Lesson 19 Section 11.1

Sequences

Exam 2 Study Guide Posted on Course Home Page

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LESSONS 11 TO 20
EXCLUDES LESSON 15.

Sequences

Example 1 :

$$\boxed{\frac{1}{3} = 0.3333..}$$

$a_1 = 0.3$

$a_2 = 0.33$

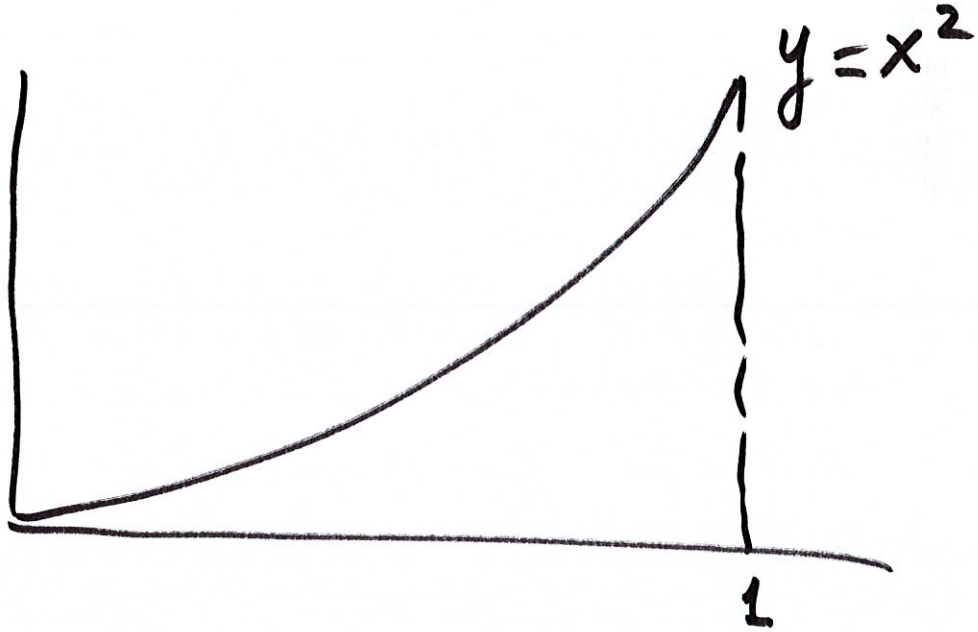
$a_3 = 0.333$

$\vdots$

$a_n = 0.\underbrace{3333\cdots 3}_{n \text{ terms}}$

$$\lim_{n\to\infty} a_n = 0.33333\ldots\ldots = \frac{1}{3}$$

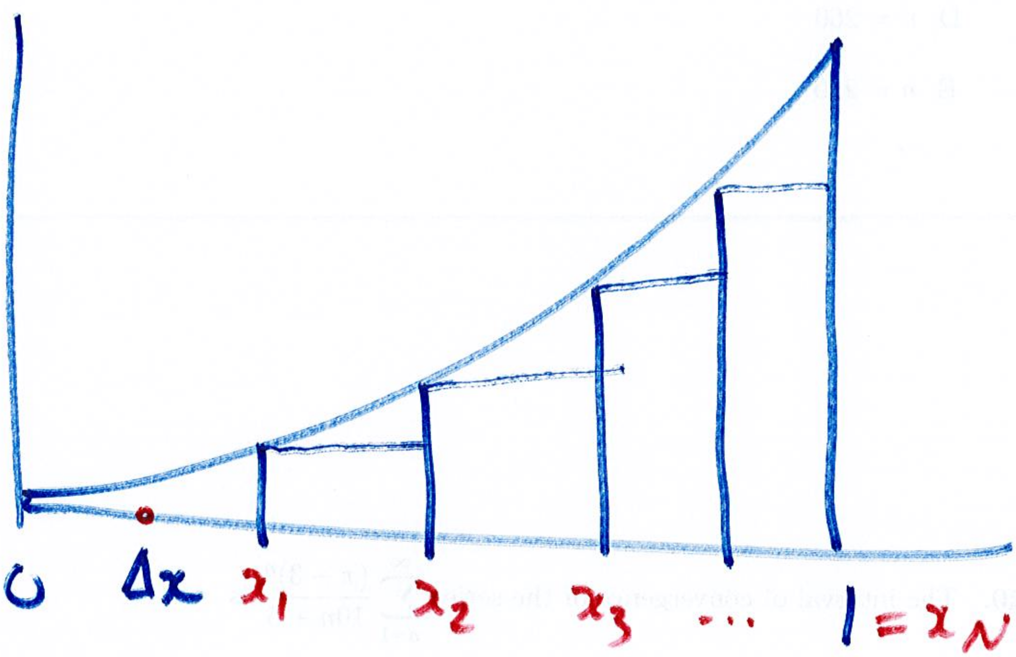
Integrals as Limit of Sequences.



$$\int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

Fundamental theorem of Calculus.

Step 1. Divide the interval $[0, 1]$ into N equal parts



$$\Delta x = \frac{1-0}{N} = 1/N.$$

Step 2. Pick a point x_j^* in $[x_{j-1}, x_j]$

Step 3: Form the sum

$$S_N = [f(x_1^*) + f(x_2^*) + \dots + f(x_N^*)]\Delta x$$

Thus gives us a sequence

$S_1, S_2, S_3, S_4 \ldots, S_N, \ldots$

$$\lim_{N \to \infty} S_N = \int_0^1 x^2 \, dx$$

Given a list

$a_1, a_2, a_3, a_4, \ldots a_n, \ldots$

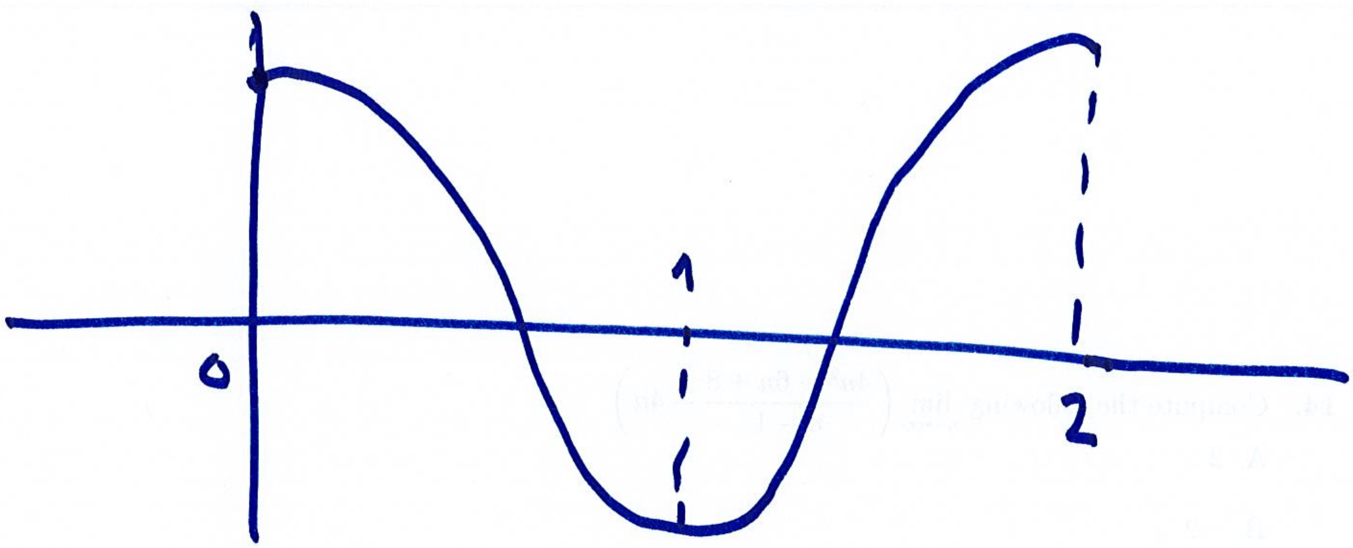
in a given order.

Task: What happens to

a_n when $n \to \infty$.

$$\lim_{n\to\infty} a_n = 0$$

Ex: $f(x) = \cos(\cancel{x}\pi)$



$$\boxed{f(n) = \cos(n\pi)}$$

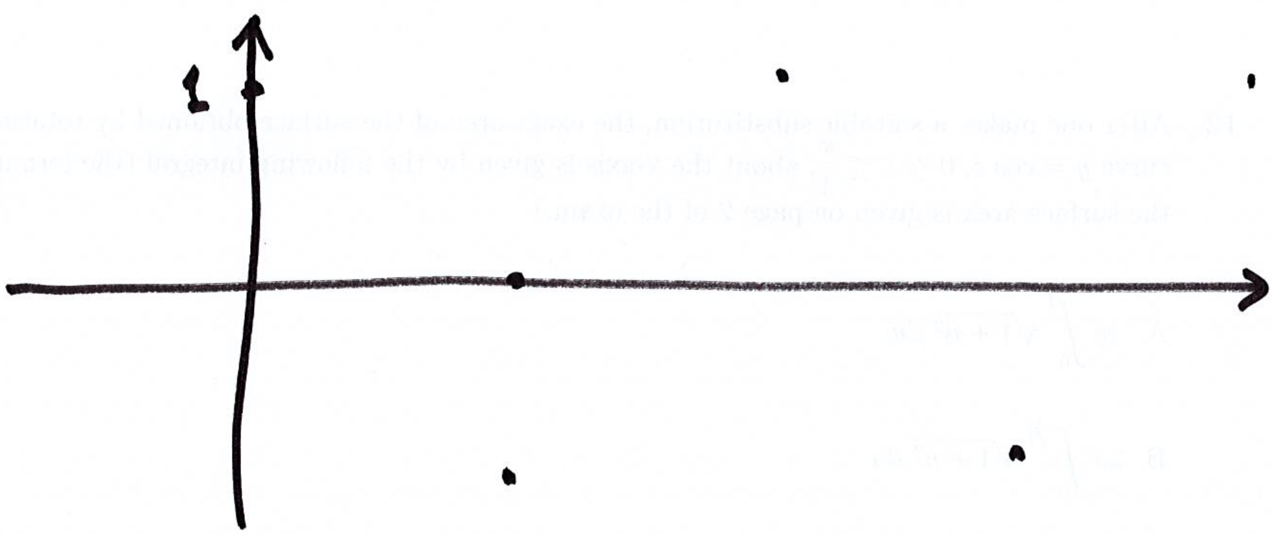
$$\boxed{a_n = f(n) = \cos n\pi}$$

$$a_0 = \cos(0\pi) = 1$$

$$a_1 = \cos\pi = -1$$

$$a_2 = \cos 2\pi = 1.$$

$$a_3 = \cos 3\pi = -1$$

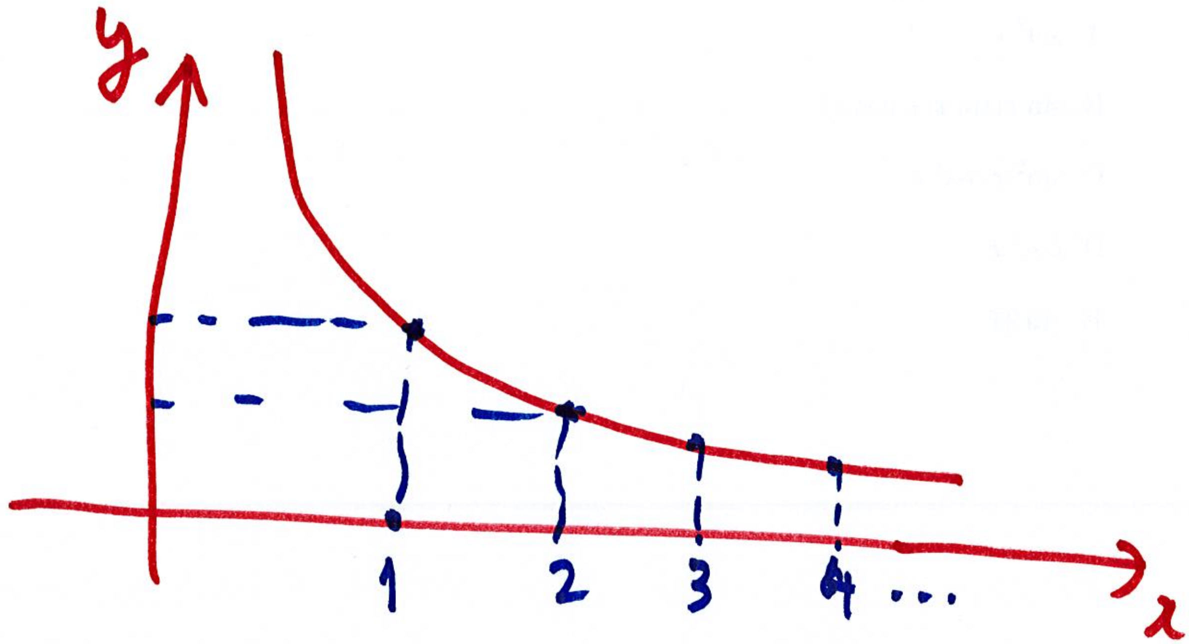


Question: What happens to

$$a_n = \cos(n\pi)$$

when $n \to \infty$?

Examples: $f(x) = \frac{1}{x}$



Extract a sequence from this

$a_n = f(n)$ $\qquad$ $a_n = 1/n$

$a_1 = f(1) = \frac{1}{1} = 1$

$a_2 = f(2) = 1/2$

$a_3 = 1/3$

If $n \to \infty$, with only even values

$n = 2, 4, 6, \ldots$ $\boxed{n = 2K}$

$a_n = \cos(n\pi) = \cos(2K\pi) = 1$

$n = 1, 3, 5, 7, 9, \ldots$ $n = 2K+1$

$a_n = \cos(\underbrace{2K\pi} + \pi) = \cos\pi = -1$

In this case we say that the limit of a_n as $n \to \infty$ does not exist.

Increasing and decreasing Sequences.

a_n is <u>increasing</u> if

$\cancel{a_{k+1} > a}$

$a_{n+1} > a_n$ if $n = 1, 2, \ldots$

When $\cancel{a_{n+0}}$ $a_{n+1} < a_n$ we say that a_n is decreasing.

If $a_{n+1} \geq a_n$.

Non-Decreasing.

$a_{n+1} \leq a_n$

Non-increasing

Limits of Sequences

$$a_n = \frac{n^2+5n+1}{n^2+3}.$$

$$a_1 = \frac{1+5+1}{1+3} = 7/4$$

$$a_2 = \frac{4+10+1}{4+3} = \frac{15}{7}$$

$$\vdots$$

$$\lim_{n\to\infty} a_n = ?$$

Given a sequence $a_1, a_2, a_3 \ldots$
We say that

$$\lim_{n \to \infty} a_n = L.$$

If a_n is arbitrarily close to L for $\underline{\underline{n}}$ large.

If there exists

$f(x)$ such that

$$a(n) = f(n)$$

and $\boxed{\lim_{x \to \infty} f(x) = L}$

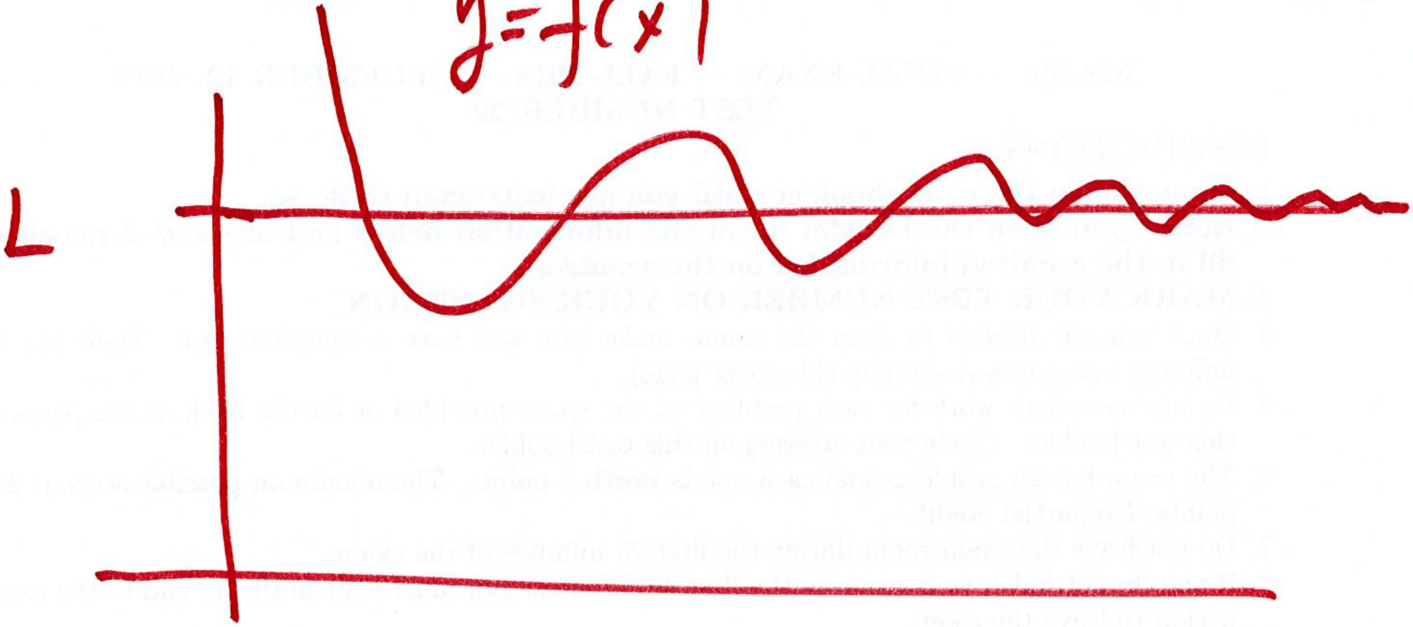
then $\lim_{n \to \infty} a_n = \lim_{n \to \infty} f(n) = L.$

<u>In this example</u>

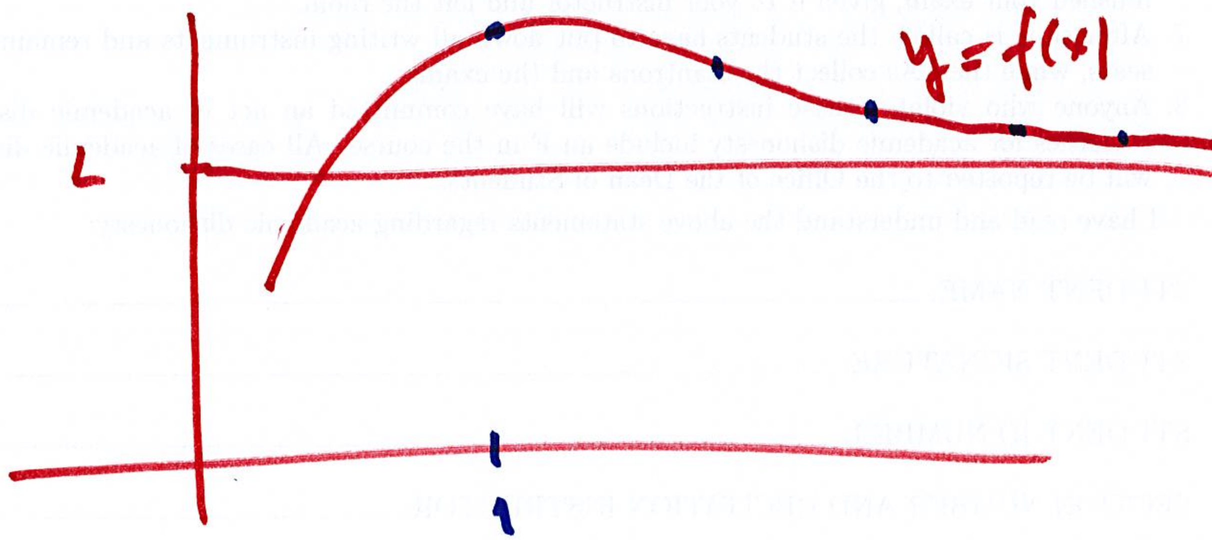
$$f(x) = \frac{x^2 + 5x + 1}{x^2 + 3}$$

$$a_n = f(n) = \frac{n^2 + 5n + 1}{n^2 + 3}.$$

$y = f(x)$



$$\lim_{x \to \infty} f(x) = L$$



$$\lim_{x\to\infty} \frac{x^2+5x+1}{x^2+3} =$$

$$= \lim_{x\to\infty} \frac{\cancel{x^2}\left(1+5/x+1/x^2\right)}{\cancel{x^2}\left(1+3/x^2\right)}$$

$$= \lim_{x\to\infty} \frac{1+5/x+1/x^2}{1+3/x^2} = 1.$$

Conclusion $\displaystyle\lim_{n\to\infty} \frac{n^2+5n+1}{n^2+3} = 1$

$$\lim_{x\to\infty} \frac{\overbrace{x^2+5x+1}^{\to\infty}}{\underbrace{x^2+3}_{\to\infty}} = \lim_{x\to\infty} \frac{2x+5}{2x}$$

$$= \lim_{x\to\infty} \frac{2}{2} = 1.$$

We say that for a sequence
a_n:

$$\lim_{n \to \infty} a_n = \infty$$

if a_n gets arbitrarily large

as $n \to \infty$

Ex: $a_n = n$

$a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $\ldots$

Example: $a_n = \frac{n^2+3n}{n+1}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2(1+3/n)}{n(1+1/n)}$$

$$= \lim_{n\to\infty} \underline{\underline{n}} \; \frac{1+3/n}{1+1/n} = \infty$$

$\lim_{n\to\infty} a_n = -\infty$ if

a_n gets arbitrarily large and negative as $n \to \infty$.