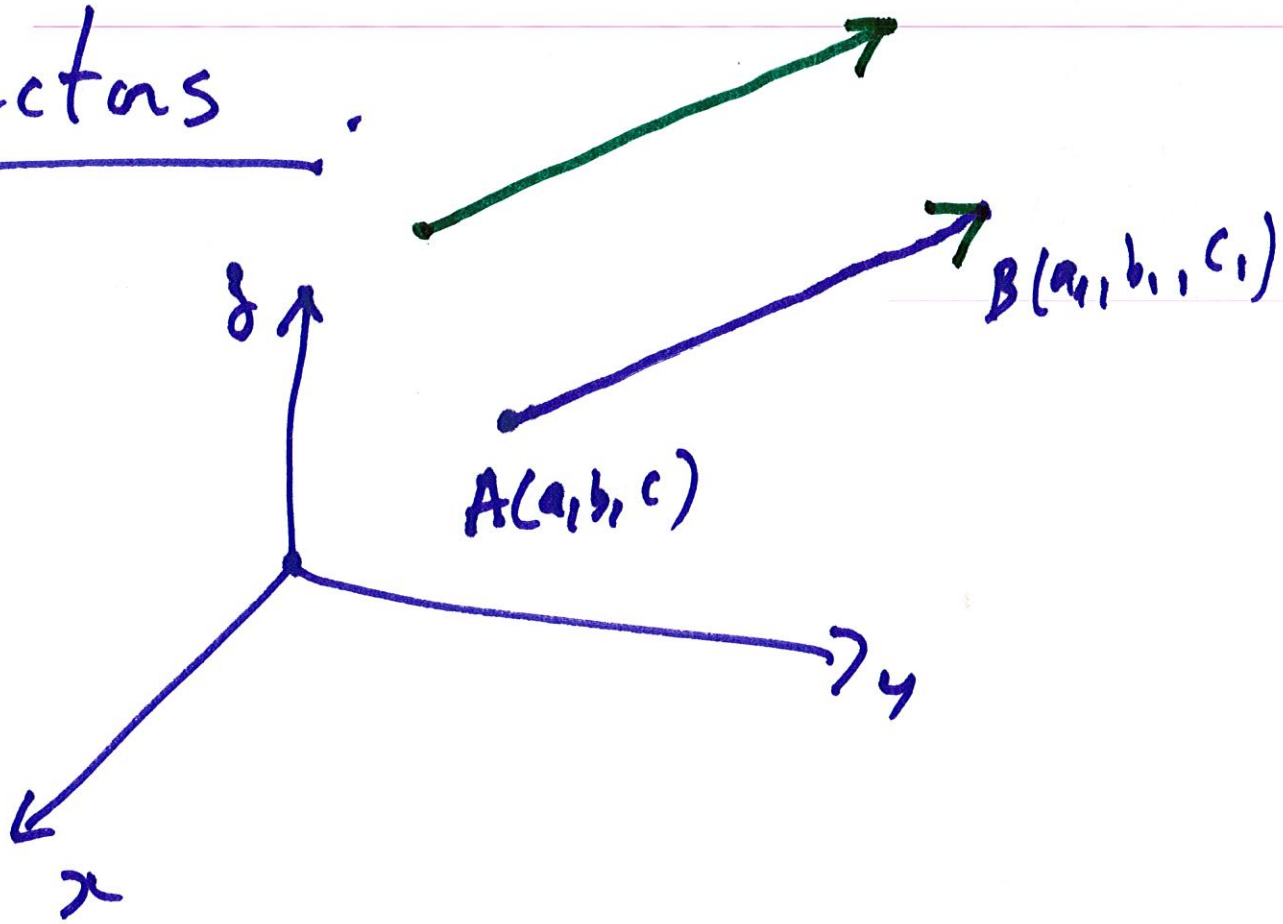
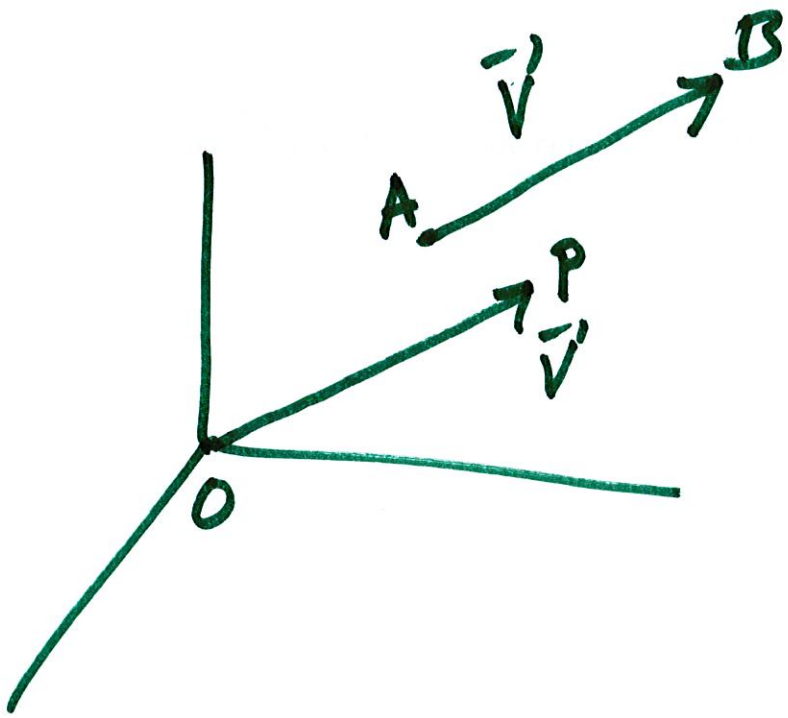
Lesson 2 Sections 12.2 and beginning of 12.3.

Vectors.



$\vec{V} = \overrightarrow{AB}$

Vectors which differ by Translations are the Same.



$\bar{OP} = \bar{AB}$.

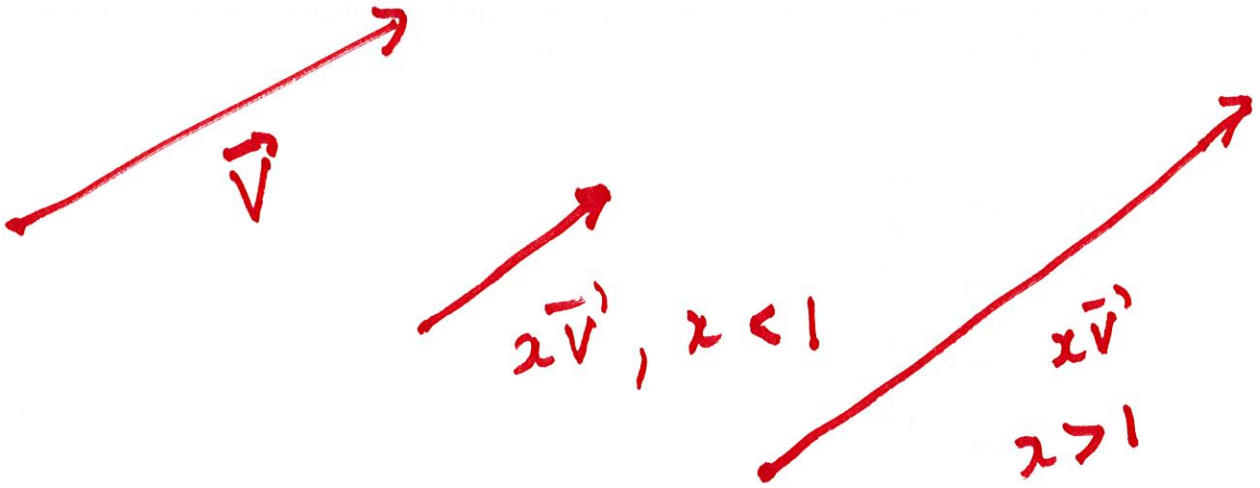
The length of a vector.

$|\bar{V}|$ = length of the segment.

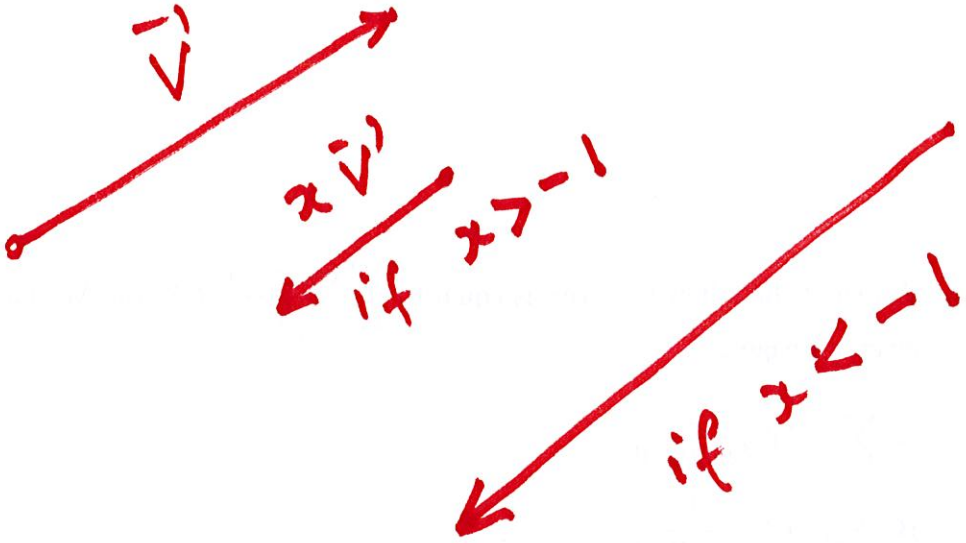
Operations with Vectors.

1) Multiplication by numbers (scalars).

$x > 0$

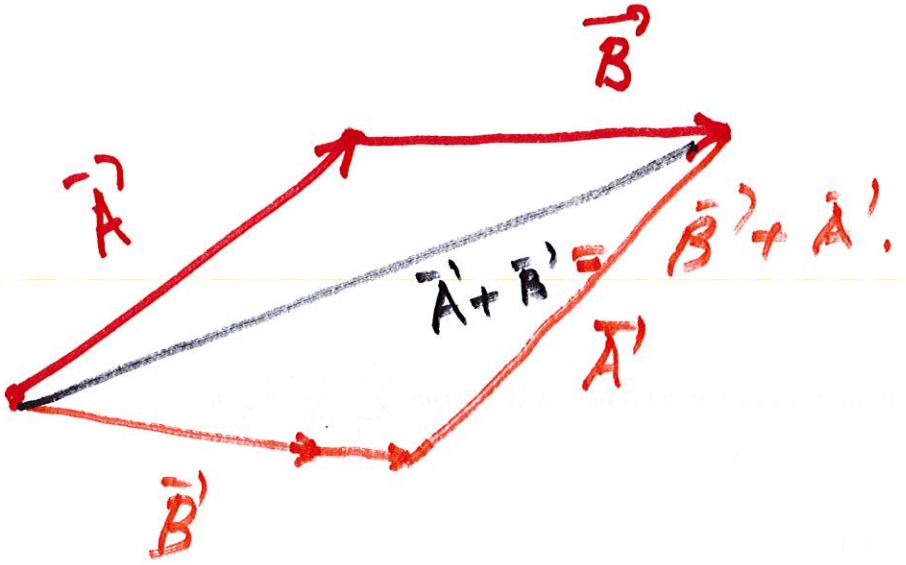


$x < 0$

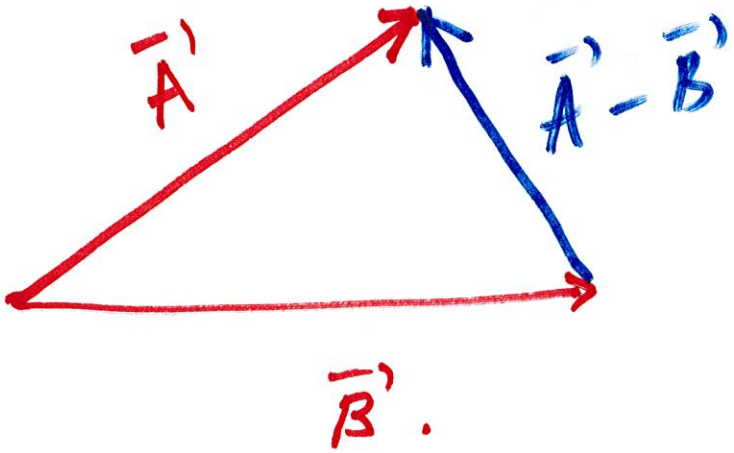


Addition.

$\vec{A} + \vec{B}$.

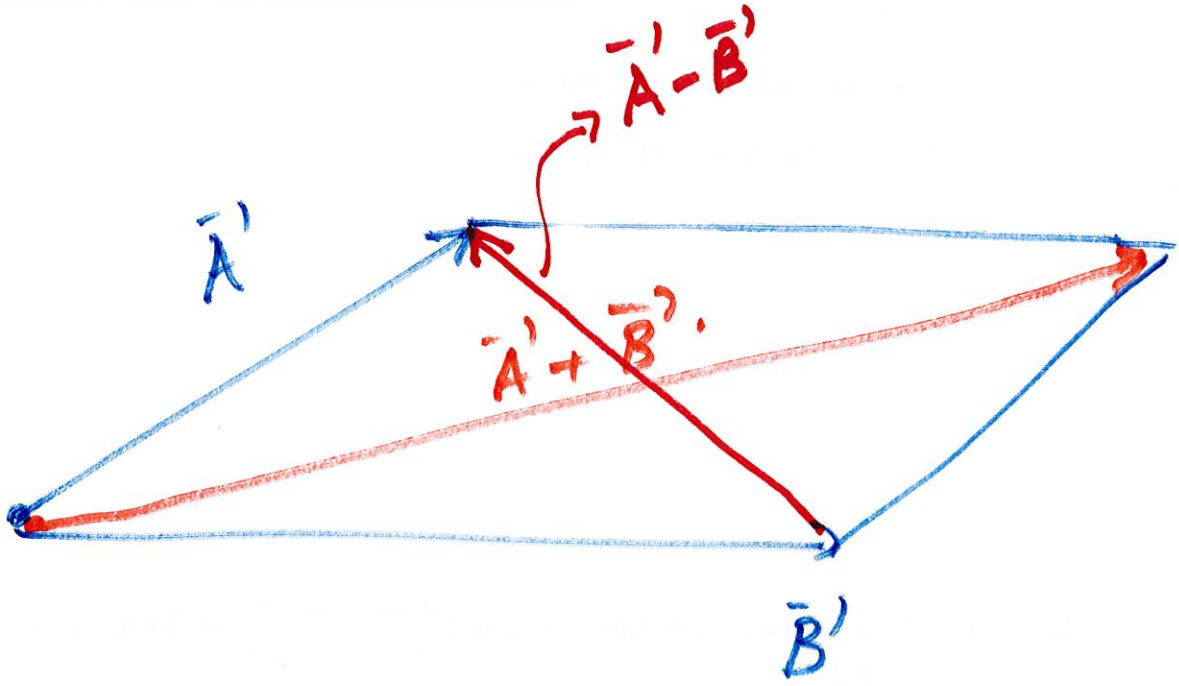


Subtraction:



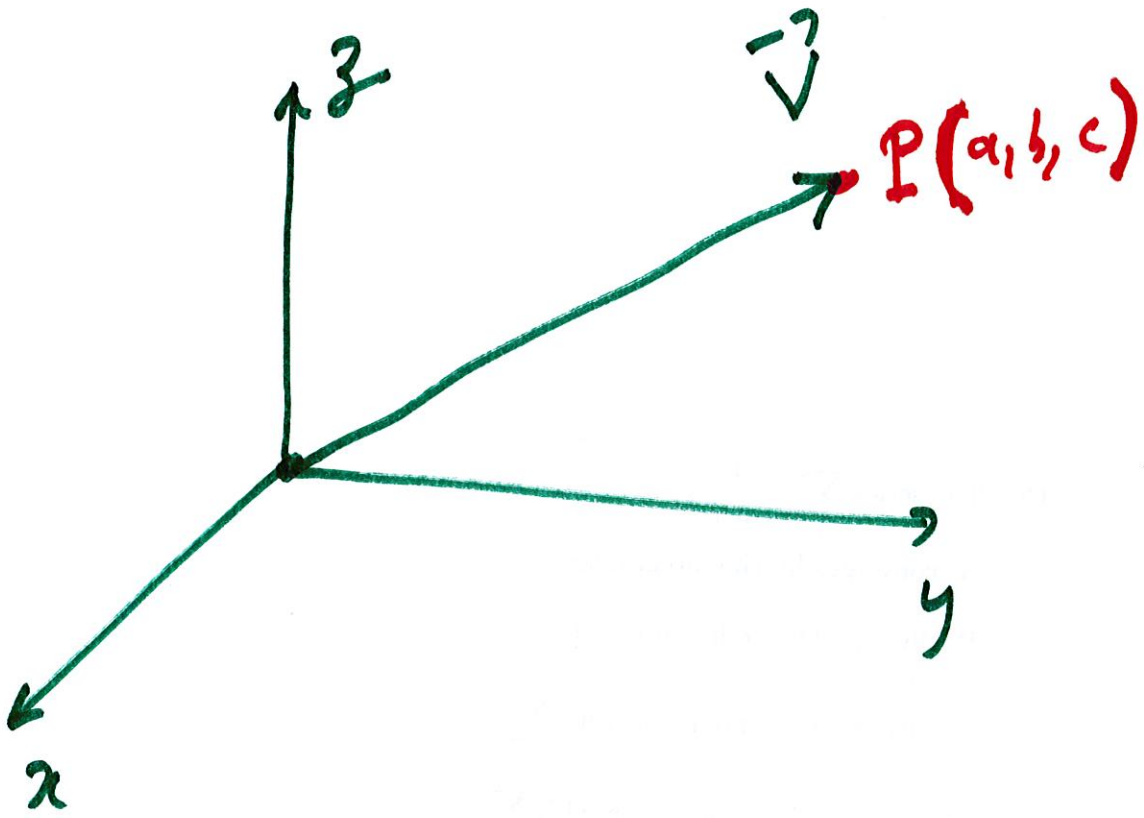
$$\vec{B}' + (\vec{A}' - \vec{B}') = \vec{A}'.$$

Parallelogram



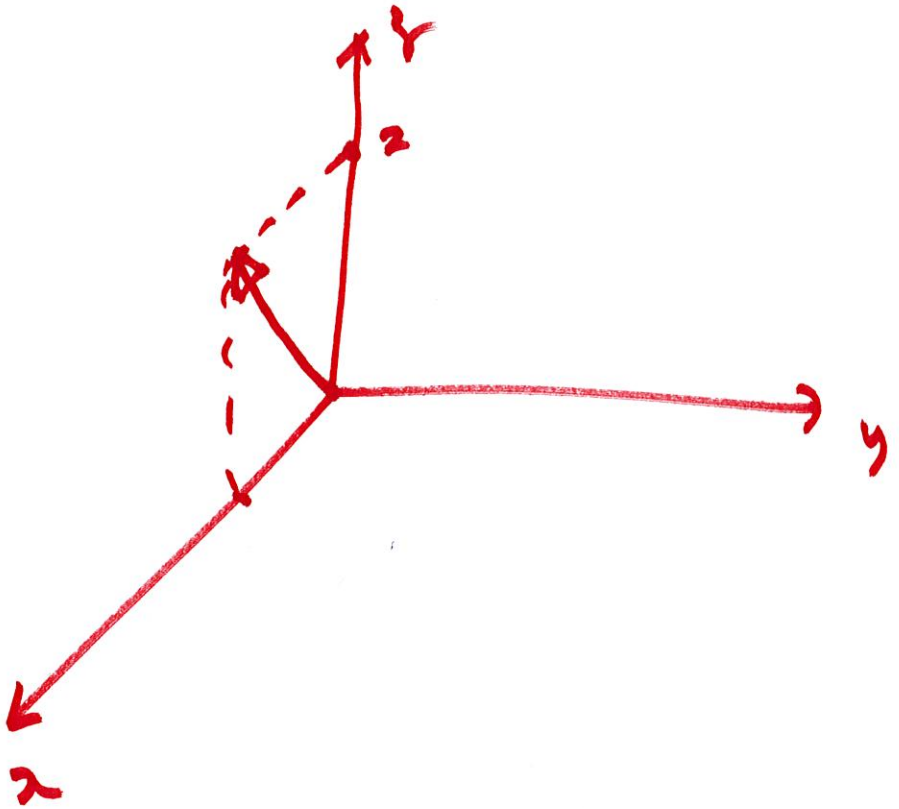
Components of Vectors

$\vec{V}$

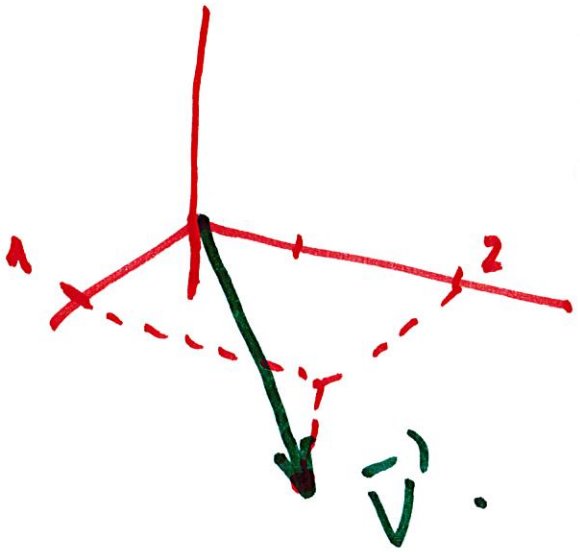


$\vec{V} = \langle a, b, c \rangle = \overrightarrow{OP}$.

Examples: $\bar{v} = \langle 1, 0, 2 \rangle$



$\bar{v} = \langle 1, 2, -1 \rangle$

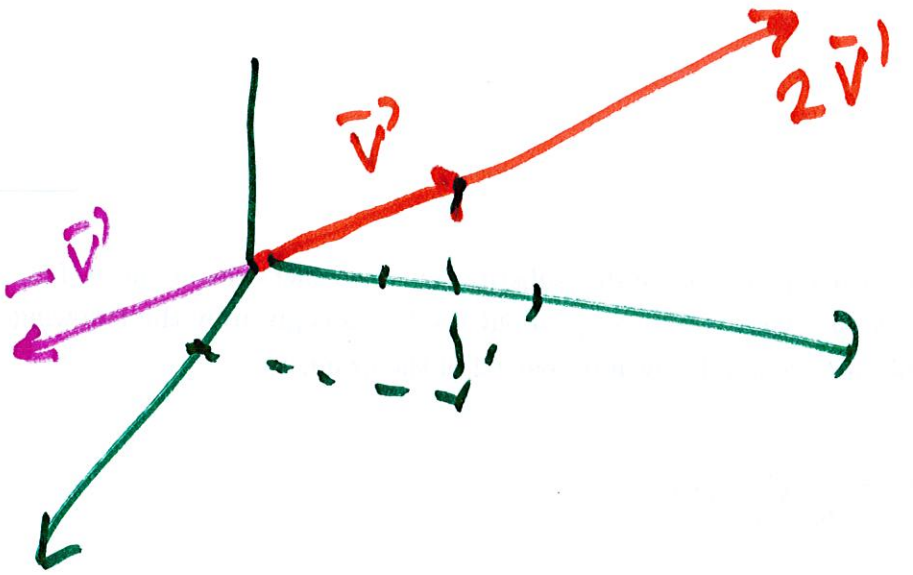


Operations of Vectors

$\bar{v} = \langle a, b, c \rangle$ x a real number.

$x\bar{v} = \langle xa, xb, xc \rangle$

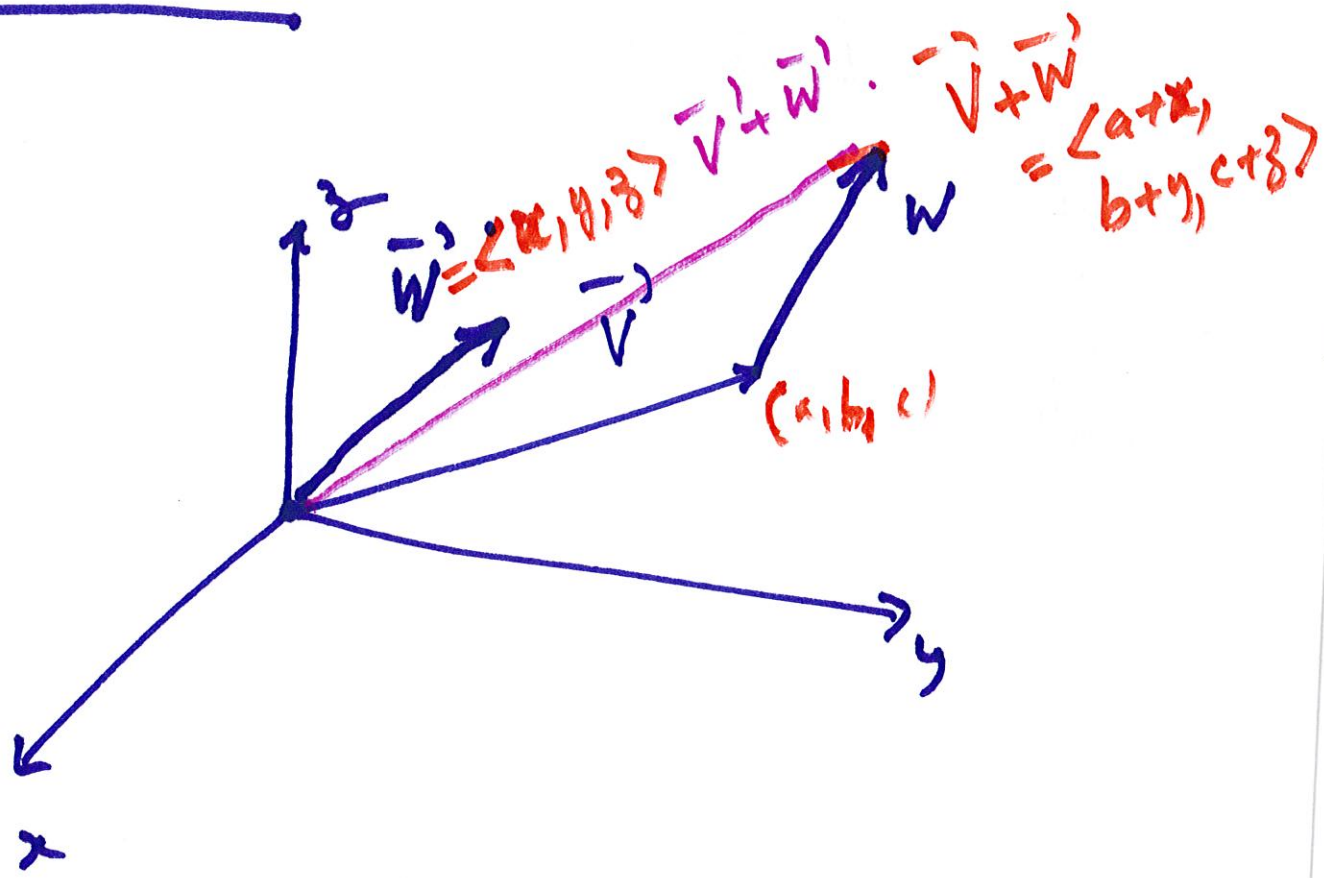
Ex:



$\bar{v} = \langle 1, 2, 1 \rangle$ $\quad -\bar{v} = \langle -1, -2, -1 \rangle$

$2\bar{v} = \langle 2, 4, 2 \rangle$.

Addition:



$\vec{v} = \langle a, b, c \rangle$

$\vec{w} = \langle x, y, z \rangle$

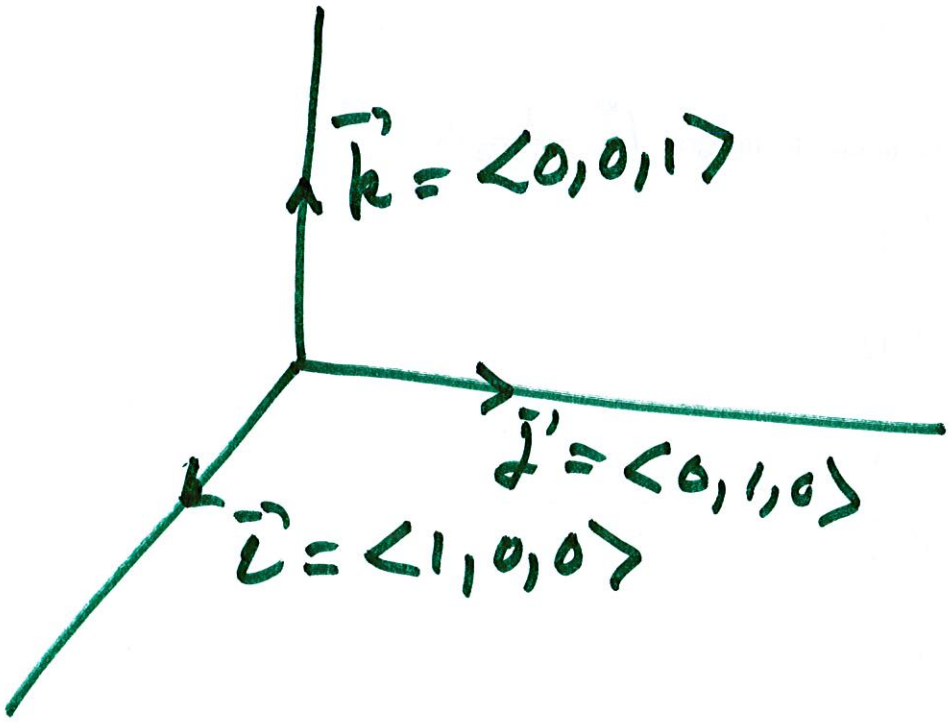
$\vec{v} + \vec{w} = \langle a + x, y + b, z + c \rangle$

Example: $\vec{v} = \langle 1, 0, 1 \rangle$

$\vec{w} = \langle 2, 3, 4 \rangle$

$\vec{v} + \vec{w} = \langle 3, 3, 5 \rangle$

Special Vectors



$$\bar{V} = \langle a, b, c \rangle =$$

$$= \langle a, 0, 0 \rangle + \langle 0, b, 0 \rangle + \langle 0, 0, c \rangle$$

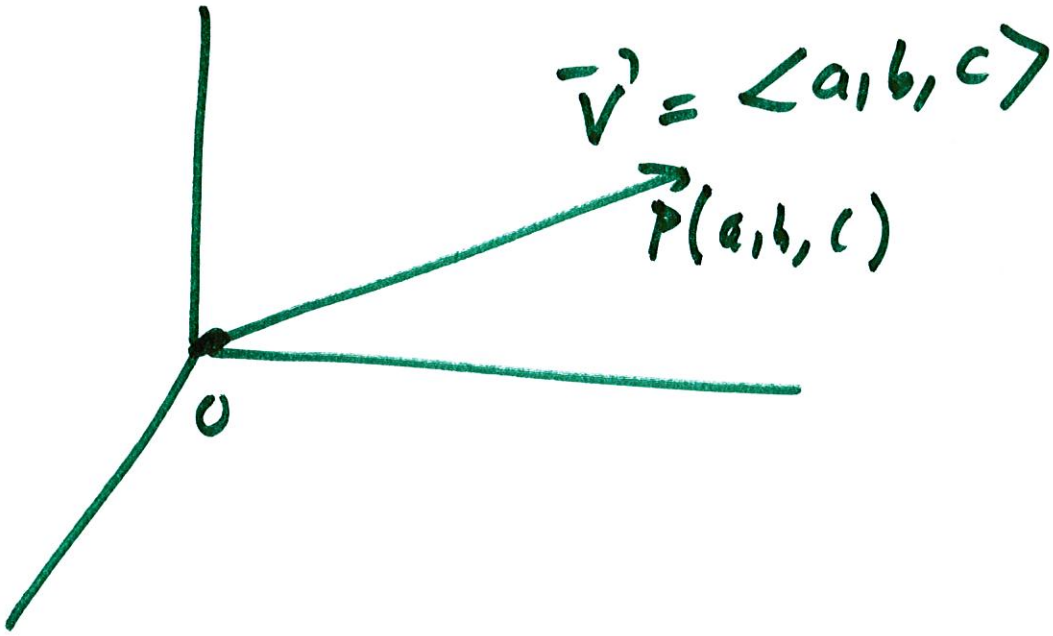
$$= a\langle 1, 0, 0 \rangle + b\langle 0, 1, 0 \rangle + c\langle 0, 0, 1 \rangle$$

$$= a\bar{i} + b\bar{j} + c\bar{k}.$$

$$\boxed{\langle a, b, c \rangle = a\bar{i} + b\bar{j} + c\bar{k}}$$

$$\langle 1, 2, 5 \rangle = \bar{i} + 2\bar{j} + 5\bar{k}.$$

Length of Vectors



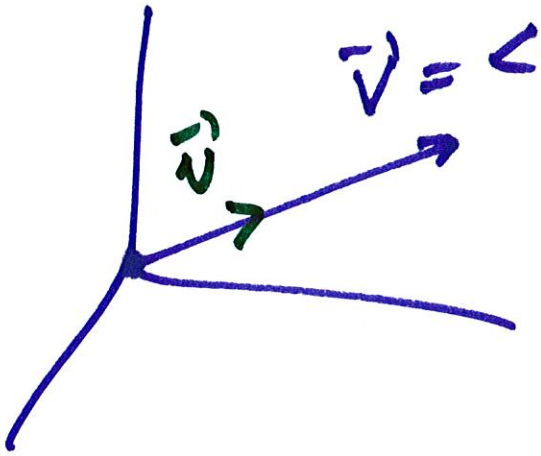
$\bar{v} = \overline{OP}$, $|\bar{v}| = |\overline{OP}| = \sqrt{a^2+b^2+c^2}$

If $\bar{v} = \langle a, b, c \rangle$, $|\bar{v}| = \sqrt{a^2+b^2+c^2}$.

Example: $\bar{v} = \langle 0, 1, 2 \rangle$

$$|\bar{v}| = \sqrt{0^2+1^2+2^2} = \sqrt{5}.$$

Unit Vectors.



Find a vector of length 1 which points in the same direction of $\vec{v}$.

$$\vec{v}' = \frac{1}{|\vec{v}|} \vec{v}.$$

Example: $\bar{v} = \langle 1, 5, 6 \rangle$

$$|\bar{v}| = \sqrt{1 + 25 + 36} =$$

$$= \sqrt{62}$$

$$\boxed{\bar{u} = \frac{1}{\sqrt{62}} \langle 1, 5, 6 \rangle}$$

$$|\bar{u}| = \sqrt{\frac{1^2}{62} + \frac{5^2}{62} + \frac{6^2}{62}}$$

$$= \frac{\sqrt{62}}{\sqrt{62}} = 1.$$

Vector of length 102 which points in the direction of $\langle 1, 1, 1 \rangle$.

$$\vec{v} = \langle 1, 1, 1 \rangle$$

$$|\vec{v}| = \sqrt{3}$$

$$\vec{u} = \frac{1}{\sqrt{3}} \langle 1, 1, 1 \rangle$$

$$\vec{w} = \frac{102}{\sqrt{3}} \langle 1, 1, 1 \rangle$$

Projections and the Dot Product

$|\vec{v}| \cdot \cos\theta$