

Use of a Table of Integrals

$$\int \frac{\sec^2 x}{1 - \tan^2 x} dx$$

Set $u = \tan x$, $du = \sec^2 x dx$

$$\int \frac{\sec^2 x dx}{1 - \tan^2 x} = \int \frac{du}{1 - u^2}$$

$$= \frac{1}{2} \ln \left| \frac{u+1}{u-1} \right| + C$$

$$\int \frac{\sec^2 x}{1 - \tan^2 x} dx = \frac{1}{2} \ln \left| \frac{1 + \tan x}{-1 + \tan x} \right| + C$$

$$\int \frac{\ln(10 + \sqrt{x})}{\sqrt{x}} dx$$

$$u = 10 + \sqrt{x} = 10 + x^{1/2}$$

$$du = \frac{1}{2} x^{-1/2} dx$$

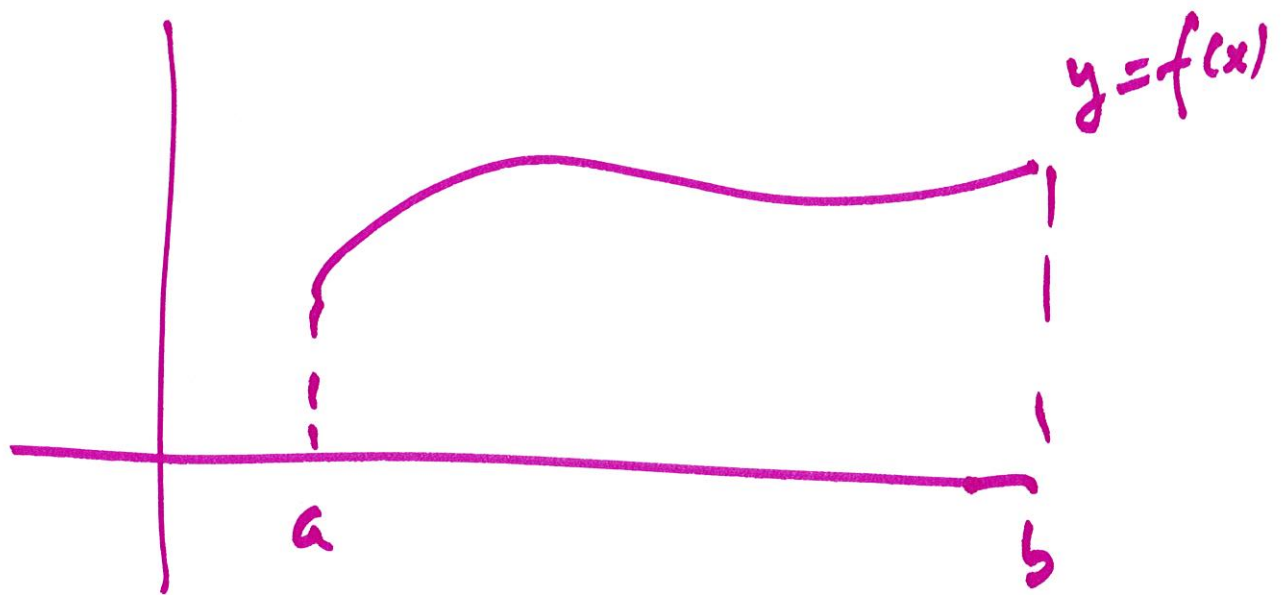
$$= \frac{1}{2} \frac{dx}{\sqrt{x}}$$

$$\int \frac{\ln(10 + \sqrt{x})}{\sqrt{x}} dx = 2 \int \ln u \, du$$

$$= 2(u \ln u - u) + C$$

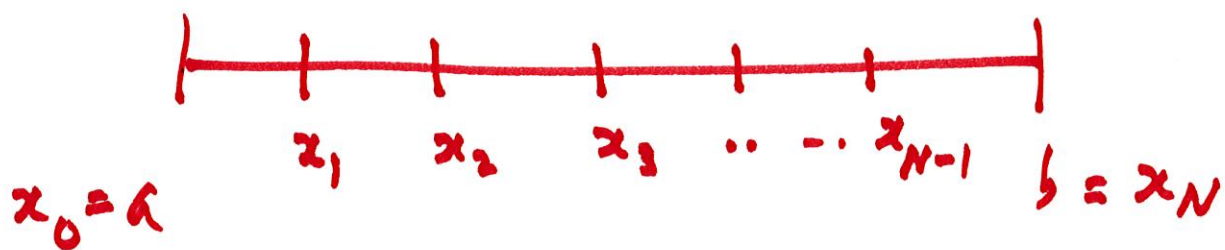
$$\int \frac{\ln(10 + \sqrt{x})}{\sqrt{x}} dx = 2 \left((10 + \sqrt{x}) \ln(10 + \sqrt{x}) - (10 + \sqrt{x}) \right) + C.$$

Approximate Integration



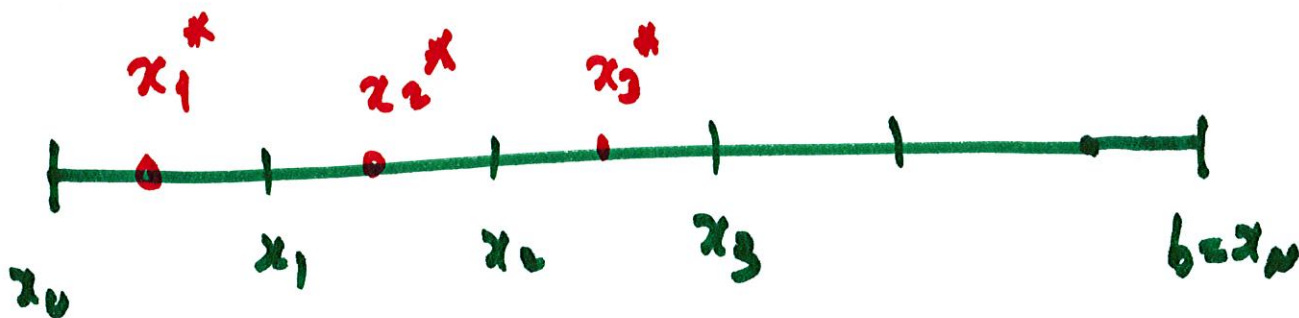
Task: Write a computer program to compute $\int_a^b f(x) dx$.

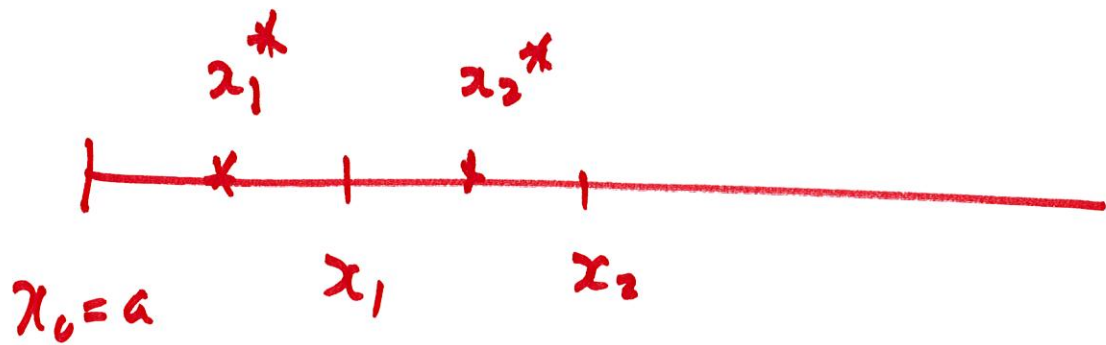
Step 1: Divide $[a, b]$ into N equal parts.



$$\Delta x = \frac{b-a}{N}$$

Step 2: Inside each interval
pick x_j^* in $[x_{j-1}, x_j]$



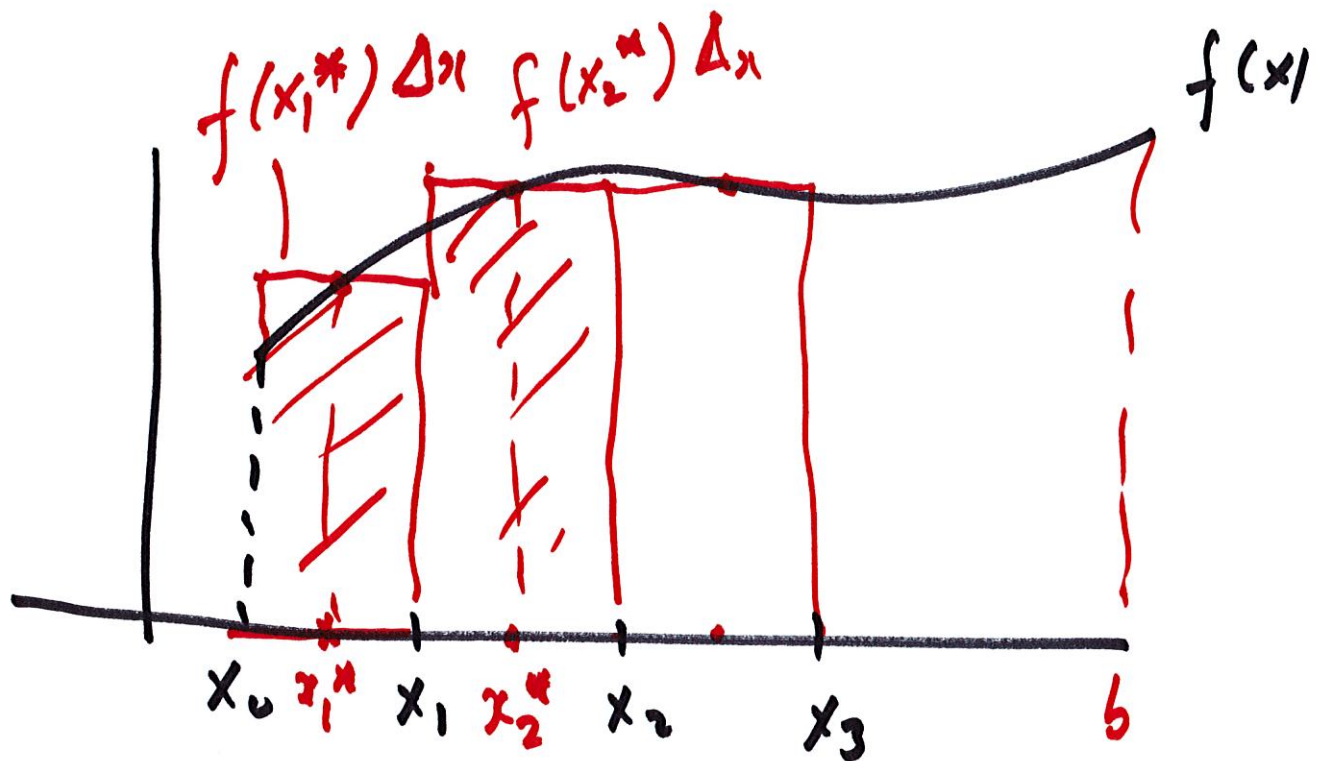


When x_j^* is the mid point
of the interval.

Thus approximation is
called the mid point
approximation.

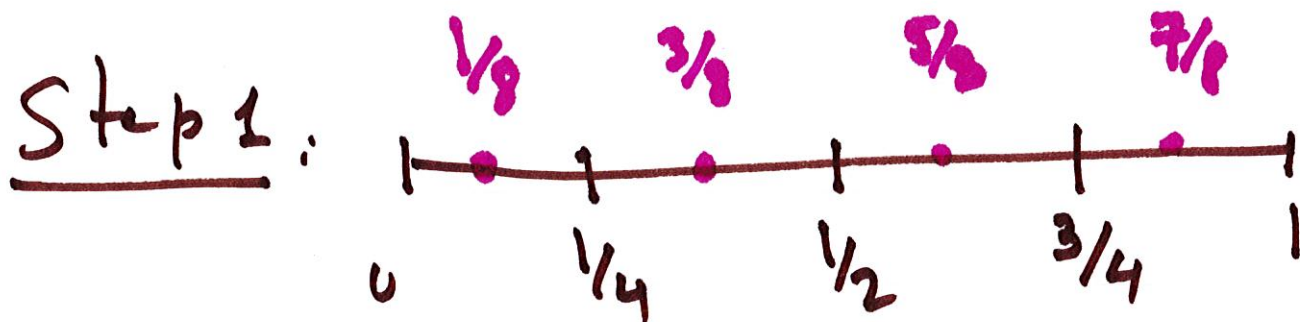
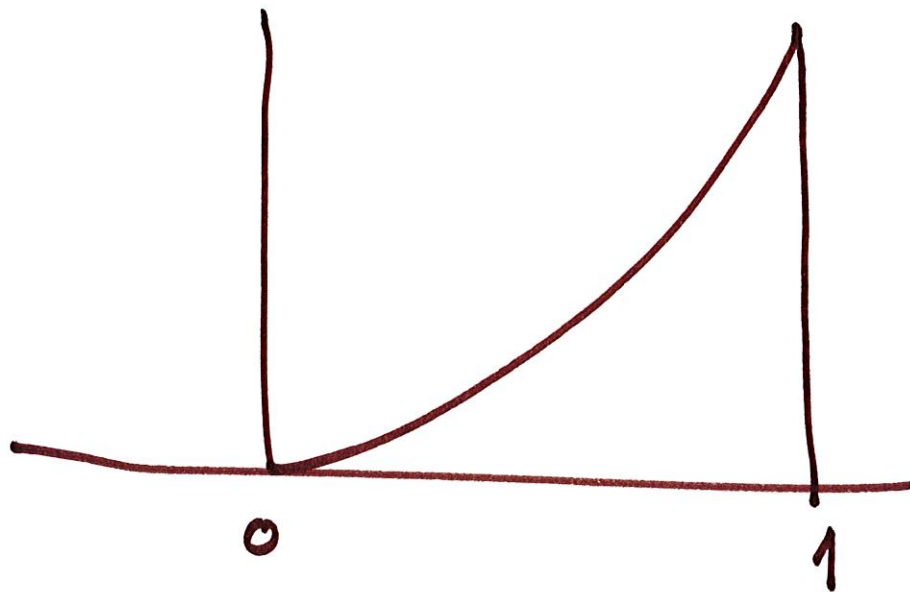
Step 3 :

$$S_N = f(x_1^*) \Delta x + f(x_1^*) \Delta x \\ + f(x_3^*) \Delta x + \dots + f(x_{N-1}^*) \Delta x.$$



$$\int_a^b f(x) dx$$

Example: Use the midpoint
approximation with $N = 4$.
to estimate $\int_0^1 x^2 dx = \frac{1}{3}$.



$$\Delta x = \frac{1 - 0}{4} = \frac{1}{4}.$$

Step 3 :

$$S_4 = f(x_1^*) \underline{\underline{\frac{1}{4}}} + f(x_2^*) \underline{\underline{\frac{1}{4}}}$$

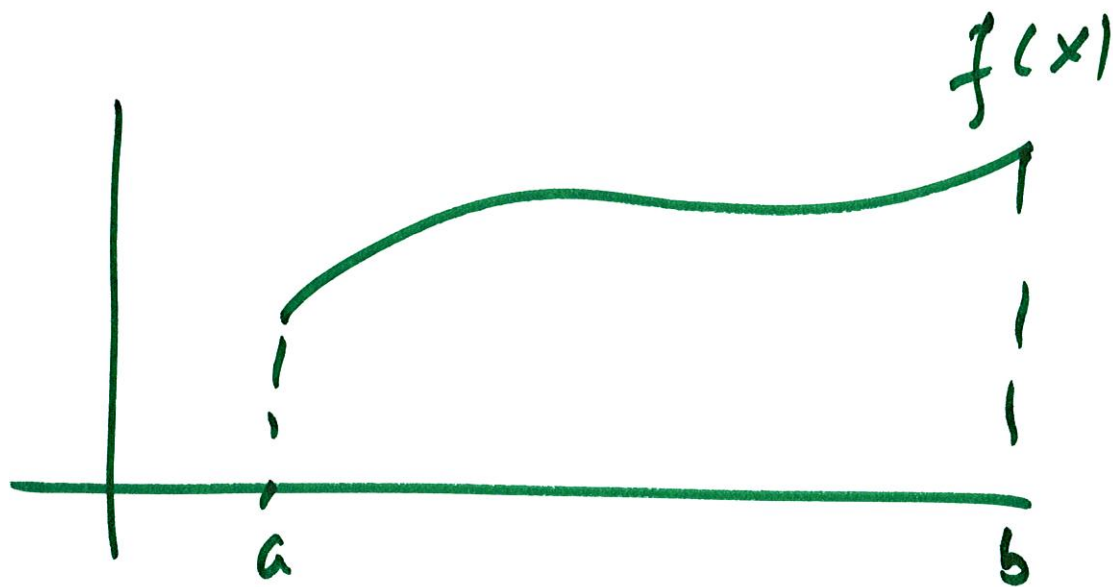
$$+ f(x_3^*) \underline{\underline{\frac{1}{4}}} + f(x_4^*) \underline{\underline{\frac{1}{4}}}$$

$$= \frac{1}{4} \left(\left(\frac{1}{8}\right)^2 + \left(\frac{3}{8}\right)^2 + \left(\frac{5}{8}\right)^2 + \left(\frac{7}{8}\right)^2 \right)$$

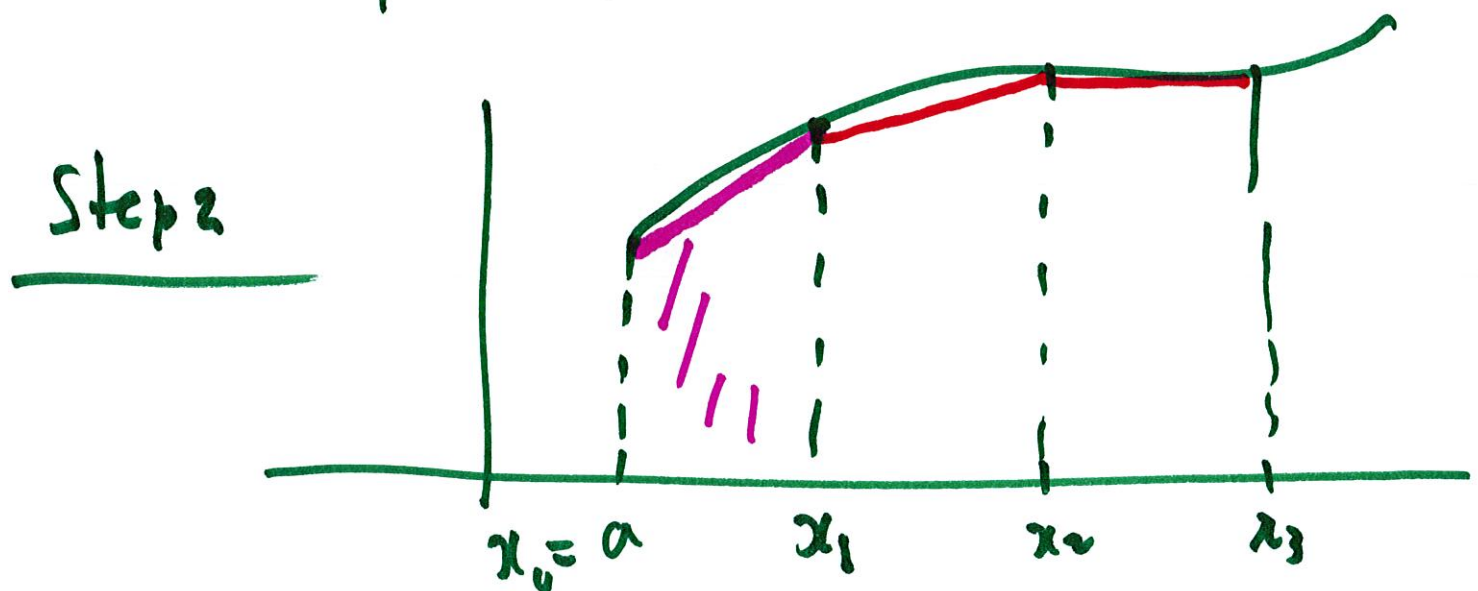
$$= \frac{1}{4} \cdot \frac{1}{64} (1 + 9 + 25 + 49)$$

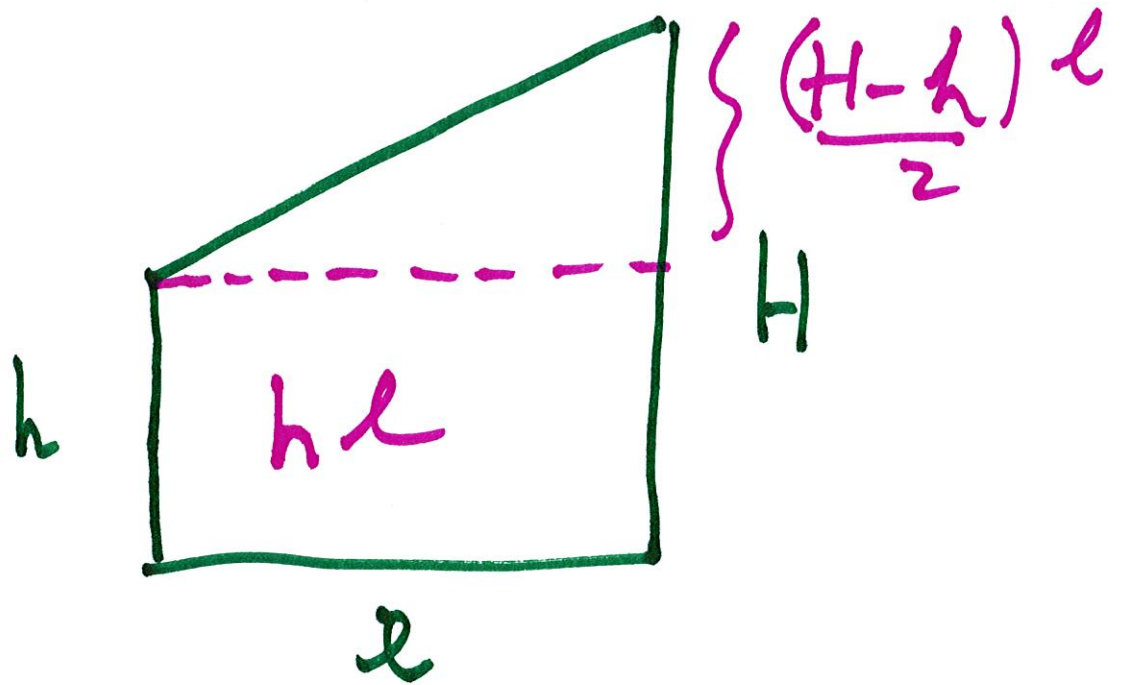
$$= \frac{1}{4} \cdot \frac{1}{64} \cdot 84 = \frac{21}{64}$$

The Trapezoidal Rule

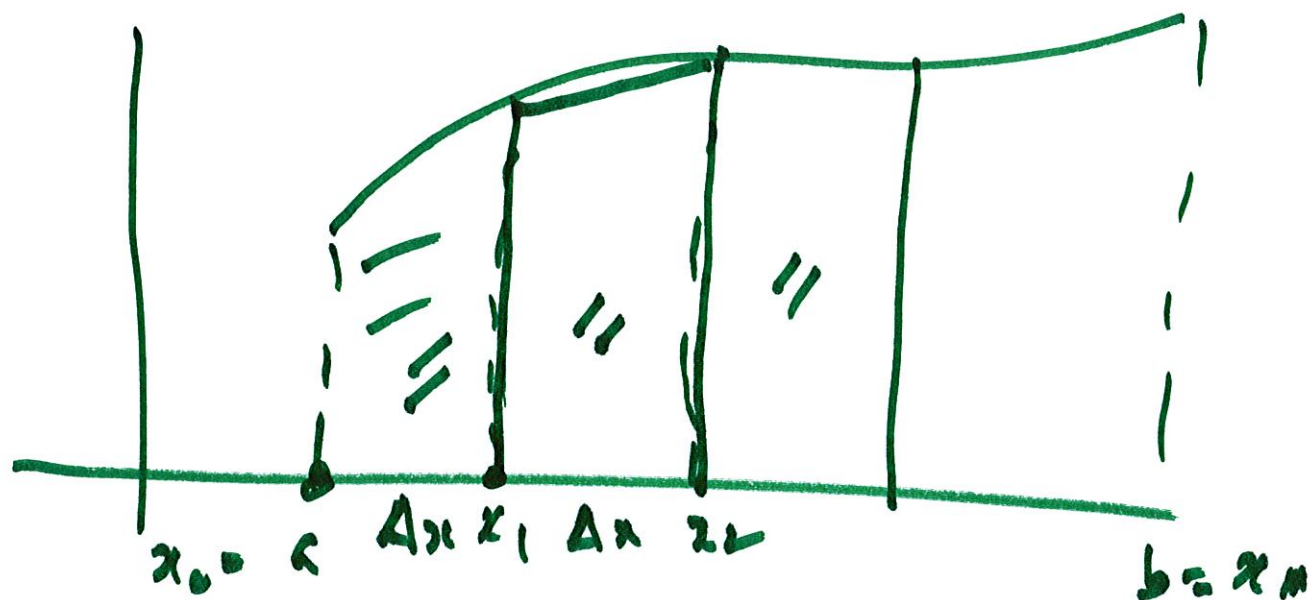


Step 1: Divide $[a, b]$ into N equal parts





$$\begin{aligned} & h \cdot l + \left(\frac{H-h}{2} \right) \cdot l \\ &= l \left[h + \frac{H-h}{2} \right] \\ &= l \left[\frac{h+H}{2} \right] \leftarrow \end{aligned}$$



$$\left(\frac{f(x_1) + f(x_0)}{2} \right) \Delta x +$$

$$\left(\frac{f(x_2) + f(x_1)}{2} \right) \Delta x + \dots$$

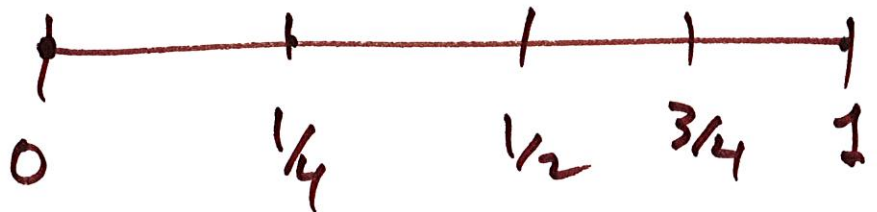
$$+ \left(\frac{f(x_{n-1}) + f(x_n)}{2} \right) \Delta x$$

$$= \Delta x \left(\frac{f(x_0)}{2} + f(x_1) + f(x_2) + \dots + f(x_{n-1}) + \frac{f(x_n)}{2} \right).$$

$$S_N = \Delta x \left(\frac{f(x_0)}{2} + f(x_1) + f(x_2) + \dots + f(x_{N-1}) + \frac{f(x_N)}{2} \right)$$

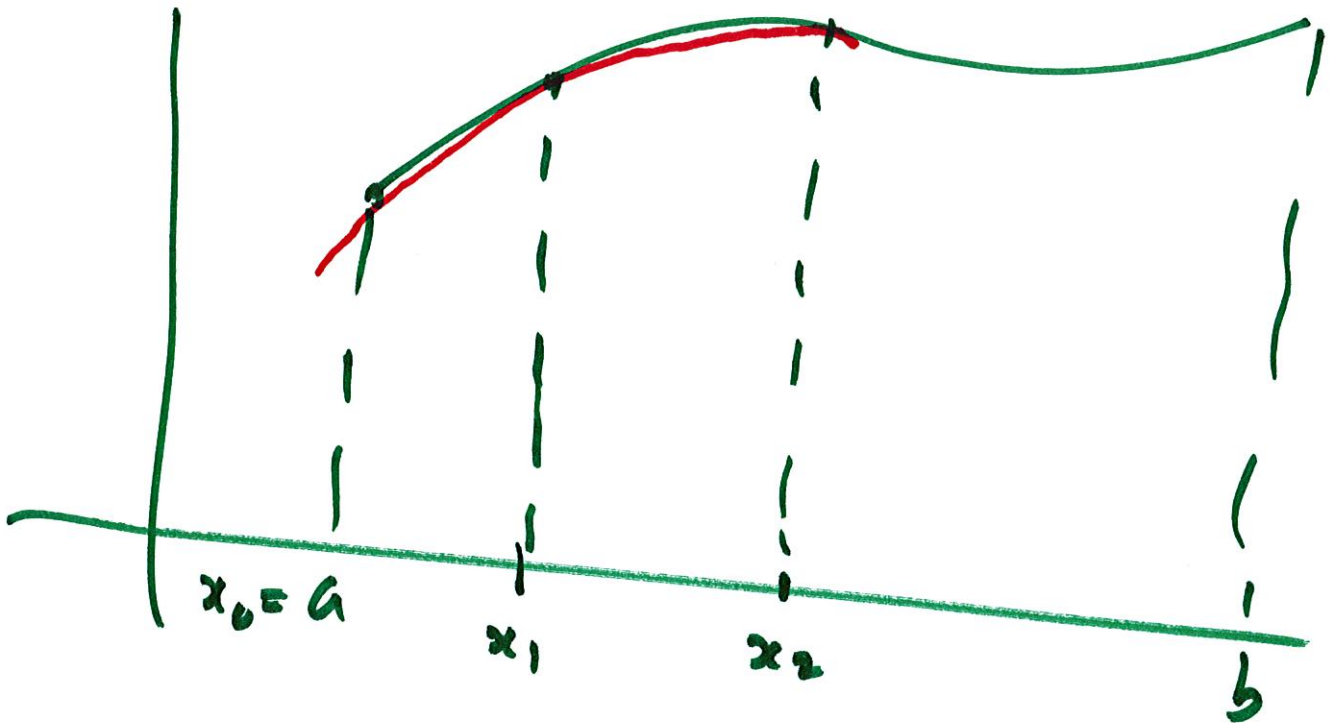
Ex: $\int_0^1 x^2 dx$

$N=4$:



$$S_4 = \frac{1}{4} \left(0 + \frac{1}{16} + \frac{1}{4} + \frac{9}{16} + \frac{1}{2} \right)$$

Simpson's Rule



N equal parts, N even

A

$$S_N = \frac{\Delta x}{3} \left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{N-1}) + f(x_N) \right).$$