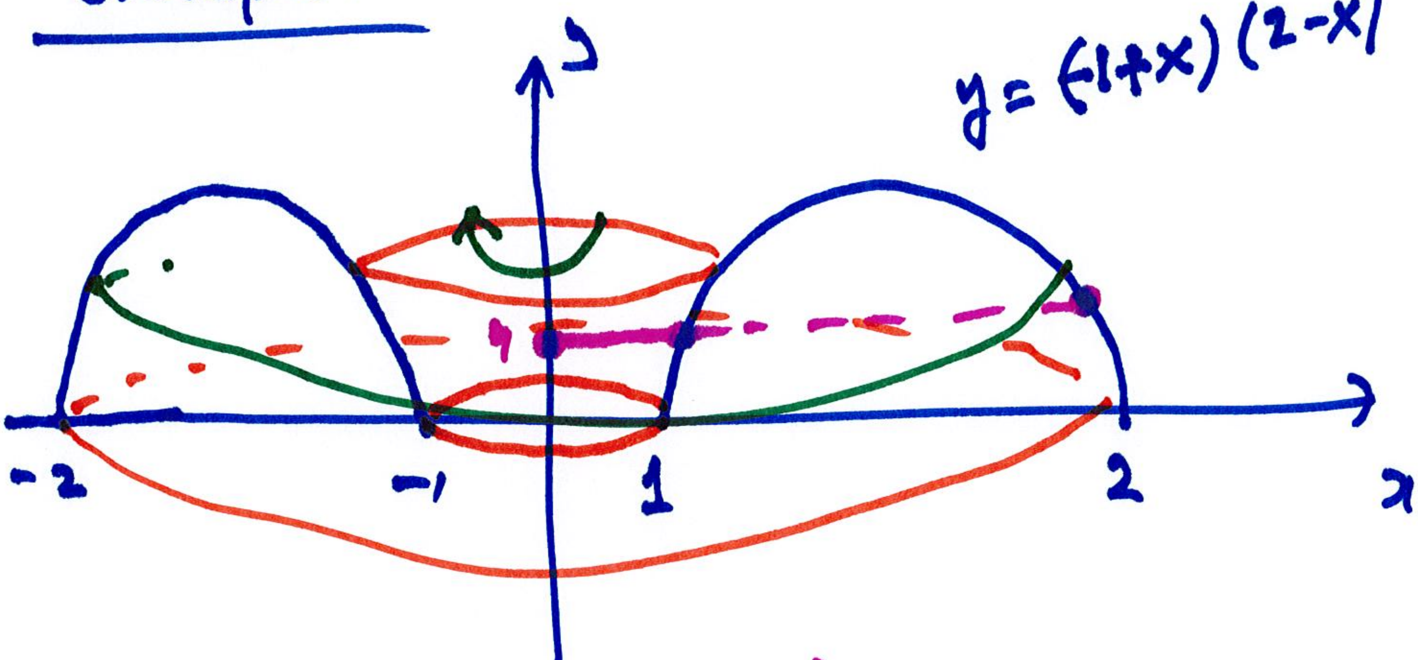
Lesson 7 Section 6.3

Volumes by Cylindrical Shells.

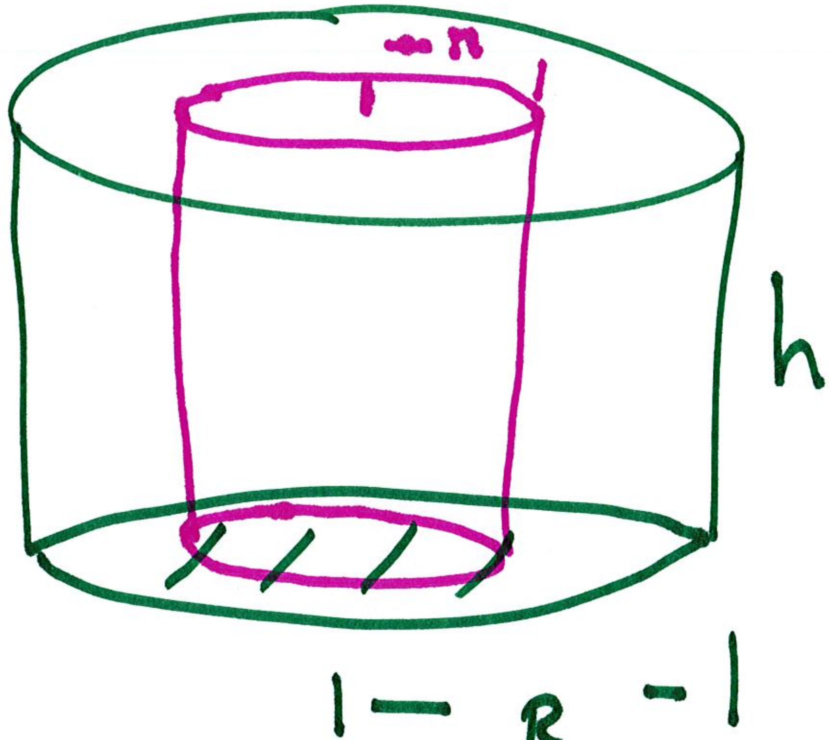
Example:



$$y = (x-1)(2-x)$$

$$y = 2x - x^2 - 2 + x = 3x - x^2 - 2$$

$$\boxed{-x^2 + 3x - 2 - y = 0}$$



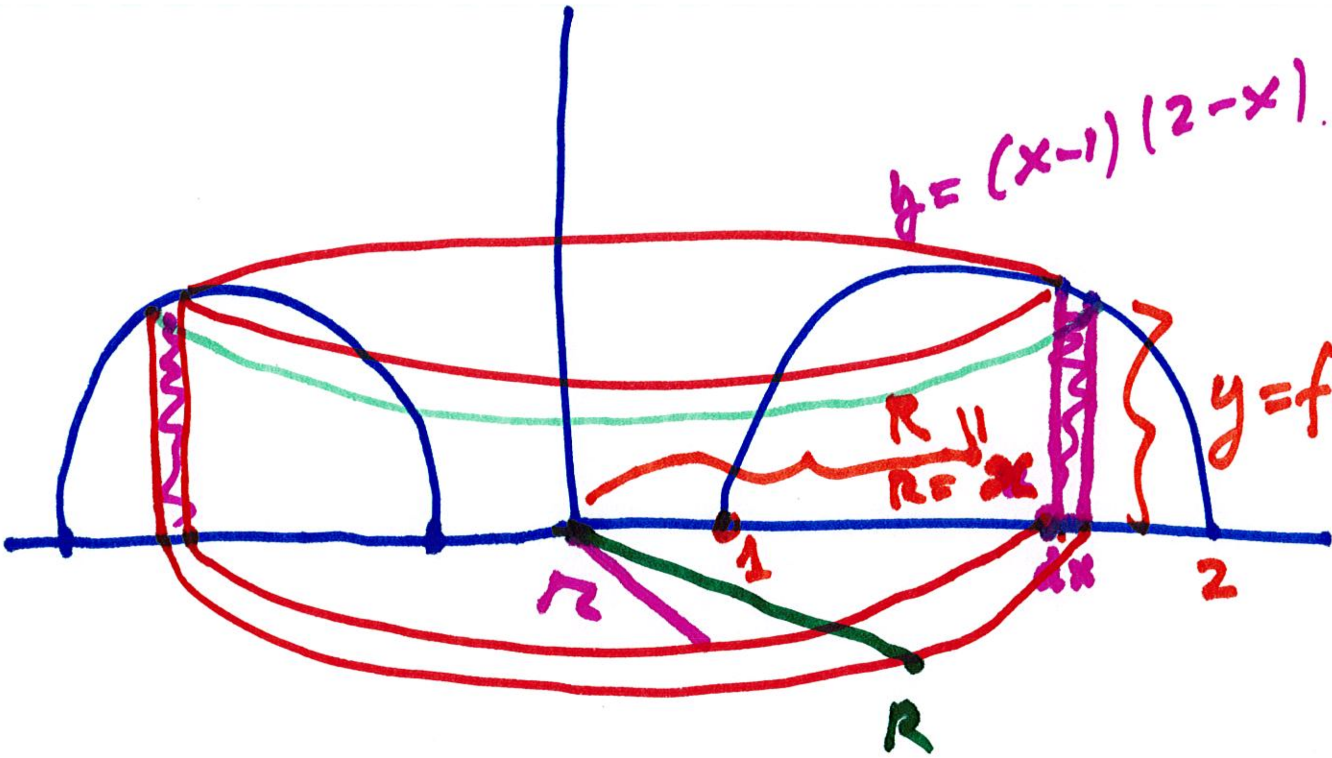
$$V = \pi R^2 \cdot h - \pi r^2 h$$

$$V = \pi h \left(R^2 - r^2\right)$$

$$= \pi h (R+r)(R-r)$$

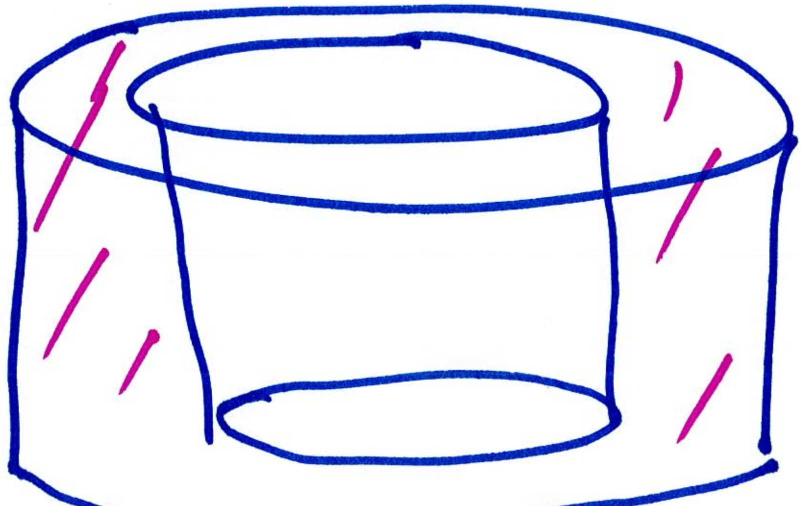
$$R - r = dx.$$

$$V = \pi h . \left(\underbrace{R+r}_{2x}\right) dx.$$



$r + R \cong 2R$

Cylindrical shell



Conclusion: $\boxed{dV = 2\pi \cdot x \cdot f(x)\, dx}$

$$V = \int_1^2 2\pi\, x\, f(x)\, dx$$

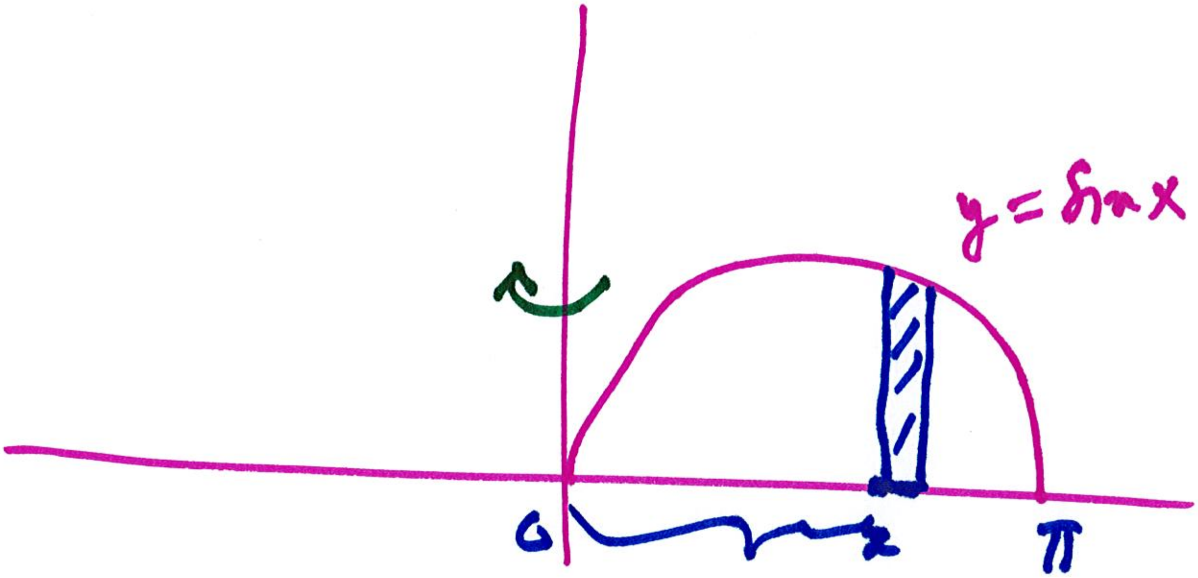
$$f(x) = (x-1)(2-x)$$

$$V = \int_1^2 2\pi x\,(x-1)(2-x)\, dx$$

$$= 2\pi \int_1^2 x\,(2x - x^2 - 2 + x)\, dx$$

$$= 2\pi \int_1^2 x\,(3x - x^2 - 2)\, dx$$

$$= 2\pi \int_1^2 (3x^2 - x^3 - 2x)\, dx$$



$$dV = 2\pi \cdot \underbrace{\text{radius}}_{x} \cdot \underbrace{\text{height}}_{\sin x} \, dx$$

$$dV = 2\pi \cdot x \cdot \sin x \, dx$$

$$V = 2\pi \int_0^{\pi} x \sin x \, dx.$$

$$\frac{d}{dx}(x \cos x) = \cos x - x \sin x$$

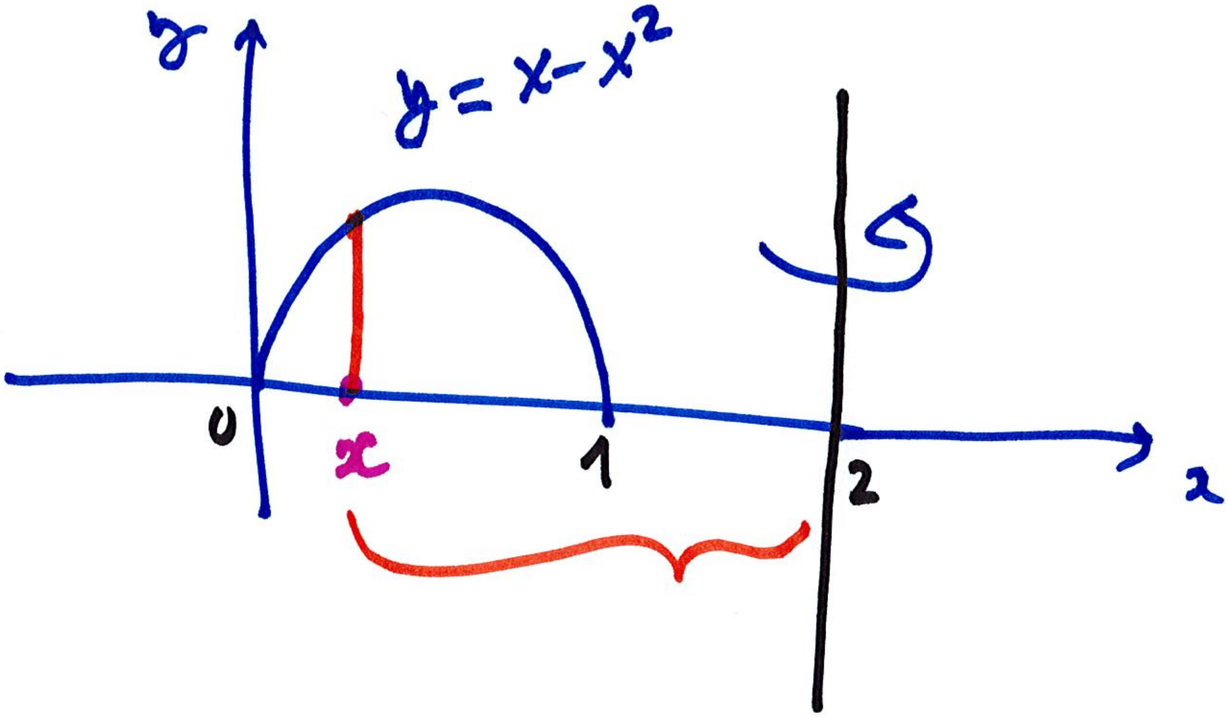
$$x \sin x = -\frac{d}{dx}(x \cos x) + \cos x.$$

$$\underbrace{\int_0^\pi x \sin x \, dx} = \int_0^\pi \left[-\frac{d}{dx}(x \cos x) + \cos x \right] dx$$

$$-\left(x \cos x \Big|_0^\pi \right) + \sin x \Big|_0^\pi$$

$$= -\pi \cos \pi = \pi.$$

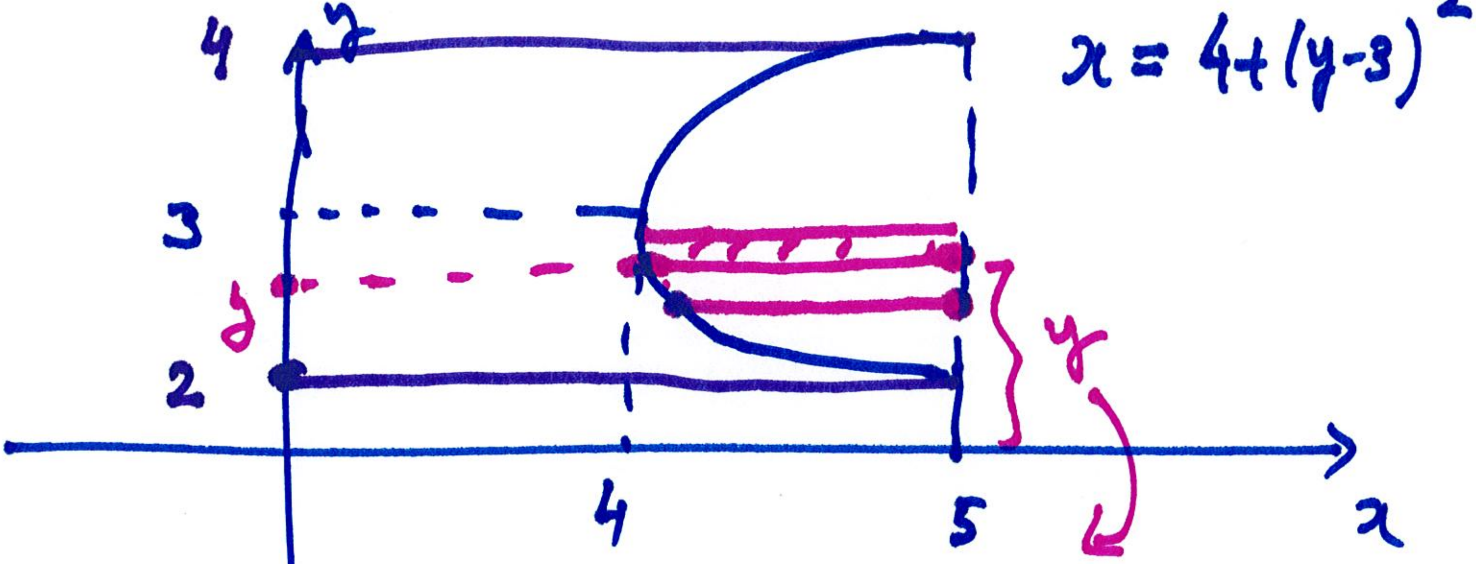
$$V = 2\pi . \pi = 2\pi^2.$$



$$dV = 2\pi \cdot \underbrace{\text{radius}}_{2-x} \cdot \underbrace{\text{height}}_{x-x^2} \cdot dx$$

$$dV = 2\pi\,(2-x)(x-x^2)\,dx.$$

$$\boxed{V = \int_0^1 2\pi\,(2-x)(x-x^2)\,dx}$$



When $x = 5 = 4 + (y-3)^2$

$(y-3)^2 = 1$

$y - 3 = 1$

or $y - 3 = -1$

$$dV = 2\pi \cdot \underbrace{\text{radius}}_{y} \cdot \text{height} \cdot dy$$

$$\cancel{\text{height} = \left(5 - \left(4 + (y+3)^2\right)\right)}$$

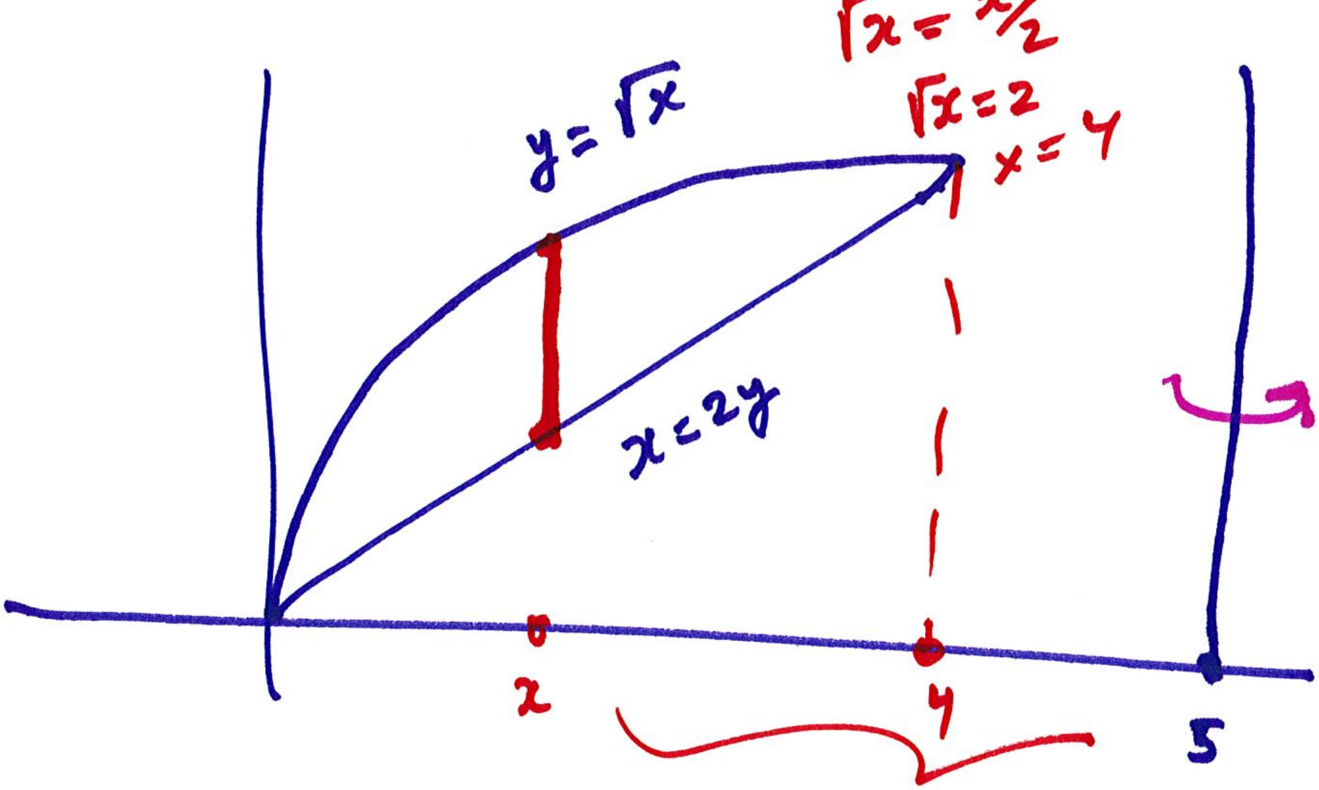
$$\text{height} = 5 - \left(4 + (y-3)^2\right)$$

$$= 1 - (y-3)^2.$$

$$V = 2\pi \int_{2}^{4} y\left(1-(y-3)^2\right) dy$$

$$= 2\pi \int_{2}^{4} y\left(1-y^2+6y-9\right) dy$$

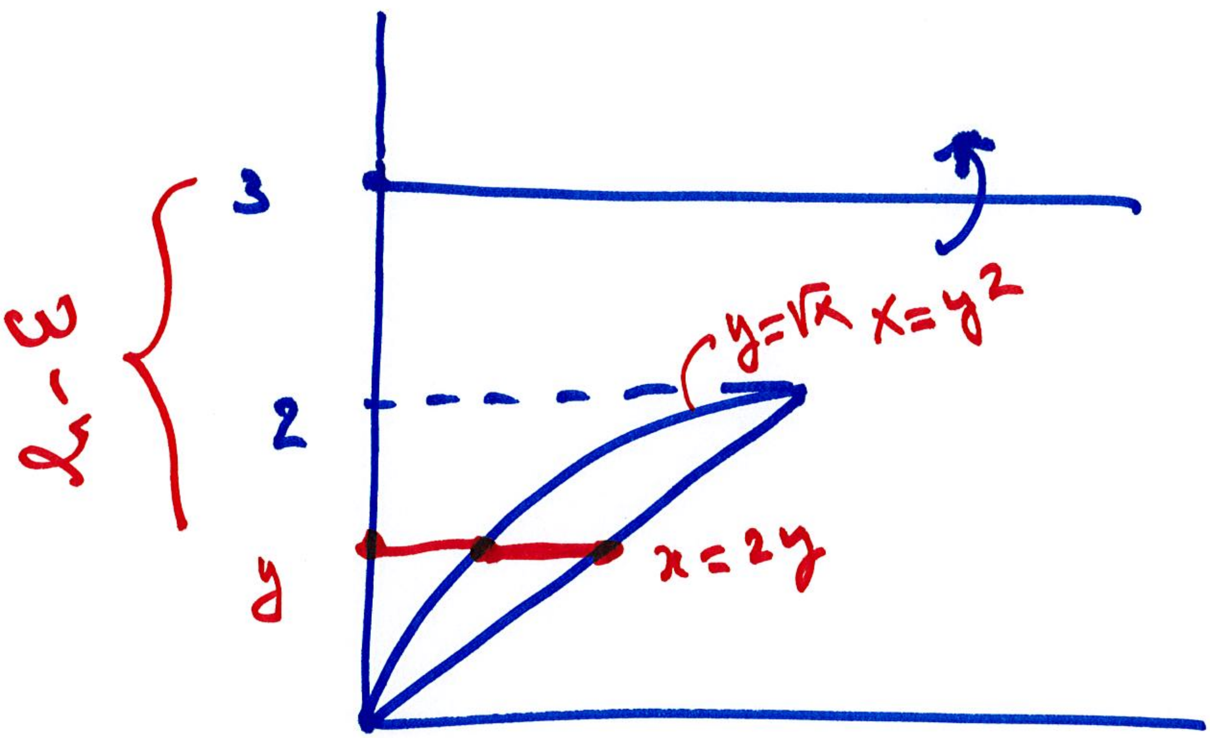
$$= 2\pi \int_{2}^{4} \left(6y^2-8y-y^3\right) dy$$



$$dV = 2\pi \cdot \text{radius} \cdot \text{height} \cdot dx$$

$$= 2\pi \cdot (5-x) \cdot \left(\sqrt{x} - x/2\right) dx$$

$$V = 2\pi \int_0^4 (5-x)\left(\sqrt{x} - x/2\right) dx$$



$$dV = 2\pi \cdot \text{Radius} \cdot \text{height} \cdot dy$$

$$= 2\pi \cdot (3-y)(2y-y^2)\,dy.$$

$$V = 2\pi \int_0^2 (3-y)(2y-y^2)\,dy.$$