MA 16600
Exam 1

Covers Lessons 1 – 10

Tue., Feb. 5, 2019

6:30 p.m.–7:30 p.m.

INSTRUCTOR

All Instructors

LOCATION

ELLT 116

REGULAR CLASS

ON FEB 8

Rescheduled Homework:

Lesson 10 – Due Feb 6

Lesson 11 – Due Feb 8.

Lesson 10 Section 7.2

Trigonometric Integrals

Basic ones:

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\int \sec^2 x \, dx = \tan x + C.$$

$$\int \tan x\,dx = \int \frac{\overbrace{\sin x\,dx}^{=-du}}{\underbrace{\cos x}_{=u}}$$

$u = \cos x, \quad du = -\sin x\,dx$

$$\int -\frac{du}{u} = -\ln|u|$$

$$\int \tan x\,dx = -\ln|\cos x| + C$$

$$= \ln\left|\frac{1}{\cos x}\right| + C$$

$$\boxed{\int \tan x\,dx = \ln|\sec x| + C}$$

We want to integrate the following functions.

$$\int \cos 3x \sin 5x \, dx$$

$$\int \cos^2 x \, dx$$

$$\int \cos^4 x \, dx$$

$$\int \sin^4 x \, dx$$

$$\int \cos 4x \cos 6x \, dx.$$

Integrals of the type

$$\int \cos ax \sin bx \, dx$$

Easy case: $\int \cos x \sin x \, dx$ $\quad a=b=1$

$\sin x = u, \quad du = \cos x \, dx$

$$\int \sin x \cos x \, dx = \int u \, du$$

$$= \frac{u^2}{2} + C = \underline{\tfrac{1}{2} \sin^2 x + C.}$$

$$\int \cos x \sin x \, dx.$$

$$\sin 2x = 2 \sin x \cos x$$

$$\int \sin x \cos x \, dx$$

$$= \int \frac{1}{2} \sin 2x \, dx$$

$$= \frac{1}{2} \int \sin 2x \, dx$$

$$= \frac{1}{2} \cdot -\frac{1}{2} \cos 2x + C$$

$$= -\frac{1}{4} \cos 2x + C$$

$\cos^2 x = 1 - \sin^2 x$

$$\boxed{\cos 2x = \cos^2 x - \sin^2 x}$$

$$\cos 2x = 1 - \sin^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$\int \sin x \cos x \, dx$$

$$= -\frac{1}{4} \cos 2x + C$$

$$= -\frac{1}{4}\left(1 - 2\sin^2 x\right) + C$$

$$= \frac{1}{2}\sin^2 x \underbrace{- \frac{1}{4} + C}_{\tilde{C}}$$

$$= \frac{1}{2}\sin^2 x + \tilde{C}$$

$$\begin{cases} \cos 2x = \underbrace{\cos^2 x} - \sin^2 x \\ \cos^2 x + \sin^2 x = 1. \end{cases}$$

$$\cos 2x = 1 - \sin^2 x - \sin^2 x$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\boxed{\sin^2 x = \frac{1 - \cos 2x}{2}}$$

$$\cos 2x = \cos^2 x - (1 - \cos^2 x)$$
$$= 2\cos^2 x - 1.$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\boxed{\cos^2 x = \frac{1+\cos 2x}{2}}$$

$$\boxed{\sin^2 x = \frac{1-\cos 2x}{2}}$$

$$\int \cos^2 x \, dx = \int \frac{1}{2}\left(1+\cos 2x\right) dx$$

$$= \frac{1}{2}x + \frac{1}{2}\int \cos 2x \, dx$$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + c.$$

$$\int \sin^2 x \, dx =$$

$$= \tfrac{1}{2} x - \tfrac{1}{4} \sin 2x + C.$$

$$\int \underbrace{\cos 3x}\ \underbrace{\sin 4x}\, dx$$

$$\begin{cases} \cos(A+B) = \cos A \cos B - \sin A \sin B \\ \cos(A-B) = \cos A \cos B + \sin A \sin B \end{cases}$$

$$\begin{cases} \sin(A+B) = \sin A \cos B + \sin B \cos A \\ \sin(A-B) = \sin A \cos B - \sin B \cos A \end{cases}$$

$$A = 3x \quad , \quad B = 4x$$

$$\sin(3x + 4x) = \underbrace{\sin 3x \cos 4x} + \underbrace{\sin 4x \cos 3x}$$

$$\sin(3x - 4x) = \underbrace{\sin 3x \cos 4x} - \underbrace{\sin 4x \cos 3x}$$

$$\sin 7x - \sin(-x) = 2 \sin 4x \cos 3x$$

$$\boxed{\sin 4x \cos 3x = \frac{1}{2}\left(\sin 7x + \sin x\right)}$$

$$\int \sin 4x \cos 3x \, dx = \frac{1}{2} \int (\sin 7x + \sin x) \, dx$$

$$= -\frac{1}{14} \cos 7x - \frac{1}{2} \cos x + C.$$

$$\int \sin 3x \cos 4x \, dx$$

$\sin(3x + \cos 4x) = \ldots$ As in previous page

$$\sin(7x) + \sin(-x)$$

$$= 2 \sin 3x \cos 4x$$

$$\sin 3x \cos 4x = \frac{1}{2}\left(\sin 7x - \sin x\right)$$

$$\int \sin 3x \cos 4x \, dx = -\frac{1}{14}\cos 7x + \frac{1}{2}\cos x + C.$$

$$\int \sec^2 x \, dx = \tan x + C$$

Compute.

$$\boxed{\sec x = \frac{1}{\cos x}}$$

$$\boxed{\int \tan^2 x \, dx}$$

$$\cos^2 x + \sin^2 x = 1.$$

Divide by $\cos^2 x$

$$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} =$$

$$\boxed{\sec^2 x = 1 + \tan^2 x}$$

So

$$\boxed{\tan^2 x = \sec^2 x - 1.}$$

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1)\, dx$$

$$= \tan x - x + C.$$

$$\int \tan^4 x \, dx =$$

$$\int \tan^2 x \tan^2 x \, dx$$

$$= \int \tan^2 x (\sec^2 x - 1) \, dx.$$

$$= \int \tan^2 x \sec^2 x \, dx$$

$$- \int \tan^2 x \, dx$$

$$= \int \underbrace{\tan^2 x \sec^2 x \, dx}_{} - \tan x + x + C.$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

set $u = \tan x$, $du = \sec^2 x \, dx$

$$\int \underbrace{\tan^2 x}_{} \underbrace{\sec^2 x \, dx}_{} = \int u^2 \, du$$

$$= \frac{1}{3} u^3 + C$$

$$= \frac{1}{3} \tan^3 x + C \; .$$

$$\int \tan^2 x \sec^2 x \, dx$$

$$= \frac{1}{3}(\tan x)^3 + c.$$

$$\int \tan^4 x \, dx =$$

$$\frac{1}{3}\tan^3 x - \tan x + x + C.$$