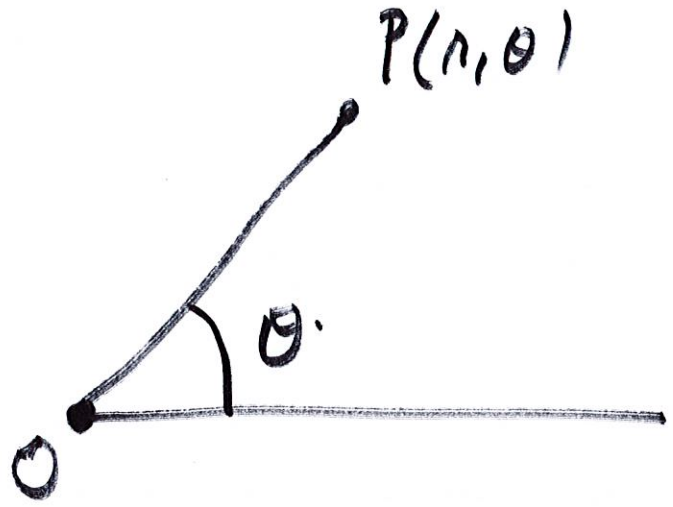


$(2, \pi/2)$; $(2, -\pi/3)$, $(3, -3\pi/4)$



$-\frac{3\pi}{4} = -\frac{\pi}{2} - \frac{\pi}{4}$

We want to extend polar coordinates for $r < 0$

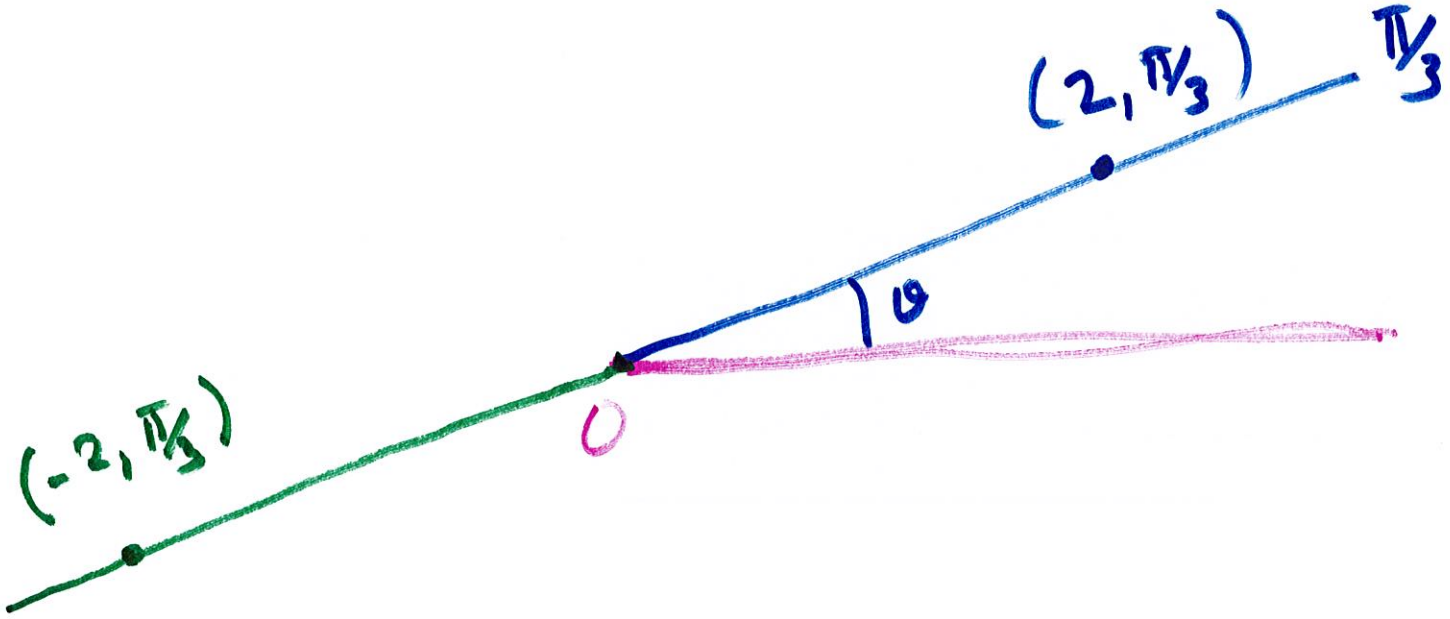


$r = |OP|$

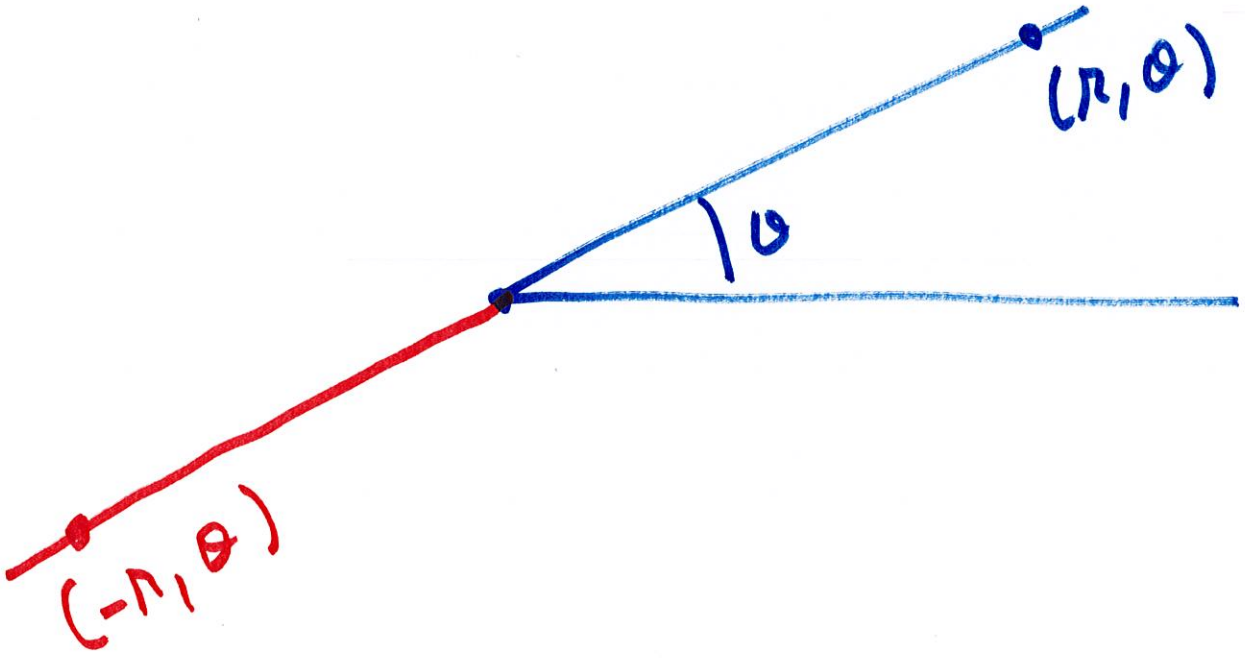
$r = 0$ is the point O.

We want to make sense of

$\left(-2, \frac{\pi}{3}\right)$.

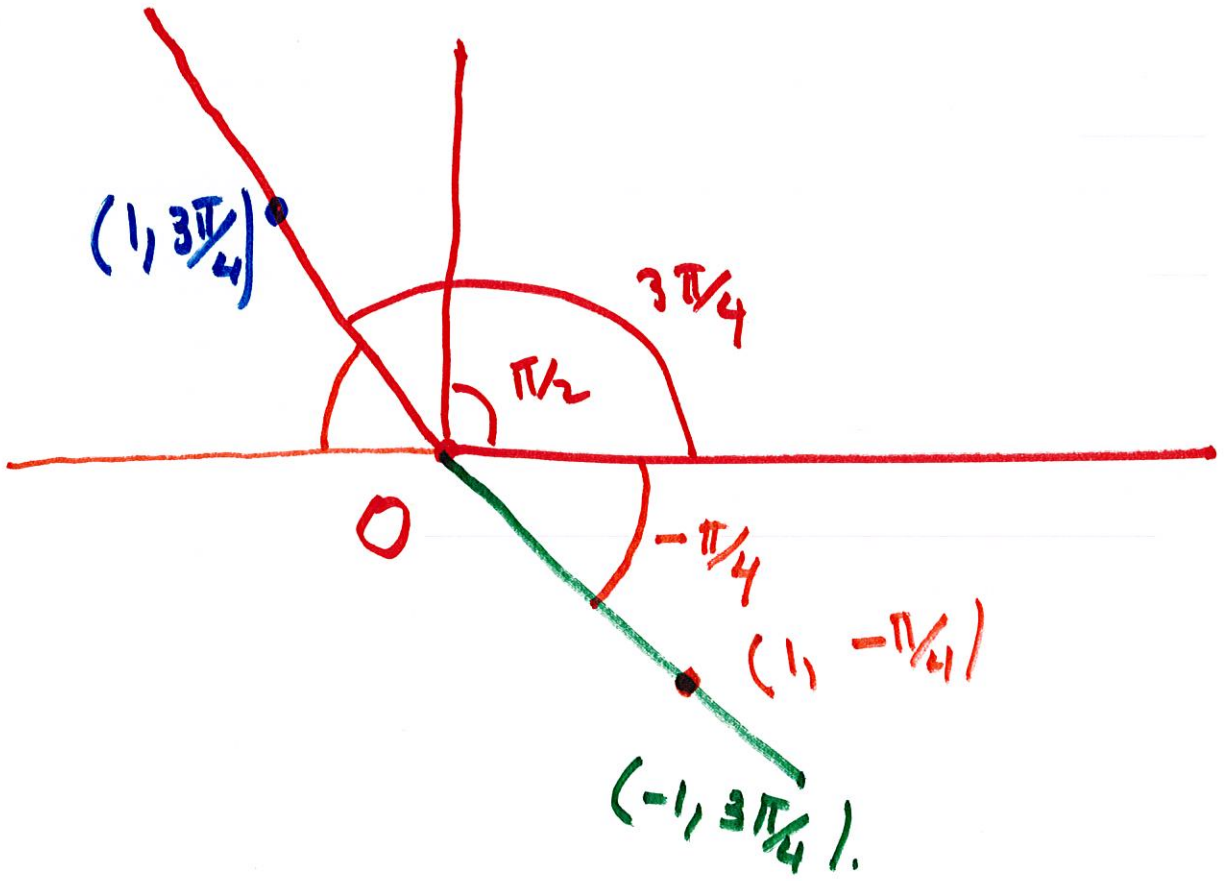


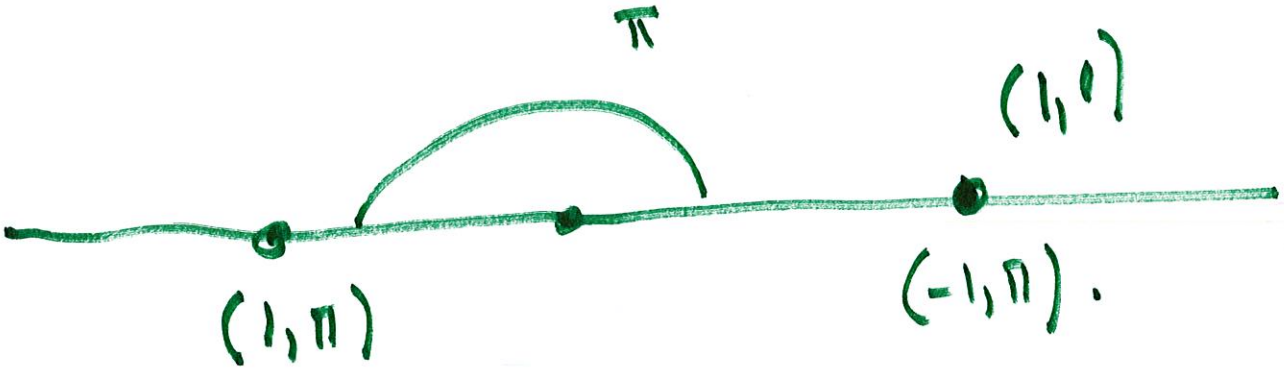
In general $(-r, \theta)$



Examples: $\left(-1, \frac{3\pi}{4}\right)$; $\left(1, -\frac{\pi}{4}\right)$

$(1, \pi)$; $(-1, \pi)$; $(1, 0)$.





$(1, 0) \neq (-1, \pi)$

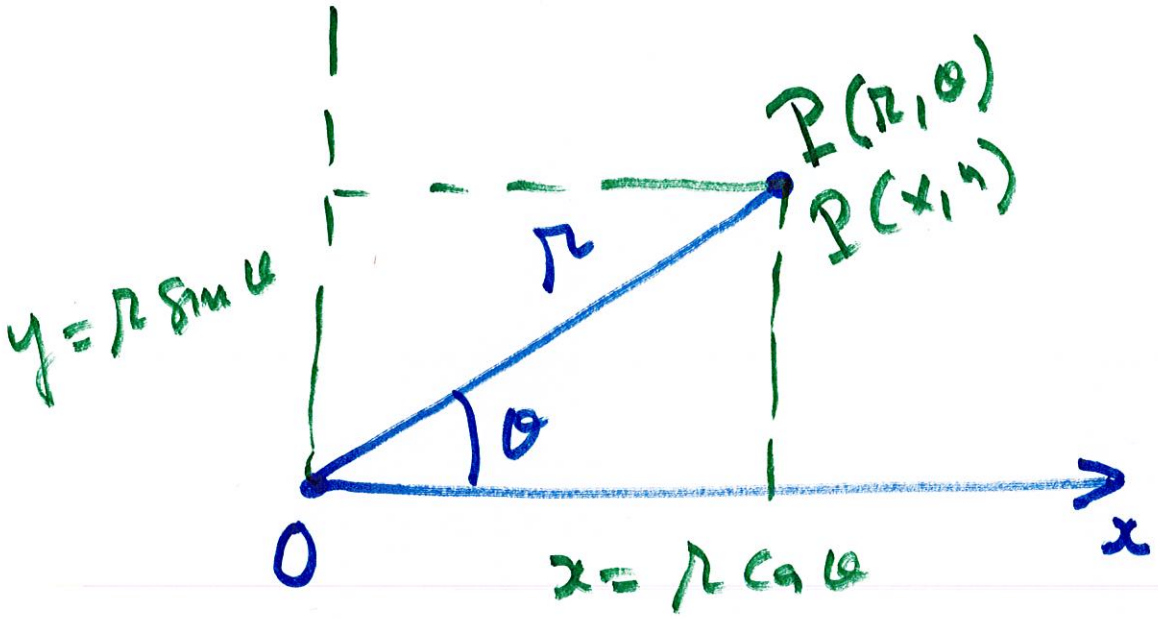
However they represent the same point on the plane.

~~$(1, 0)$~~; $(-0$

$(1, 0)$; $(1, 2\pi)$; $(1, 4\pi)$

$(-1, \pi)$, $(-1, 3\pi)$, $(-1, 5\pi)$

Cartesian Coordinates and Polar Coordinates.



The polar axis = Positive x-axis

$O = (0,0)$

$x = r\cos\theta, \quad y = r\sin\theta.$

$$r = \sqrt{x^2 + y^2}.$$

$$\cos(\theta+\pi) = \cos\theta\cos\pi - \underbrace{\sin\theta\sin\pi}_{0}$$

$$= -\cos\theta$$

$$\sin(\theta+\pi) = \sin\theta\cos\pi + \underbrace{\cos\theta\sin\pi}_{0}$$

$$= -\sin\theta$$

$$\left(r\cos(\theta+\pi),\ r\sin(\theta+\pi) \right)$$

$$= \left(-r\cos\theta,\ -r\sin\theta \right).$$

Express $(1, \pi/4)$, $(2, \pi/3)$ in Cartesian coordinates.

Express $(2, 3)$; $(-1, \sqrt{3})$ in polar coordinates.

$(1, \pi/4)$; $r = 1$, $\theta = \pi/4$

$x = r\cos\theta$, $y = r\sin\theta$

$x = \cos\pi/4$, $y = \sin\pi/4$

$x = \sqrt{2}/2$, $y = \sqrt{2}/2$

$\left(\sqrt{2}/2, \sqrt{2}/2\right)$.

$(2, \pi/3)$; $r = 2$, $\theta = \pi/3$

$x = 2\cos\pi/3$, $y = 2\sin\pi/3$

$x = 2 \cdot \sqrt{3}/2$;

$x = 2 \cdot 1/2$; $y = 2 \cdot \sqrt{3}/2$

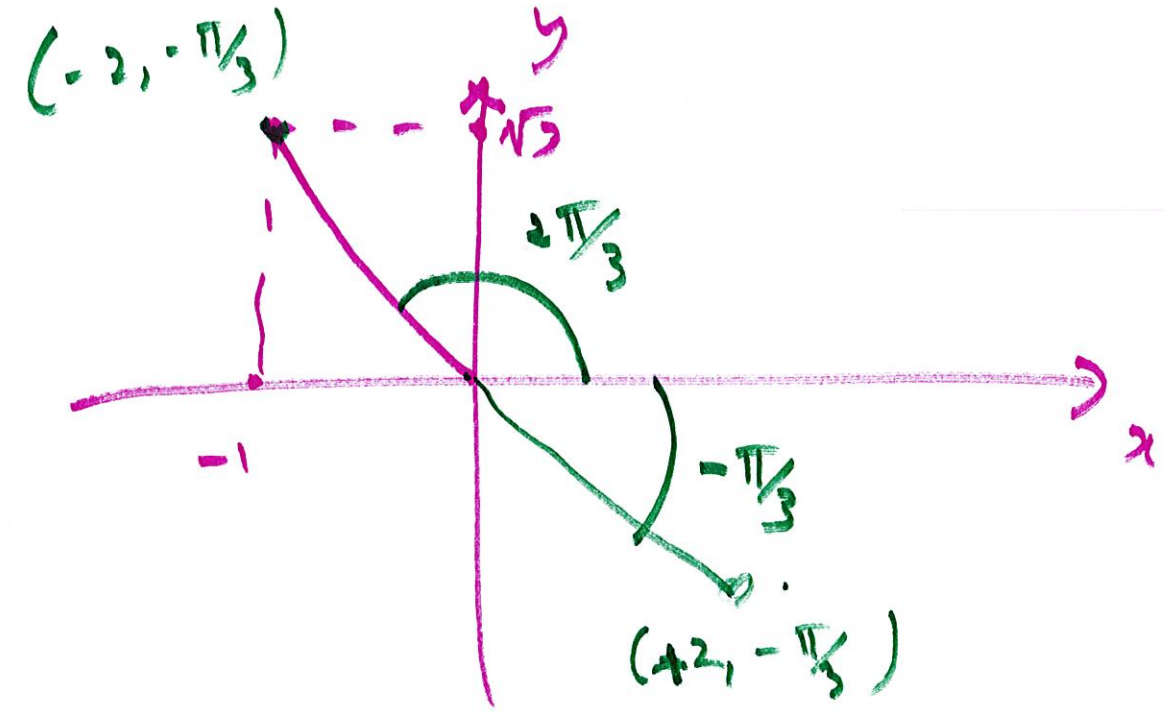
$x = 1$, $y = \sqrt{3}$.

$(1, \sqrt{3})$.

$(-1, \sqrt{3})$ in polar coordinates.

$x = -1$, $y = \sqrt{3}$.

$r = \sqrt{x^2 + y^2} = \sqrt{1+3} = 2$.



$r \cos\theta = x$; $\cos\theta = x/r = -1/2$

$r \sin\theta = y$; $\sin\theta = y/r = \sqrt{3}/2$

$\theta = 2\pi/3$.

$\theta = 2\pi/3 + 2\pi = 8\pi/3$.

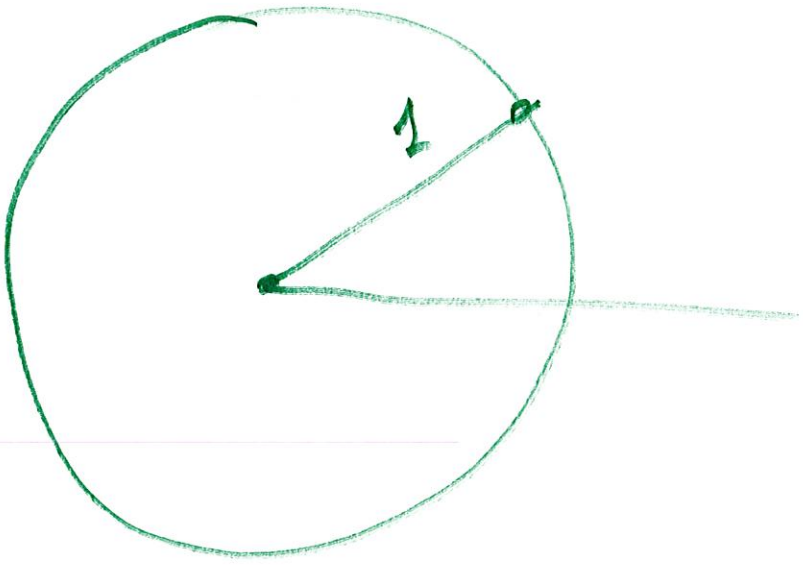
$(-2, -\pi/3)$.

Express $(-1, \sqrt{3})$ in polar coordinates with $r \geq 0$ and θ in $(0, 2\pi)$.

$r = 2, \quad \theta = 2\pi/3.$

Curves in Polar coordinates

$r = 1$



$r = \cos\theta$

multiply by r.

$$r^2 = r\cos\theta$$

$$x^2 + y^2 = x$$

$$x^2 - x + y^2 = 0$$

$$(x-1/2)^2 - 1/4 + y^2 = 0$$

$$(x-1/2)^2 + y^2 = 1/4$$