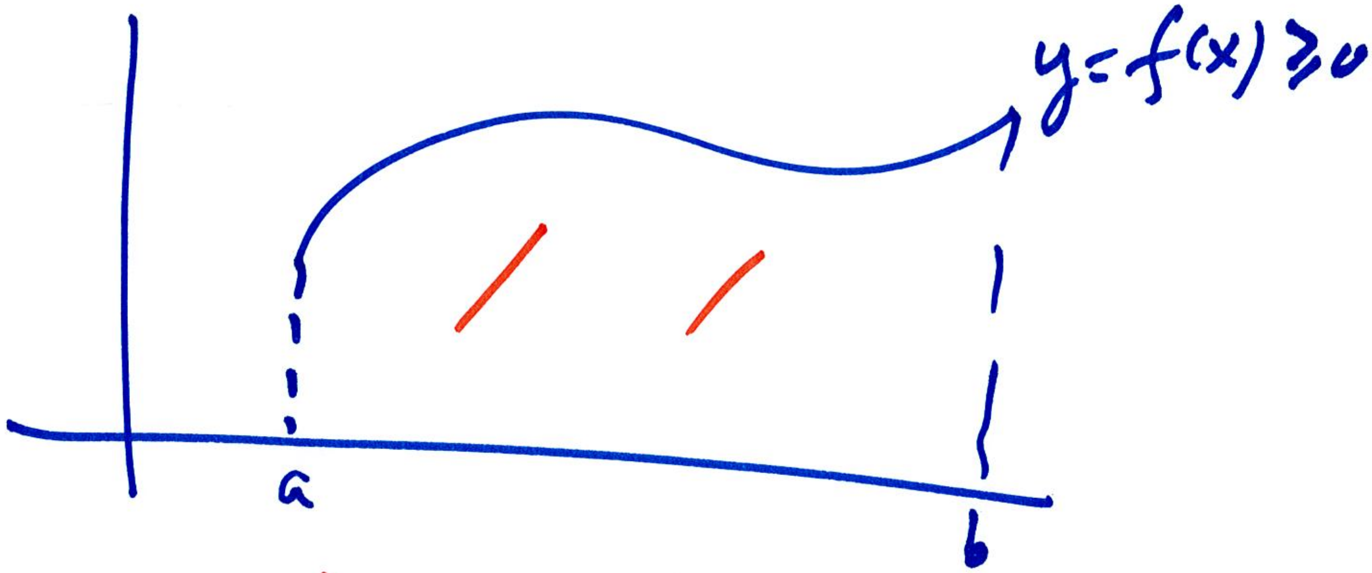
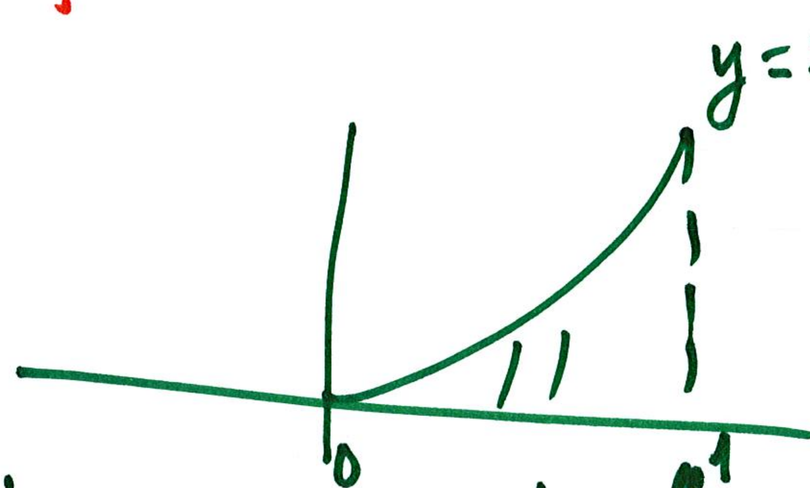
Lesson 5 Section 6.1

Area Between Curves

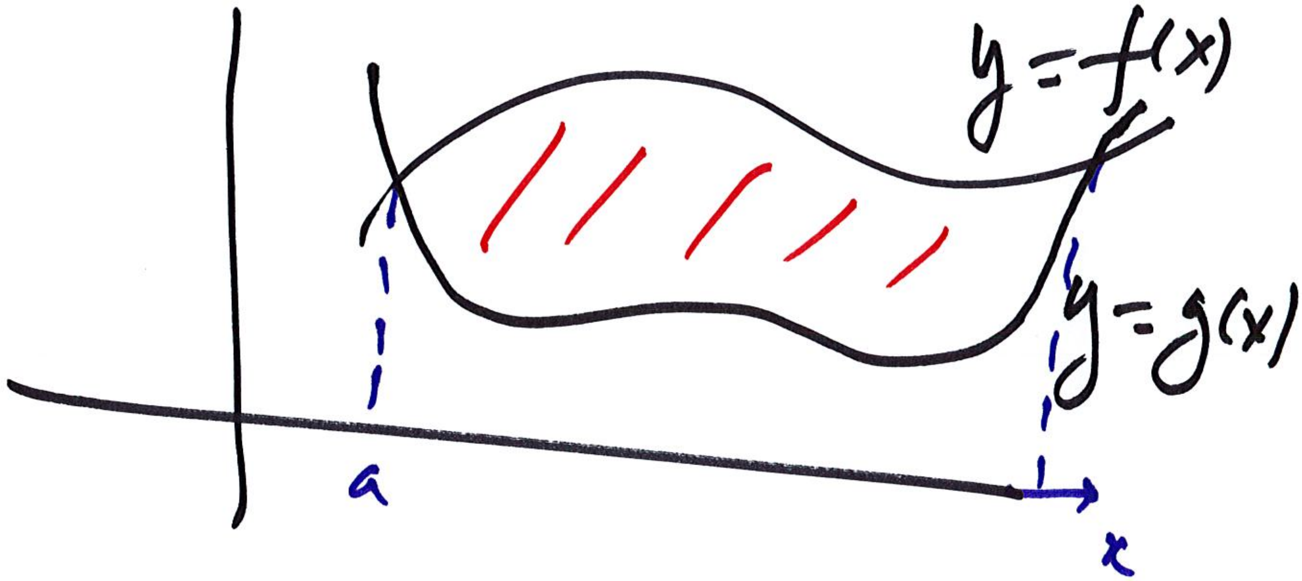


$$A = \int_a^b f(x)\,dx.$$

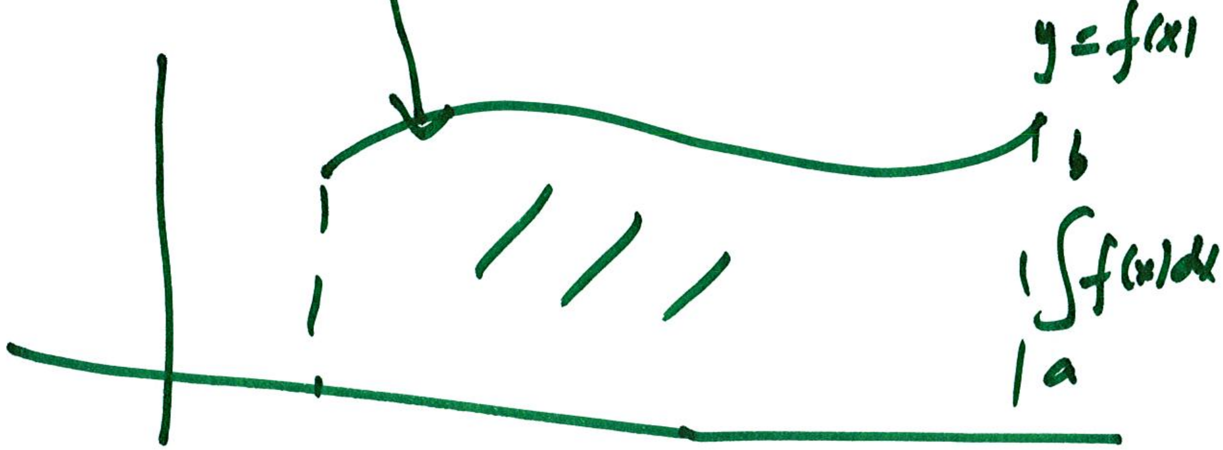
Example:

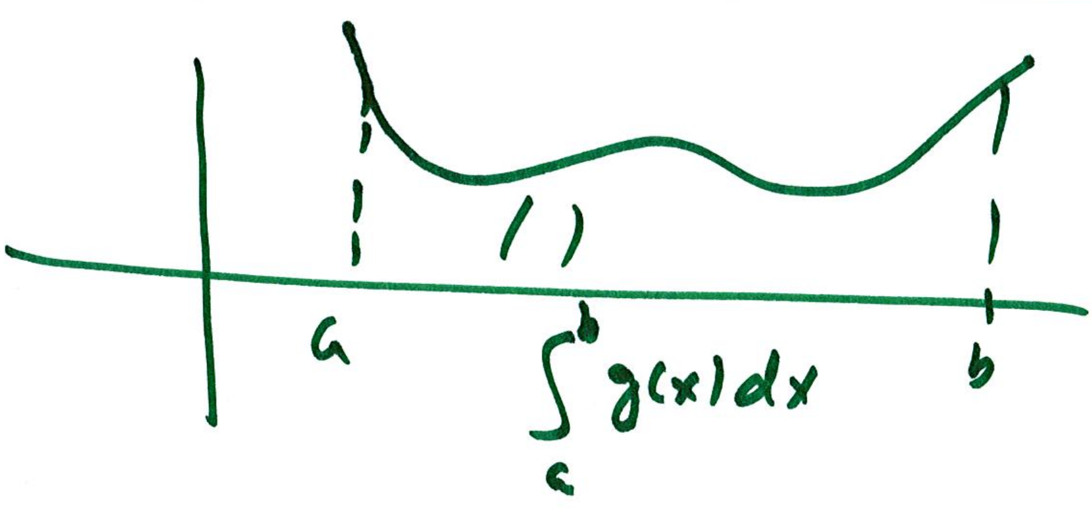


$$A = \int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}.$$



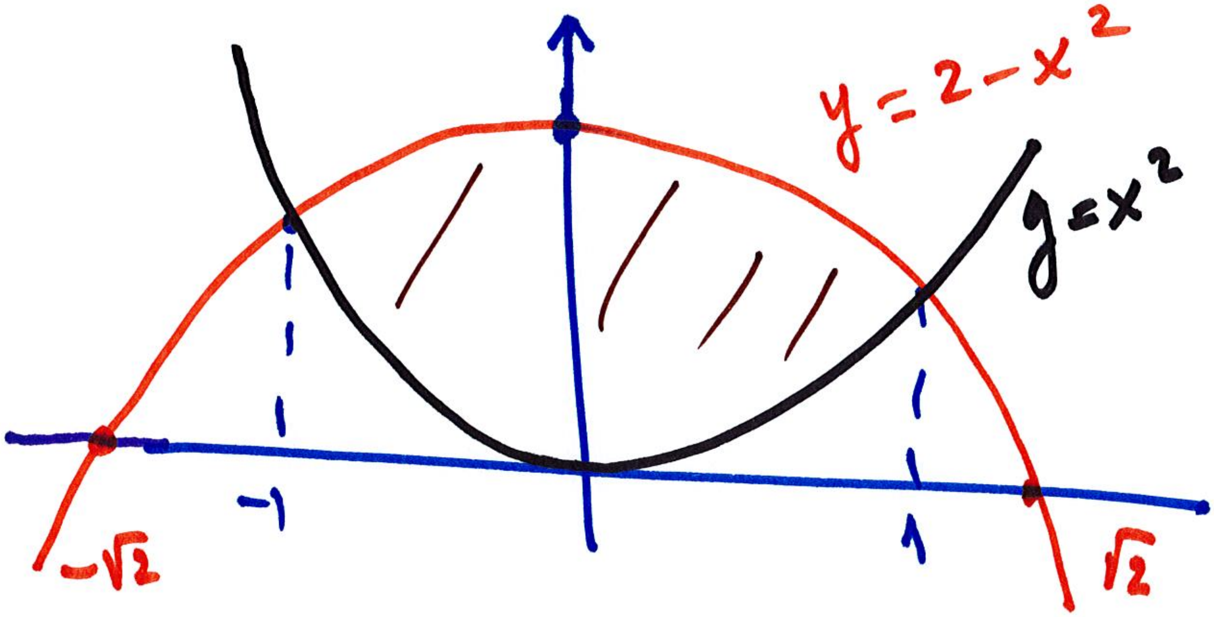
$$A = \int_a^b \left(f(x) - g(x) \right) dx$$





Find the area of "the region" bounded by.

(1) $y = 2 - x^2$, $y = x^2$.



$$\text{Area} = \int_{-1}^{1} \left[(2-x^2) - x^2 \right] dx$$

$2 - x^2 = x^2$ $\quad$ $2 = 2x^2$

$x^2 = 1$ $\quad x = -1$

$x = 1$

$$A = \int_{-1}^{1} (2 - 2x^2)\, dx$$

$$= 2 \int_{-1}^{1} \underbrace{(1 - x^2)}_{\text{even function}}\, dx.$$

$$= 2 \cdot 2 \int_{0}^{1} (1 - x^2)$$

$$= 2 \cdot \left(x - \frac{x^3}{3} \right) \Big|_{-1}^{1}$$

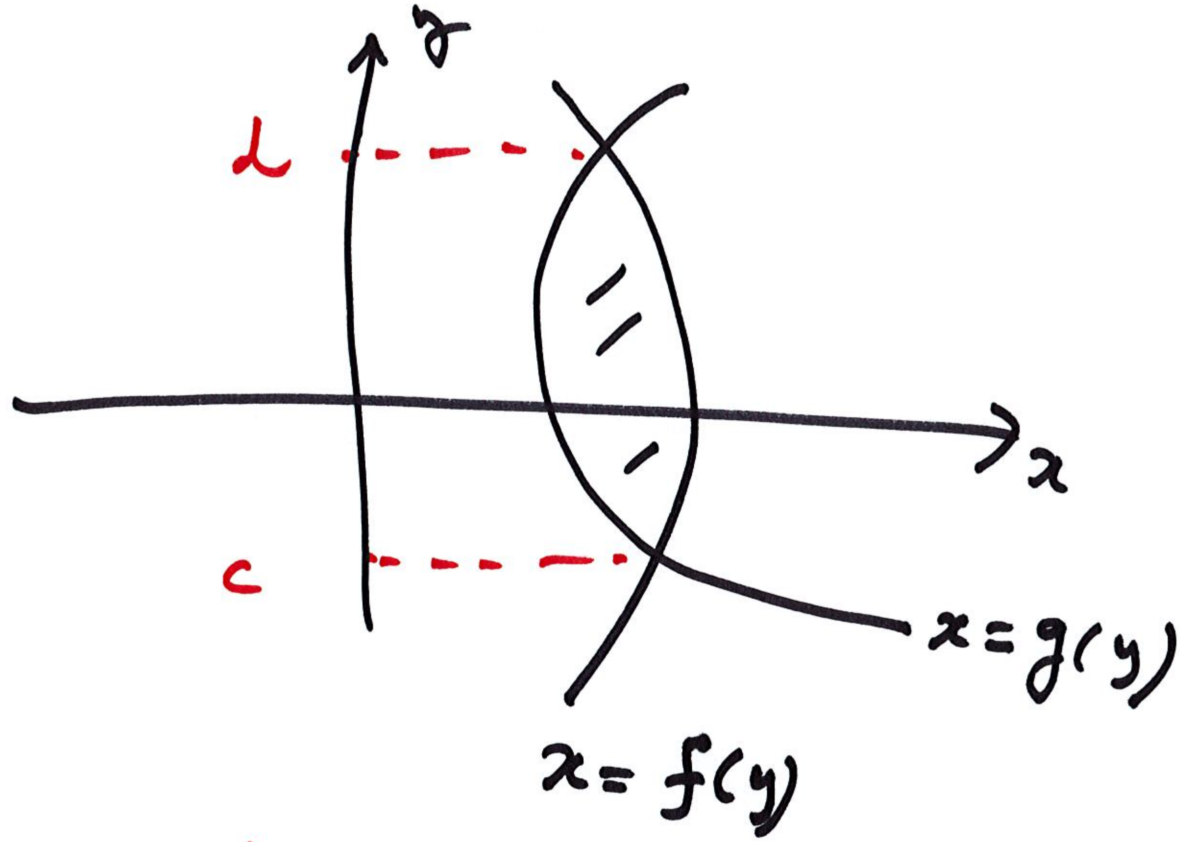
$$= 4 \cdot \left(x - \frac{x^3}{3} \right) \Big|_{0}^{1}$$

$$= 8/3$$

$$= 2 \left(\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right) \right)$$

$$= 2 \left(\left(1 - 1/3\right) + \left(1 - 1/3\right) \right)$$

$$2 \cdot 4/3 = 8/3.$$

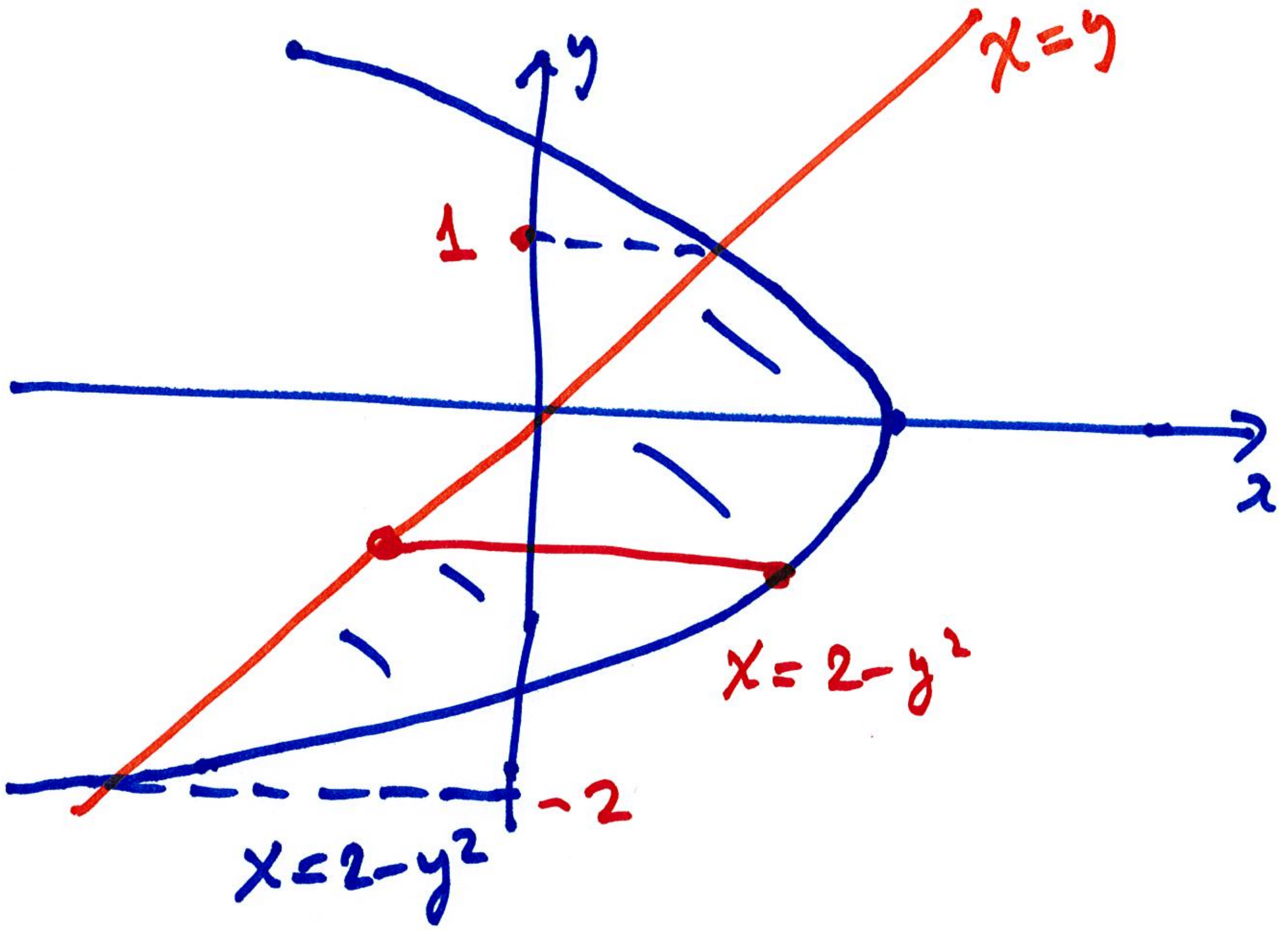


$$A = \int_c^d \left(f(y) - g(y) \right) dy$$

Example: $y = x$ and $y = \pm\sqrt{2 - x}$.

$$y = \pm\sqrt{2-x}\ .$$

$$y^2 = 2 - x \quad ; \quad x = 2 - y^2 .$$



$$A = \int_{-2}^{1} \left[2 - y^2 - y \right] dy\ .$$

The curves intersect when

$$2-y^2=y.$$

$$y^2+y-2=0$$

$$(y+2)(y-1)=0$$

$$y=-2, \quad y=1$$

$$A = \int_{-2}^{1} (2 - y^2 - \underline{y})\, dy$$

$$= 2y - \frac{y^3}{3} - \frac{y^2}{2} \Big|_{-2}^{1}$$

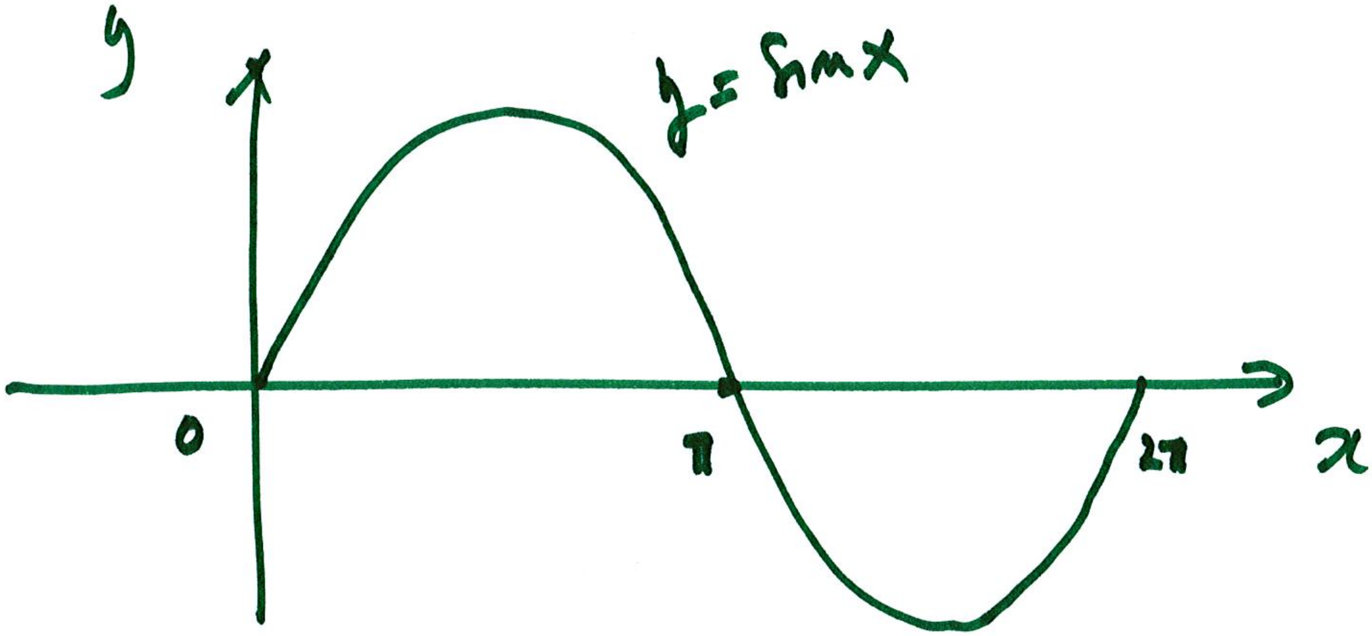
$$= \left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 + \frac{8}{3} - \frac{4}{2}\right)$$

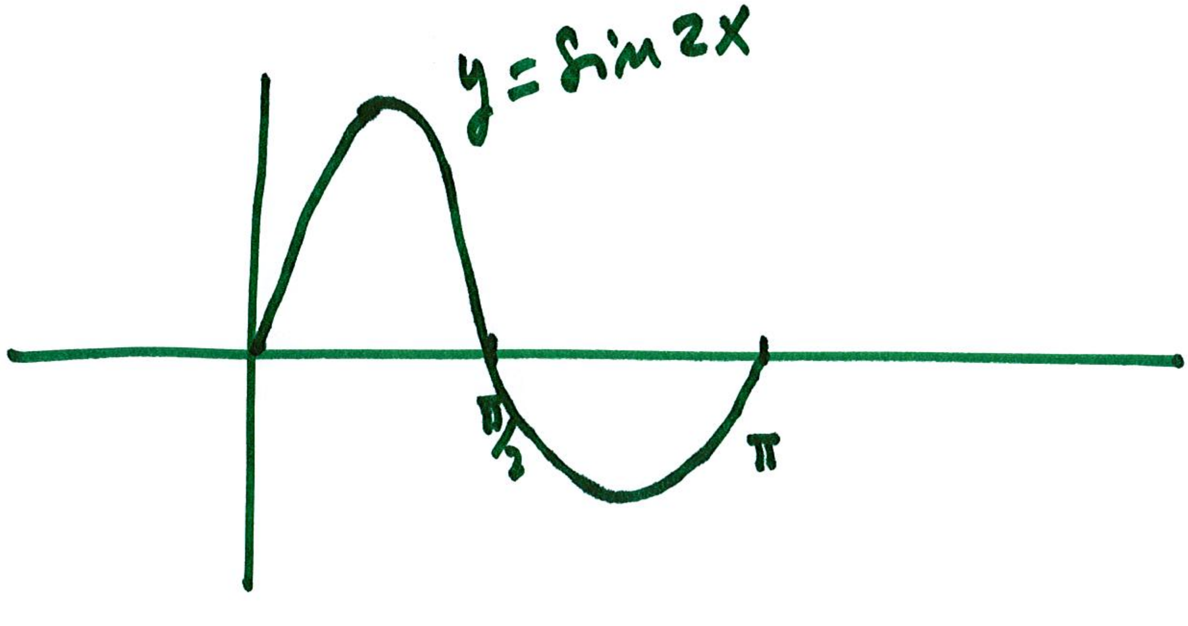
$$= \frac{7}{6} - \left(-6 + \frac{8}{3}\right)$$

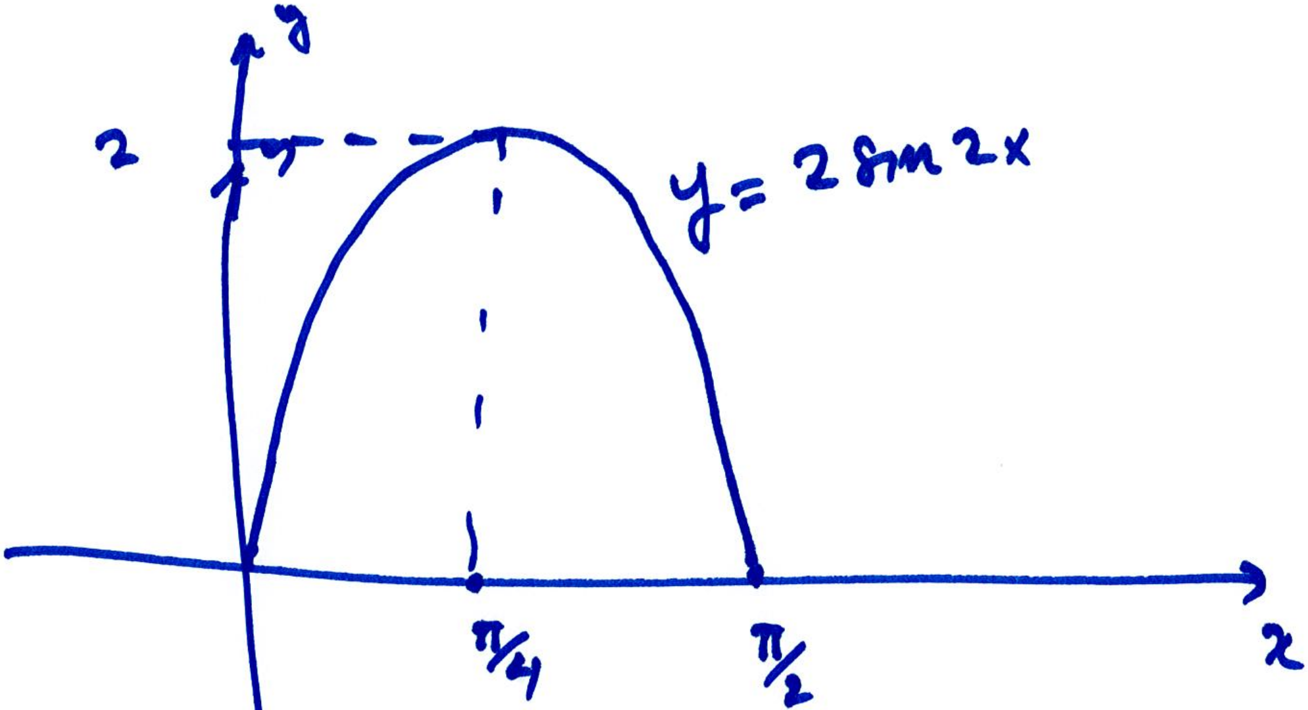
$$= \frac{7}{6} + \frac{10}{3} = \frac{27}{6}.$$

$y = 2\sin 2x$, $y = 2 - 2\sin 2x$

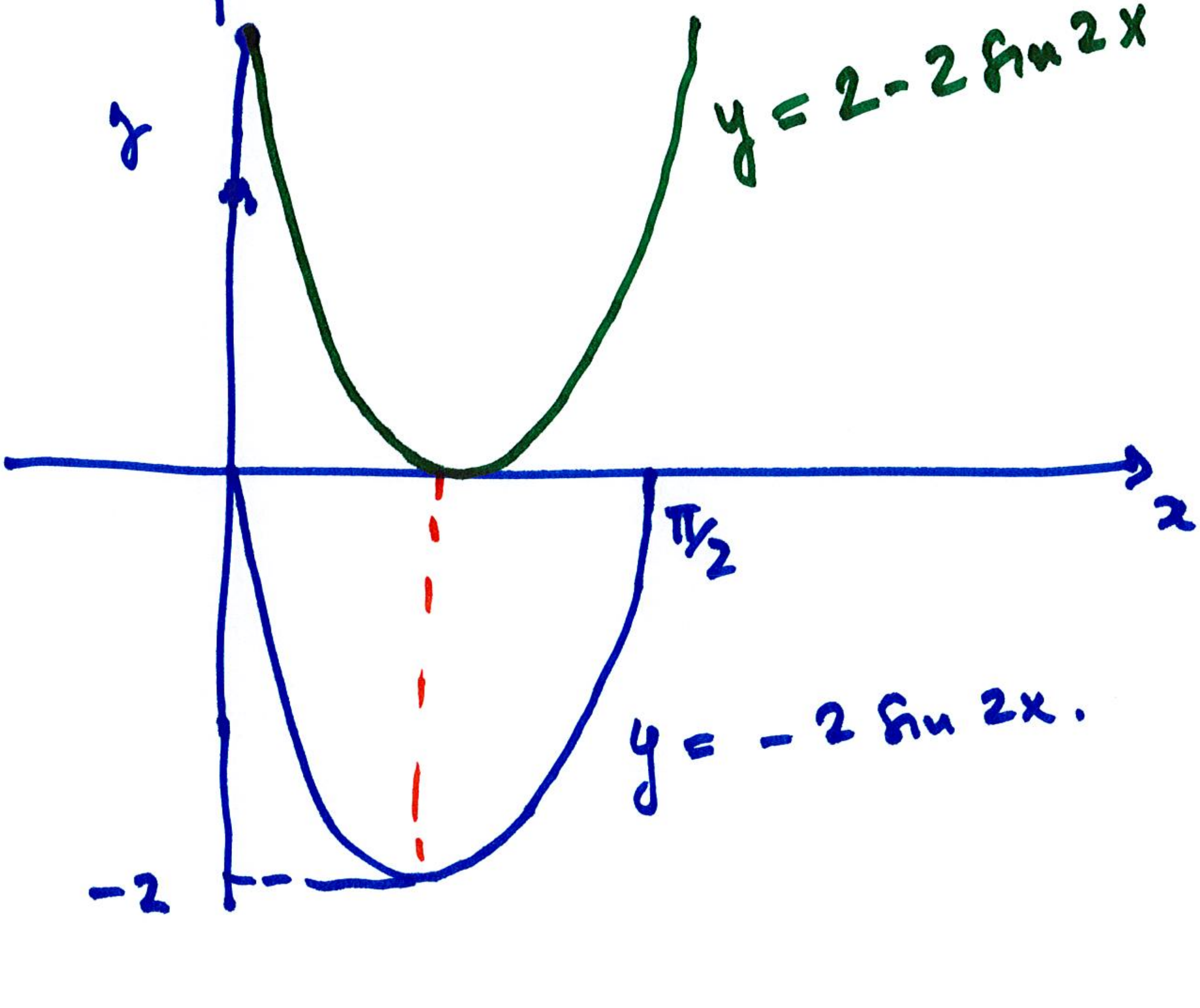
$0 \leq x \leq \pi/2$.



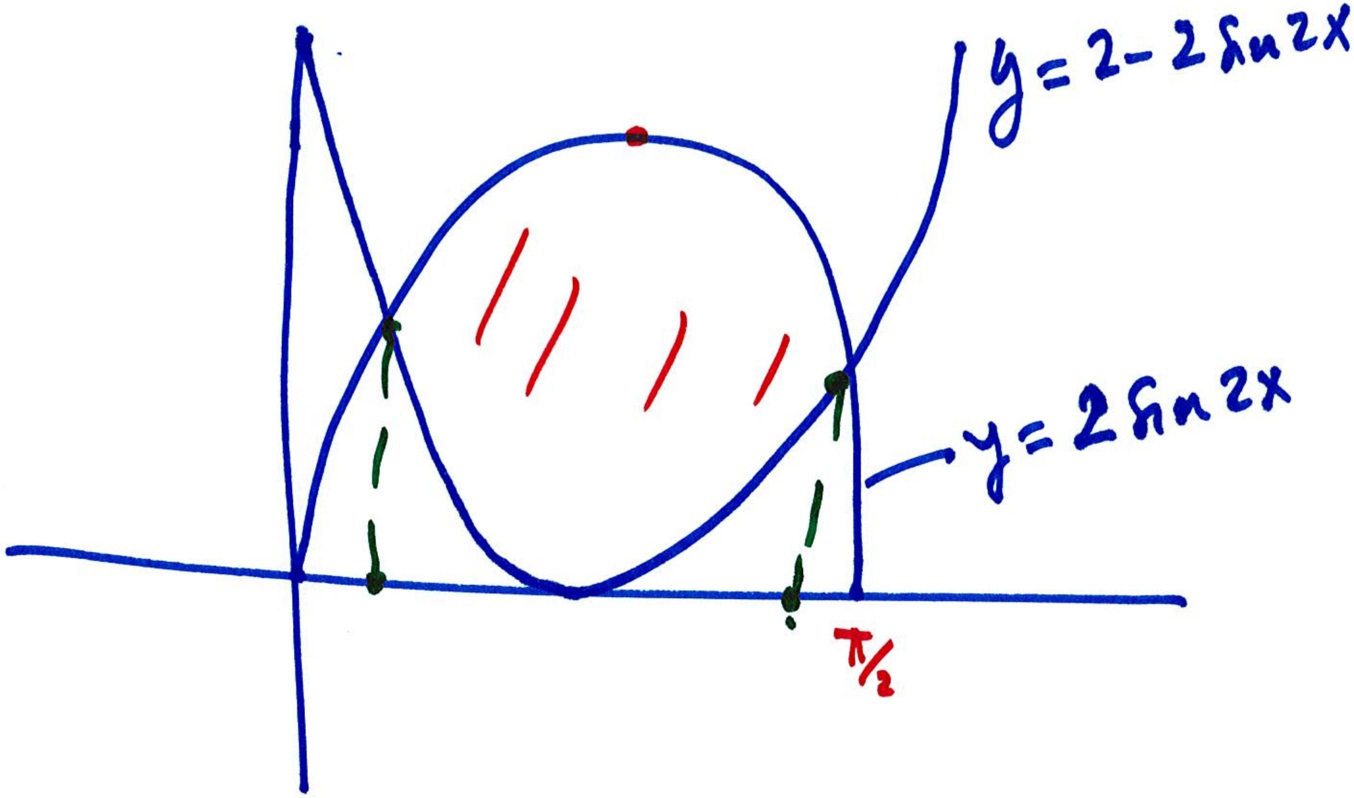




y
2
y = 2 sin 2x
π/4
π/2
x



y = 2 - 2 sin 2x
y
x
π/2
y = - 2 sin 2x.
-2



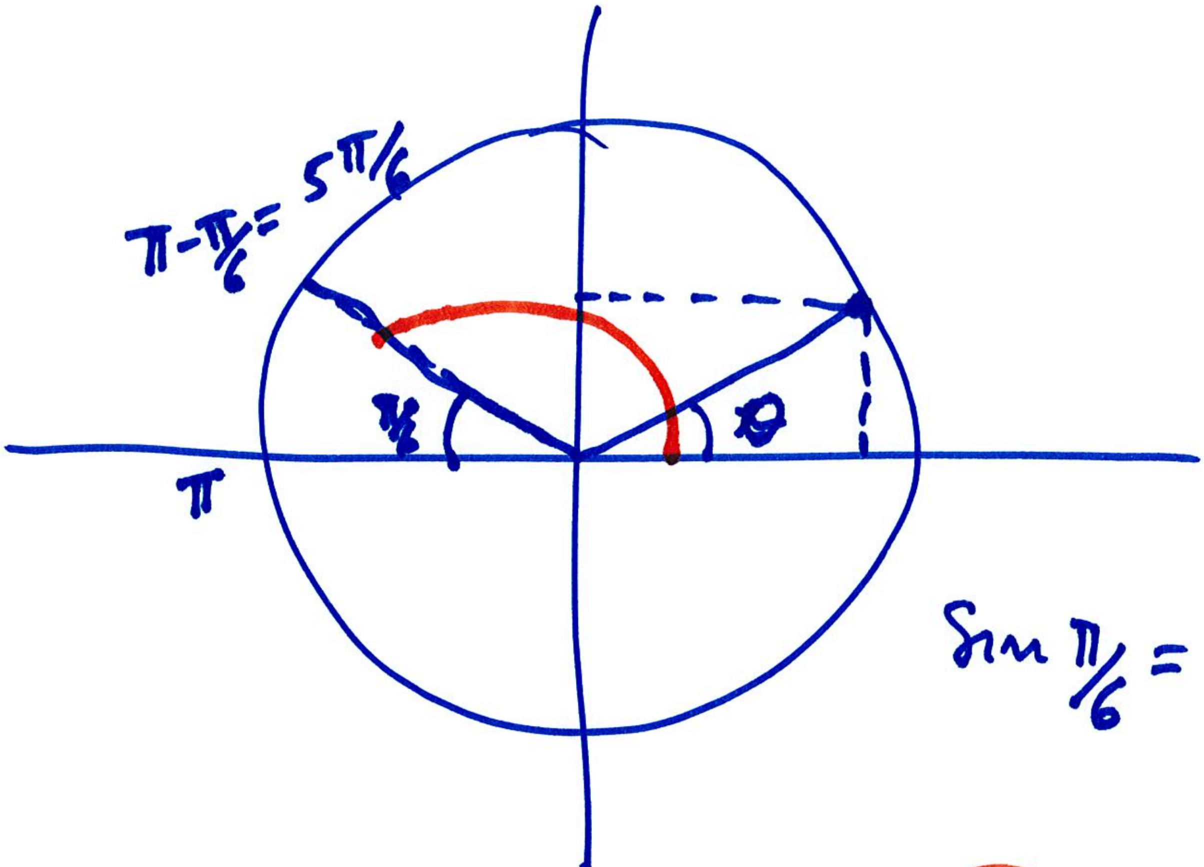
$$A = \int_{\pi/12}^{5\pi/12} \left[2\sin 2x - (2 - 2\sin 2x) \right] dx$$

The curves intersect when

$$2 - 2\sin 2x = 2\sin 2x$$

$$2 = 4\sin 2x$$

$$\boxed{\sin 2x = \frac{1}{2}}$$



$\sin \frac{\pi}{6} = \frac{1}{2}$

Two possibilities $\quad 2x = \frac{\pi}{6}$

$$2x = \frac{5\pi}{6}$$

$x = \frac{\pi}{12}, \quad x = \frac{5\pi}{12}.$

$$A = \int_{\pi/12}^{5\pi/12} (4 \sin 2x - 2) \, dx$$

$$= 4 \cdot \left(- \frac{\cos 2x}{2} \right) - 2x \Big|_{\pi/12}^{5\pi/12}$$

$$= - 2 \cos 2x - 2x \Big|_{\pi/12}^{5\pi/12}$$

$$= -2 \left(\cos \frac{5\pi}{6} - \cos \frac{\pi}{6} \right)$$

$$-2 \left(\frac{5\pi}{12} - \frac{\pi}{12} \right)$$

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}, \quad \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$A = -2\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) - 2 \cdot \frac{4\pi}{12}$$

$$= 2\sqrt{3} - \frac{2\pi}{3}.$$