

MA166 — EXAM I — FALL 2018 — SEPTEMBER 21, 2018
TEST NUMBER 11– SOLUTIONS

INSTRUCTIONS:

1. Do not open the exam booklet until you are instructed to do so.
2. Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers on this test booklet.
6. The exam has 10 problems and each one is worth 10 points. The maximum possible score is 100 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 40 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

DON'T BE A CHEATER:

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have doubts, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, given it to your instructor and left the room.
5. Do not consult notes or books.
6. **Do not handle** phones or cameras, calculators or any electronic device until after you have finished your exam, given it to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: SOLUTIONS

STUDENT SIGNATURE: _____

STUDENT ID NUMBER: _____

SECTION NUMBER AND RECITATION INSTRUCTOR: _____

1. Find the value of a such that the vectors $\vec{u} = \langle -a, 2, 1 \rangle$ and $\vec{v} = \langle -1, a, 1 \rangle$ are perpendicular

A. $a = 1$

B. $a = \frac{1}{3}$

C. $a = \frac{1}{4}$

D. $a = -\frac{1}{3}$

E. $a = -\frac{1}{4}$

$\vec{u} \cdot \vec{v} = 0$ if and only if \vec{u} and \vec{v} are perpendicular.

$$\vec{u} \cdot \vec{v} = a + 2a + 1 = 0$$

$$3a + 1 = 0$$

$$\boxed{a = -\frac{1}{3}}$$

2. Find the center and the radius of the sphere $4x^2 + 4y^2 + 4z^2 - 4x - 12y + 8z + 10 = 0$.

A. Center $(\frac{1}{2}, -\frac{5}{4}, -1)$ and radius 2

B. Center $(\frac{1}{2}, \frac{3}{4}, 1)$ and radius 2

C. Center $(\frac{1}{2}, \frac{3}{4}, 1)$ and radius 4

D. Center $(\frac{1}{2}, \frac{3}{2}, -1)$ and radius 1

E. Center $(\frac{1}{2}, \frac{3}{2}, -1)$ and radius 2

Divide the equation by 4

$$x^2 + y^2 + z^2 - x - 3y + 2z + \frac{10}{4} = 0$$

$$(x - \frac{1}{2})^2 - \frac{1}{4} + (y - \frac{3}{2})^2 - \frac{9}{4}$$

$$+ (z + 1)^2 - 1 + \frac{10}{4} = 0$$

$$(x - \frac{1}{2})^2 + (y - \frac{3}{2})^2 + (z + 1)^2 = 1$$

Center $(\frac{1}{2}, \frac{3}{2}, -1)$, Radius 1.

3. Find the area of the triangle with vertices $P(1, 1, 2)$, $Q(1, 2, 3)$ and $R(0, 1, 1)$.

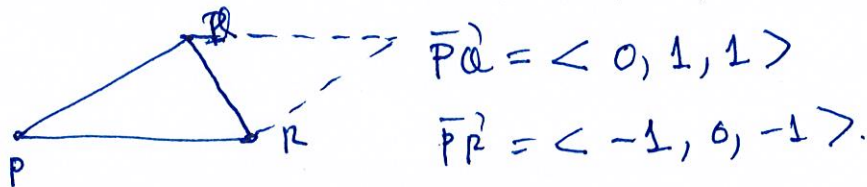
A. $\sqrt{3}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{\sqrt{3}}{3}$

D. $\frac{\sqrt{5}}{2}$

E. 2



Area of the Triangle = $\frac{1}{2}$ Area of the parallelogram = $\frac{1}{2} | \vec{PQ} \times \vec{PR} |$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i}' & \hat{j}' & \hat{k}' \\ 0 & 1 & 1 \\ -1 & 0 & -1 \end{vmatrix} = \hat{i}'(-1) - \hat{j}'(1) + \hat{k}'(1) = -\hat{i}' - \hat{j}' + \hat{k}'$$

$$| \vec{PQ} \times \vec{PR} | = \sqrt{3}$$

$$\text{Area of the triangle} = \frac{\sqrt{3}}{2}$$

4. The volume of the parallelepiped determined by the vectors $\vec{a} = \langle 0, 2, 1 \rangle$, $\vec{b} = \langle 1, -1, 2 \rangle$ and $\vec{c} = \langle 0, 1, 2 \rangle$ is equal to

A. 1

B. 3

C. 5

D. 8

E. 9

Diagram showing three vectors \vec{a} , \vec{b} , and \vec{c} originating from a common point, forming a parallelepiped.

$$\text{Volume} = | \vec{a} \cdot (\vec{b} \times \vec{c}) |$$

$$= \left| \begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 2 \\ 0 & 1 & 2 \end{vmatrix} \right|$$

$$= | -4 - 2 + 1 | = | -5 | = 5$$

7. A force of 70 N is required to hold a spring that has been stretched from its natural length of 1 m to a length of 1.5 m. How much work is done (in joules) by stretching the spring from a length of 2 m to a length of 3 m?

A. 75

B. 125

C. 150

D. 210

E. 300

$$F = kx, \quad x = 1.5 - 1 = 0.5$$

$$70 = \frac{k}{2}; \quad \boxed{k = 140}$$

$$W = 140 \int_1^2 x \, dx = 140 \cdot \frac{1}{2} (4 - 1) = 210$$

8. A cylindrical tank containing a liquid of density 10 lb/ft³ has a radius of 2 ft. Its side is 5 ft. high and the depth of the liquid is 4 ft. How much work is required (in ft.-lbs.) to pump all the liquid out over the top of the tank?

A. 240π

B. 480π

C. $1,000\pi$

D. 700π

E. 640π

Weight of the slab

$$= \pi \cdot R^2 \cdot \rho \cdot dx$$

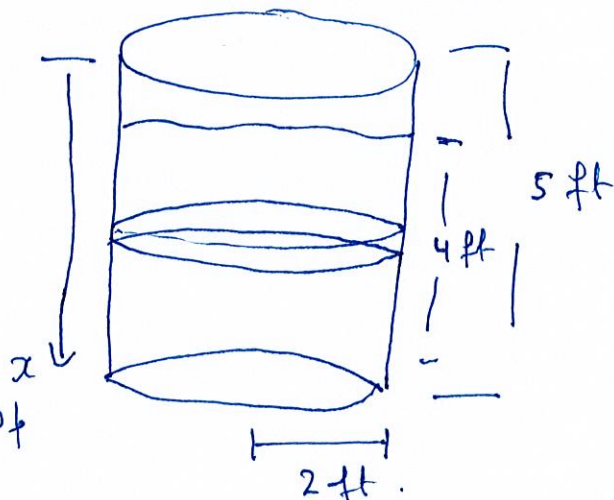
$$R = 2, \quad \rho = 10$$

$$= 40\pi \, dx$$

Work to move the slab to the top

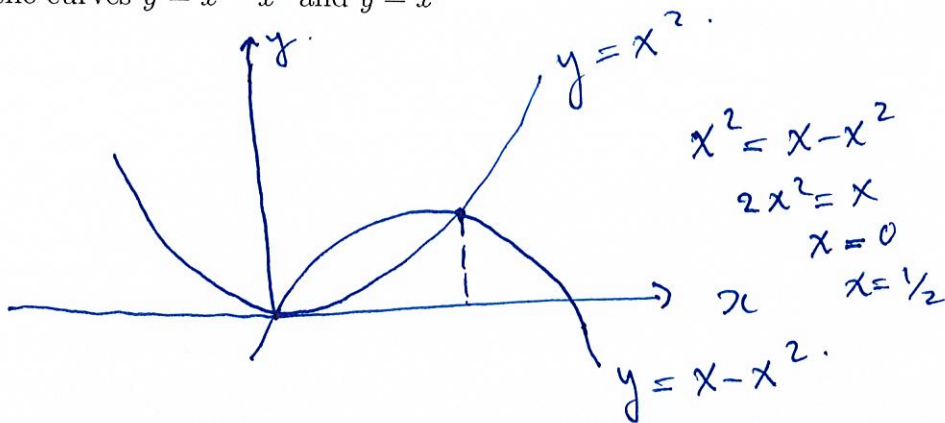
$$40\pi x \, dx$$

$$\text{Total Work} = \int_1^5 40\pi x \, dx = 40\pi \cdot \frac{1}{2} x^2 \Big|_1^5 = 20\pi (25 - 1) = 480\pi$$



5. Find the area enclosed by the curves $y = x - x^2$ and $y = x^2$

- A. $\frac{1}{3}$
 B. $\frac{1}{2}$
 C. $\frac{2}{3}$
 D. $\frac{1}{24}$
 E. $\frac{5}{3}$



$$A = \int_0^{1/2} (x - x^2 - x^2) dx$$

$$= \int_0^{1/2} (x - 2x^2) dx = \left[\frac{x^2}{2} - \frac{2x^3}{3} \right]_0^{1/2} = \frac{1}{8} - \frac{2}{3} \cdot \frac{1}{8}$$

$$= \frac{1}{8} \cdot \left(1 - \frac{2}{3} \right) = \frac{1}{24}$$

6. Find the volume of the solid generated by revolving the region bounded by $y = 5x - x^2$ and $y = 4x$ about the y -axis.

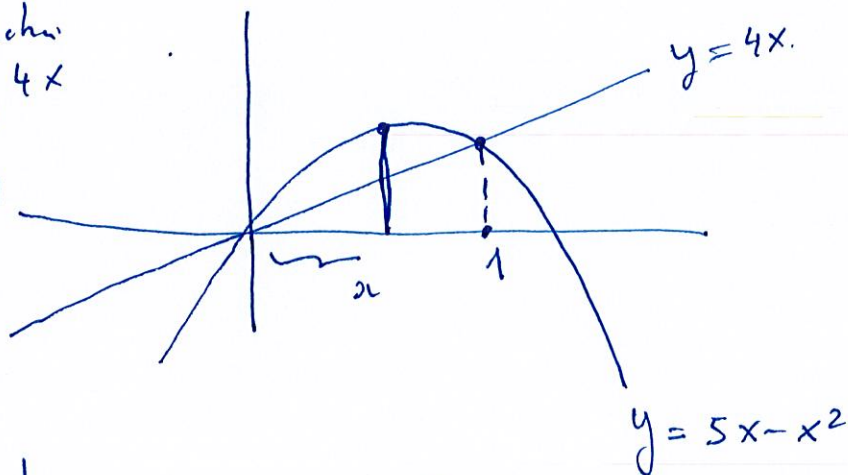
- A. $\frac{4\pi}{3}$
 B. $\frac{\pi}{4}$
 C. $\frac{2\pi}{3}$
 D. $\frac{\pi}{6}$
 E. $\frac{3\pi}{4}$

Intersection

$$5x - x^2 = 4x$$

$$x = x^2$$

$$x = 0, x = 1$$



$$V = 2\pi \int_0^1 x (-4x + 5x - x^2) dx$$

$$= 2\pi \int_0^1 (x^2 - x^3) dx = 2\pi \left(\frac{1}{3} - \frac{1}{4} \right) = \frac{\pi}{6}$$

9. If $\int_0^1 x^9 e^x dx = I$, then $\int_0^1 x^{10} e^x dx$ is equal to

A. $e - 10I$

B. $2e - 9I$

C. $e - 9I$

D. $2e - 10I$

E. $e - 11I$

Set $u = x^{10}, dv = e^x dx$

$du = 10x^9; v = e^x$

$$\int_0^1 x^{10} e^x dx = x^{10} e^x \Big|_0^1 - 10 \int_0^1 x^9 dx$$

$$= \underline{e^1 - 10I} = e - 10I.$$

10. The integral $\int_0^{\pi/3} \tan x (\sec x)^3 dx$ is equal to

A. $\frac{3}{4}$

B. $\frac{8}{3}$

C. $\frac{7}{3}$

D. $\frac{2}{3}$

E. $\frac{3}{5}$

$$\int_0^{\pi/3} \tan x (\sec x)^3 dx$$

$$= \int_0^{\pi/3} \tan x \cdot \sec x (\sec^2 x)^2 dx$$

$u = \sec x$

$du = \sec x \tan x dx$

$$= \int_1^2 u^2 du = \frac{u^3}{3} \Big|_1^2 = \frac{7}{3}$$

ANSWER KEYS:

TEST 11: 1- D, 2-D, 3-B, 4- B, 5-D, 6-D, 7-D, 8-B, 9-A, 10-C

TEST 22: 1-C, 2-B, 3-A, 4-A, 5-A, 6-C, 7-B, 8-C, 9-C, 10-A

TEST 33: 1- B, 2-A, 3-C, 4-E, 5-B, 6-A, 7-E, 8-A, 9-B, 10-E

TEST 44: 1-E, 2- E, 3-E, 4-D, 5-C, 6-E, 7-A, 8-?, 9-E, 10-D