MA 16600
Exam 3

Tue., Apr. 9, 2019 6:30 p.m.–7:30 p.m.

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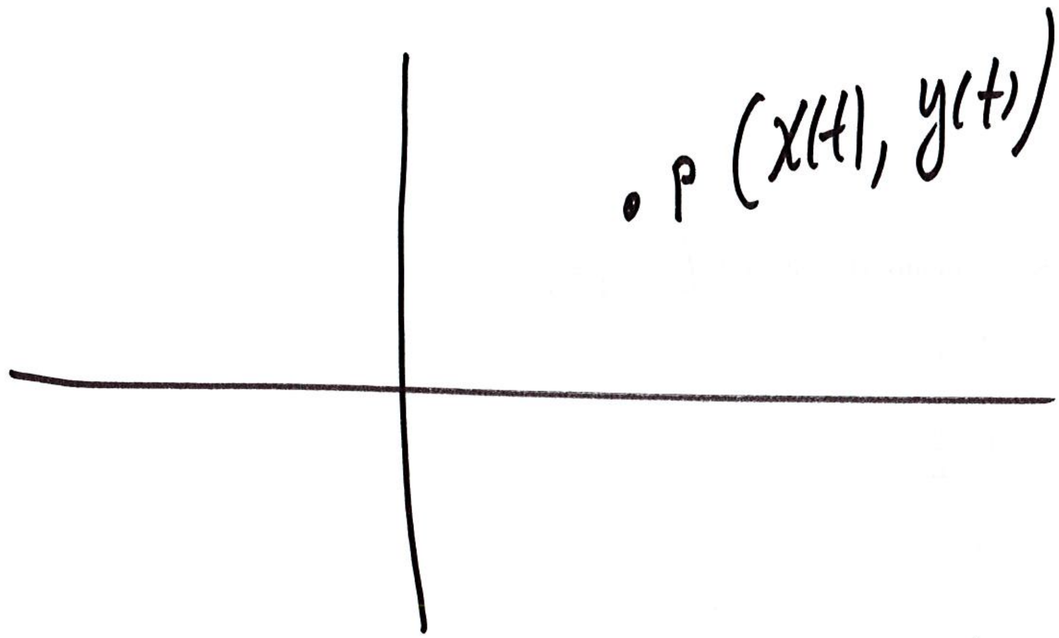
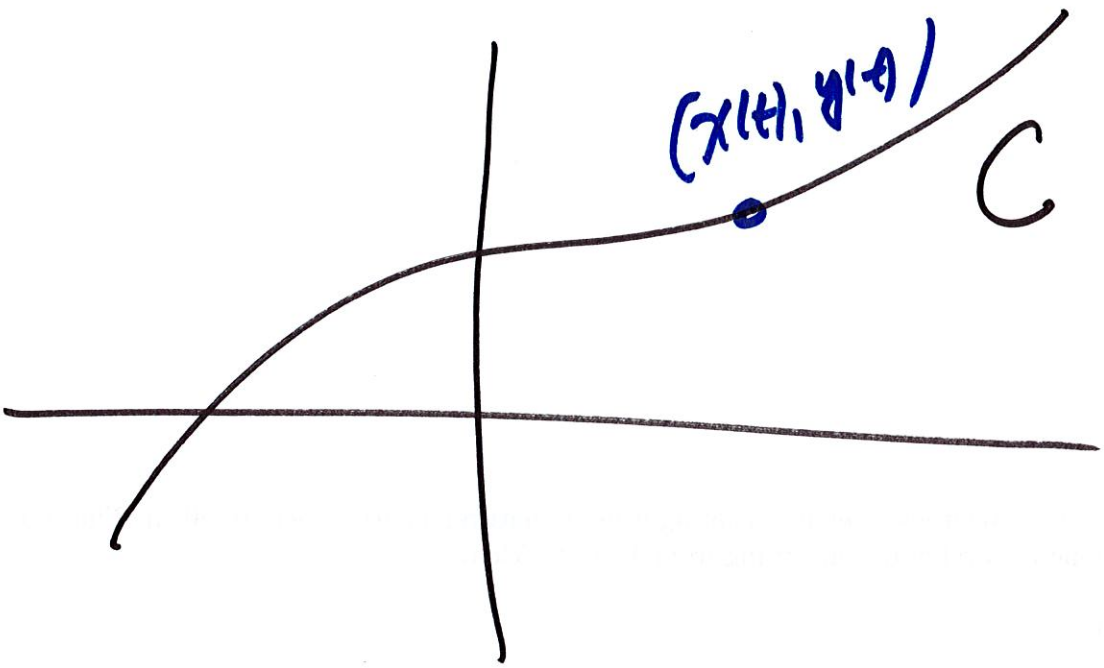
Lesson 31 (Not on the exam)

Section 10.1 Curves defined by Parametric Equations



A point p is moving along the plane and at time t its position is $(x(t), y(t))$.

As the point moves it describes a curve C



We want to analyze the C from $(x(t), y(t))$, which is called a ~~parametrization~~ PARAMETRIZATION of C. And the motion of the point.

Example 1: $x(t) = 1 + 2t$

$y(t) = 2 + t$

$-5 \leq t \leq 15.$

Find an equation for ~~sy~~ the trajectory in (x, y)

$2t = x - 1$; $\boxed{t = \frac{x-1}{2}}$

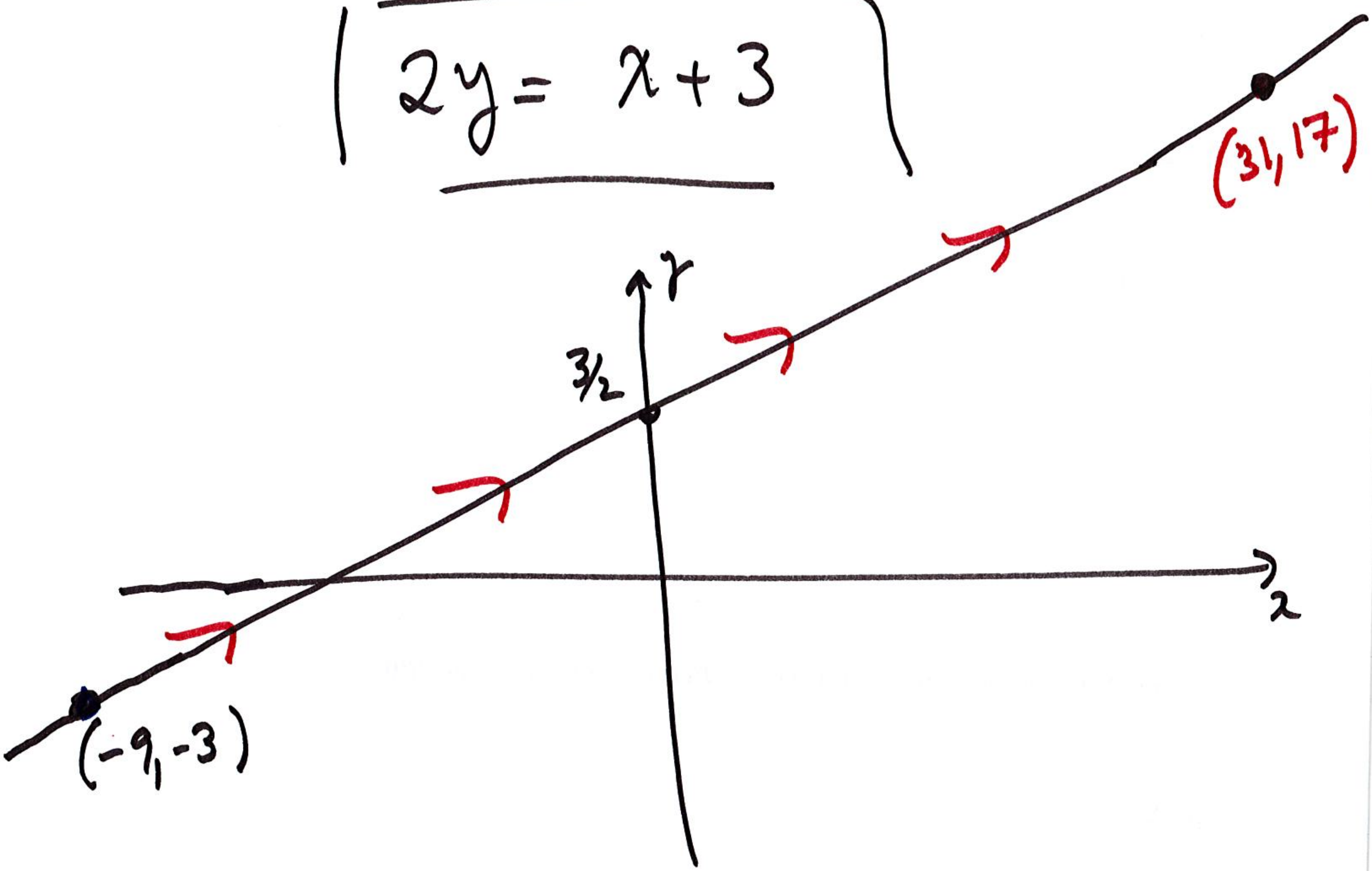
$\boxed{t = y - 2}$

$$t = \frac{x-1}{2} = y - 2$$

$$\frac{x-1}{2} = y - 2$$

$$x-1 = 2y-4$$

$$\boxed{2y = x+3}$$



The point moves along this line.

when $t=-5$; $x = 1+2t = 1-10 = -9$

$y = 2+t = 2-5 = -3$

$t = 15$; $x = 31$

$y = 17$.

$x(t) = 3\cos t, \quad y = 3\sin t.$

$0 \leq t \leq 2\pi.$

In this case we use that

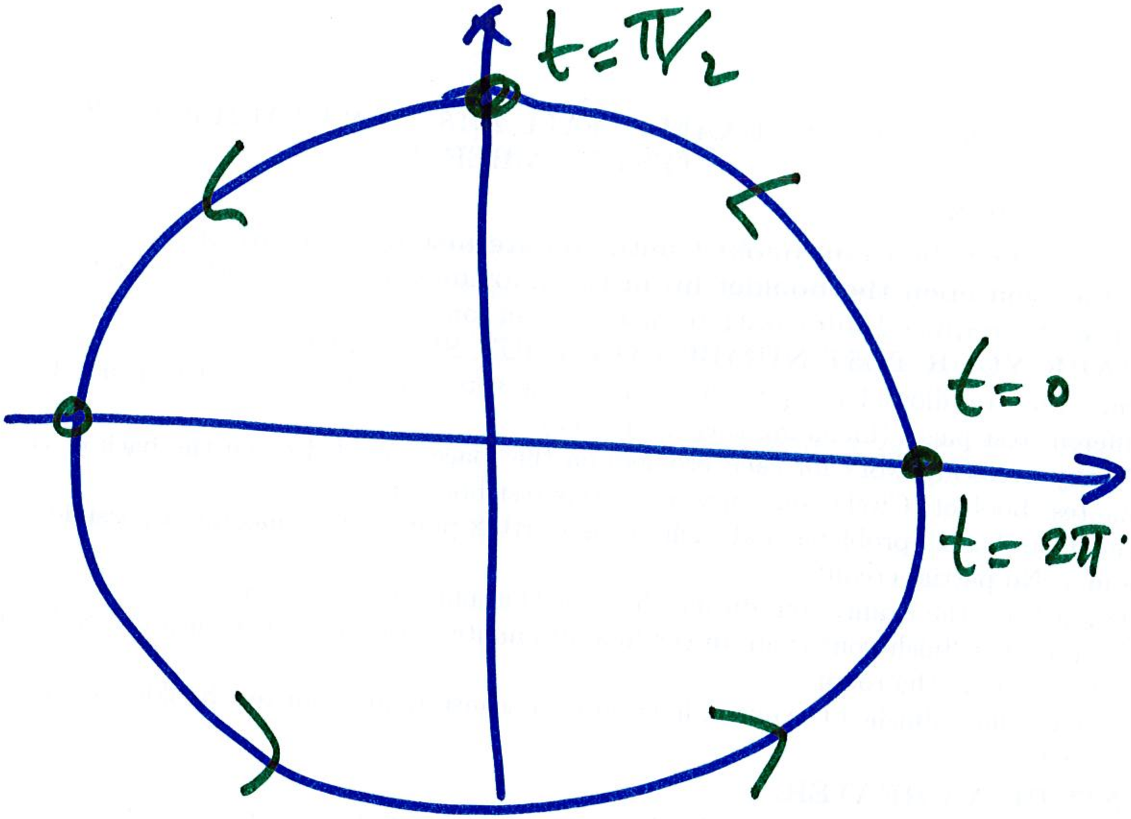
$$\cos^2 t + \sin^2 t = 1$$

$$\cos t = x/3 \; ; \quad y/3 = \sin t$$

$$\left(x/3\right)^2 + \left(y/3\right)^2 = \cos^2 t + \sin^2 t = 1$$

$$\frac{x^2}{9} + \frac{y^2}{9} = 1$$

$$\boxed{x^2 + y^2 = 9}$$



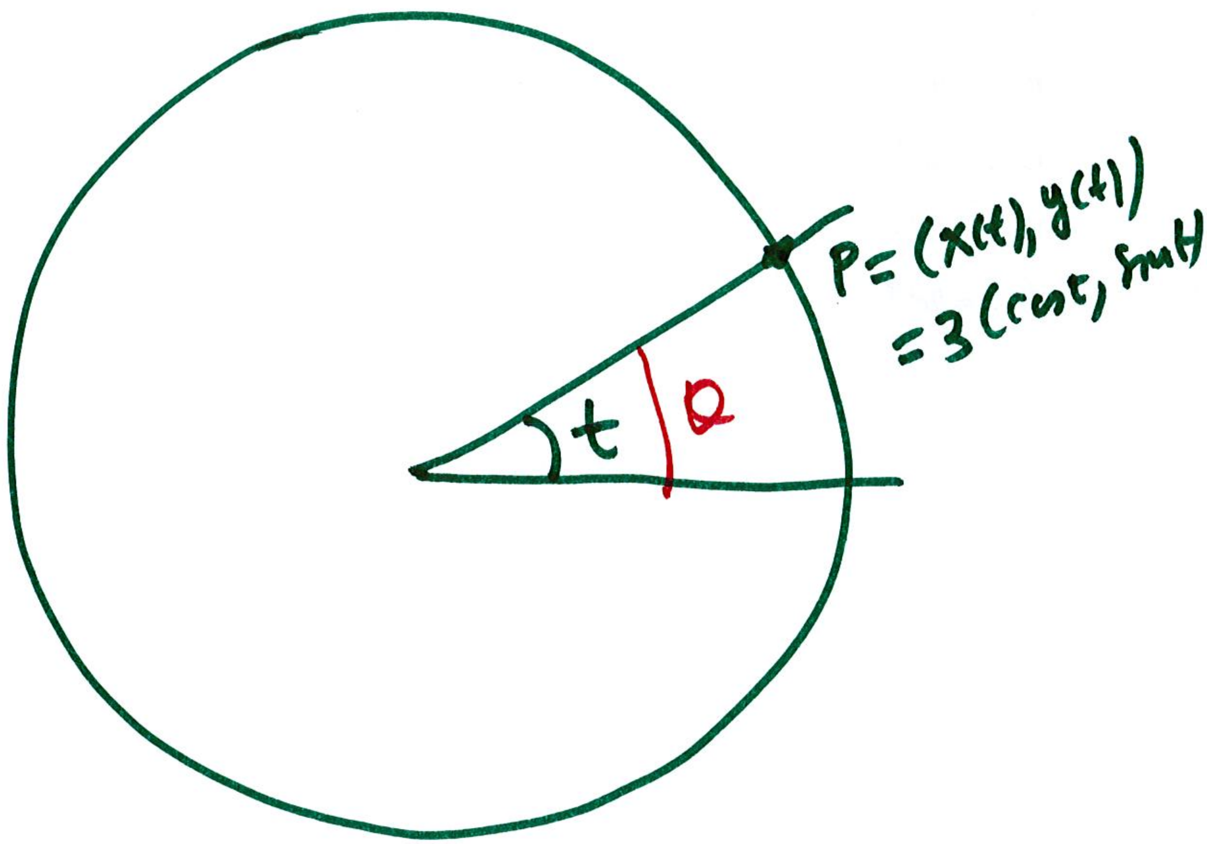
The curve described by the point is a circle centered at 0 and radius 3.

When $t=0$, $x=\cos t$, $x=1$

$y=\sin t$, $y=0$

$t=\pi/2$; $x=0$

$y=1$



$t =$ angle shown in the picture.

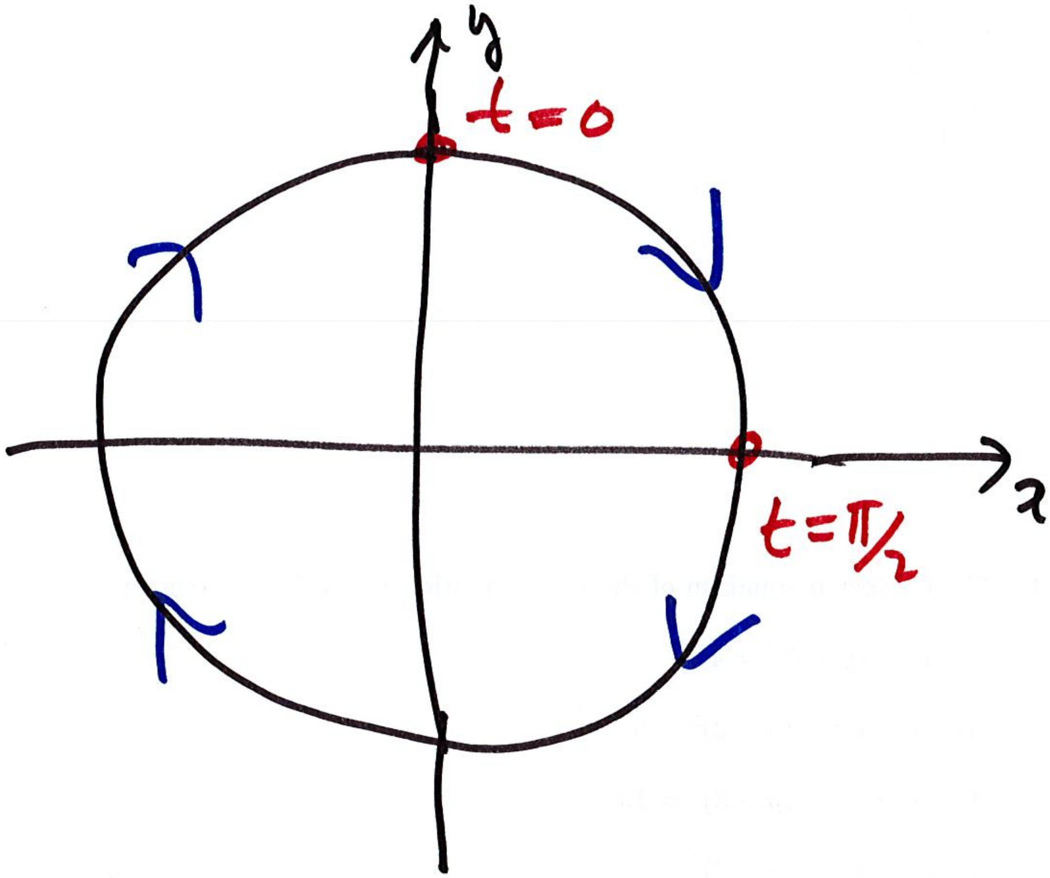
$t =$ time.

Angular speed $= 1$.

$\theta(t) = t \;,\; \frac{d\theta}{dt} = 1.$

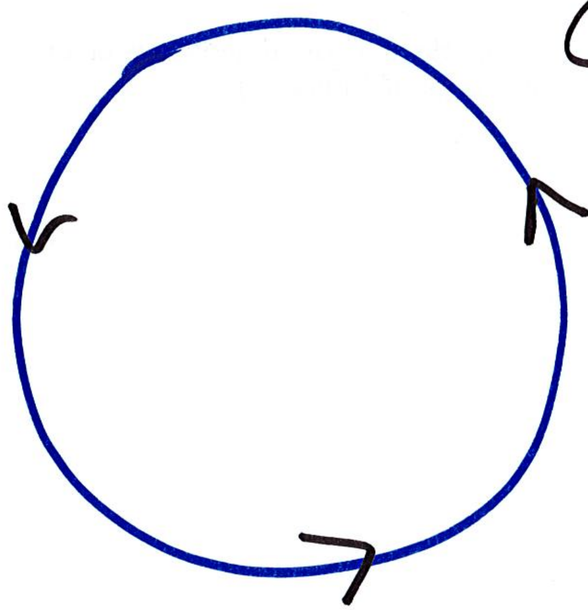
Example:

$$x = 3\sin t, \quad y = 3\cos t$$

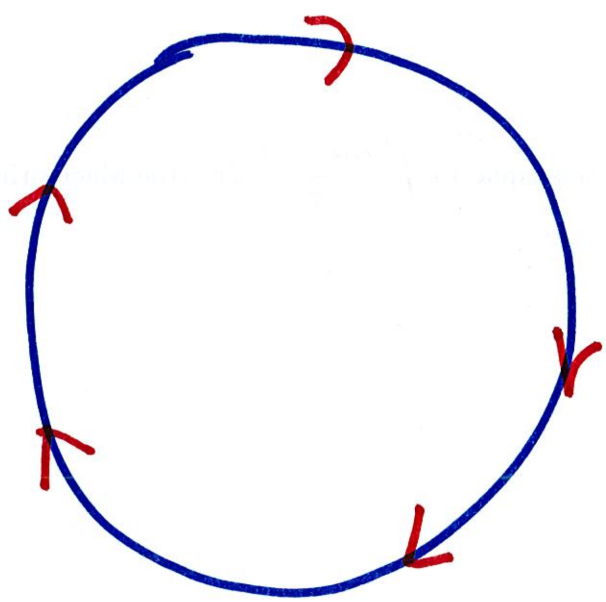


$t=0, \quad x=0, \quad y=3$

$t=\pi/2, \quad x=3, \quad y=0$



COUNTER
CLOCKWISE
ORIENTATION



CLOCKWISE
ORIENTATION

$$x = 4\cos t \; ; \quad y = 3\sin t$$

$$0 \le t \le \pi .$$

$$\cos t = x/4 \; , \quad \sin t = y/3$$

$$\boxed{\cos^2 t + \sin^2 t = 1}$$

$$\left(x/4\right)^2 + \left(y/3\right)^2 = 1$$

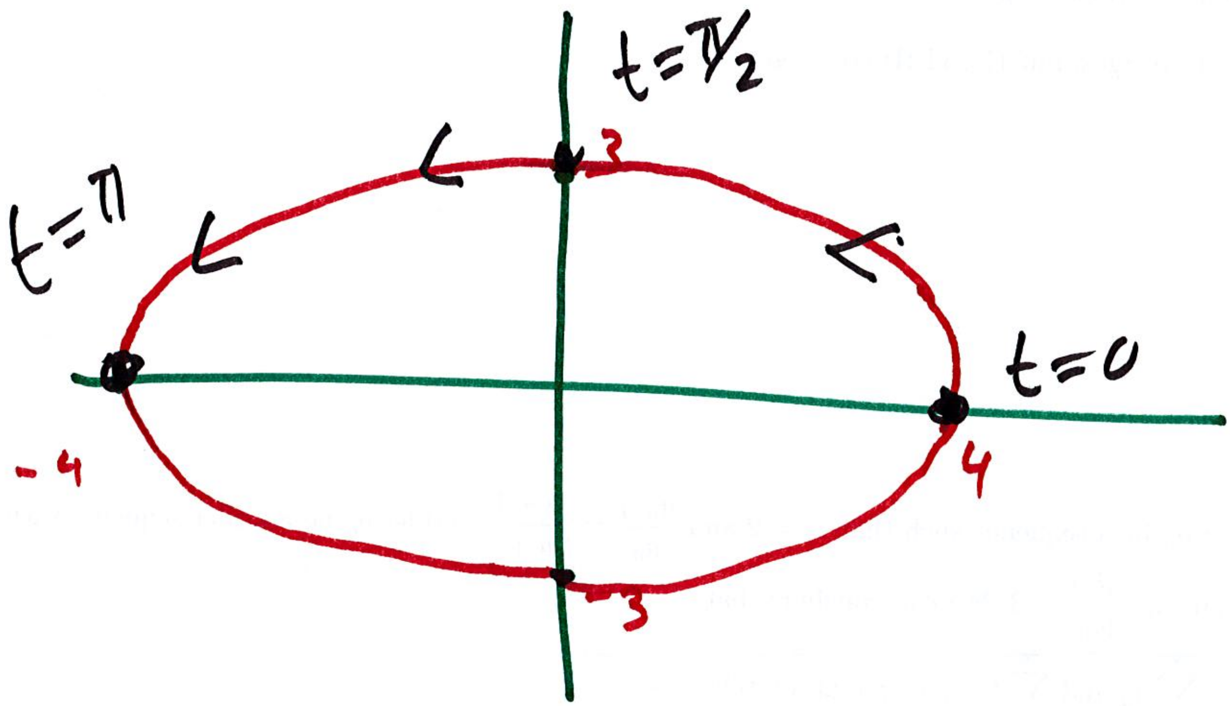
$$\boxed{x^2/16 + y^2/9 = 1}$$ Is an ellipse.

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \quad , \quad x=0, \; y^2 = 9$$

$$y = \pm 3$$

$$y=0, \; x^2 = 16$$

$$x = \pm 4$$



when $t = 0$, $x = 4$, $y = 0$

$t = \pi/2$; $x = 0$, $y = 3$

$$x = \sin t, \quad y = \cos^2 t$$

$t = 0, \quad x = 0, \quad y = 1$

$t = \pi/2, \quad x = 1, \quad y = 0$

$t = \pi, \quad x = 0, \quad y = 1$

$t = 3\pi/2; \quad x = -1, \quad y = 0$

$t = 2\pi, \quad x = 0, \quad y = 1$

$x = \sin t; \quad y = \cos^2 t.$

$0 \leq t \leq 2\pi.$

Use that $\cos^2 t + \sin^2 t = 1.$

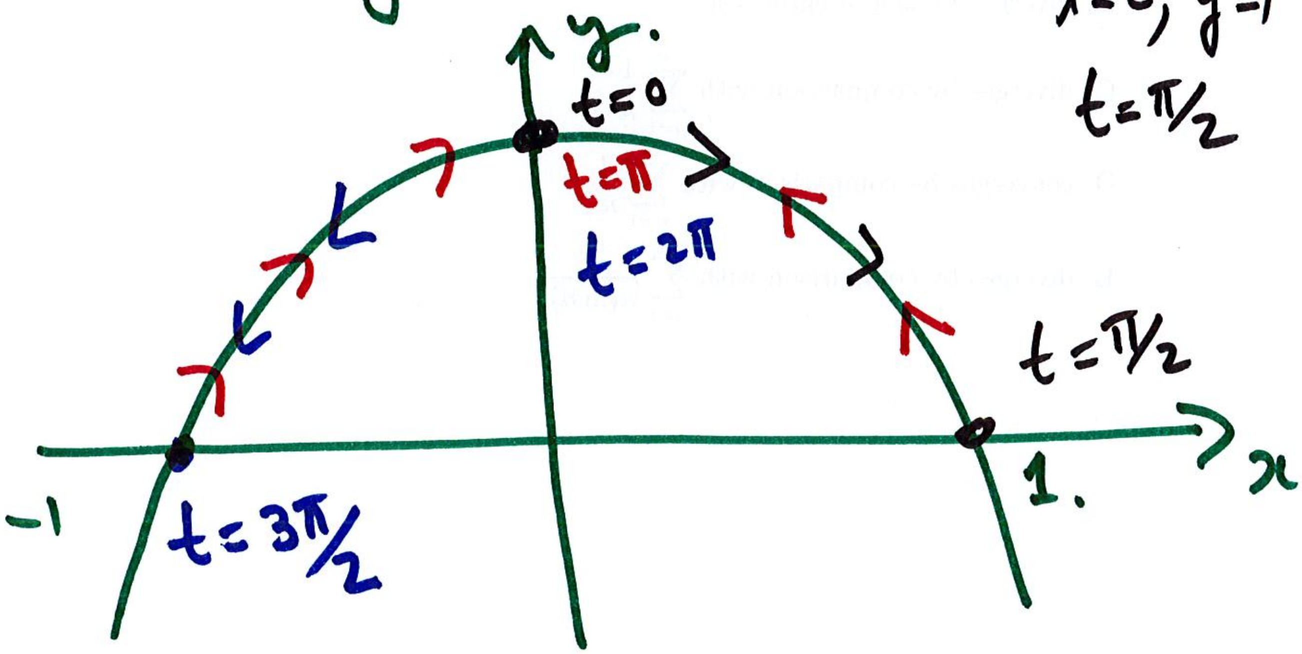
$y + x^2 = 1$

$y = 1 - x^2$

$t=0$

$x=0, y=1$

$t=\pi/2$



$x = 2 - t^2, \quad y = t + 1$

$-1 \le t \le 1$

Solve for t: $t = y - 1$

$$\boxed{x = 2 - (y-1)^2}$$

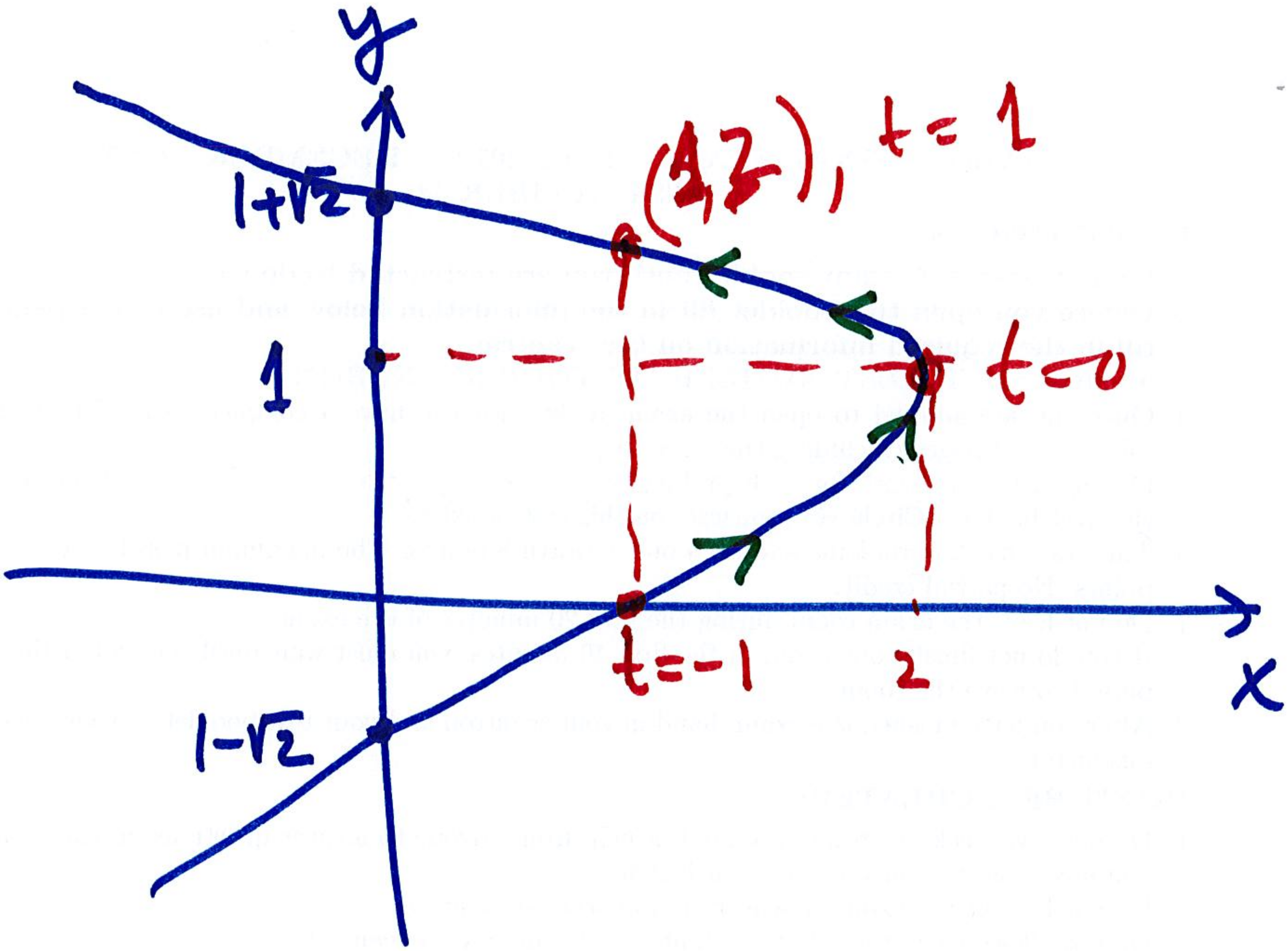
$x = 2 - (y^2 - 2y + 1)$

$$\boxed{x = 1 + 2y - y^2}$$

$\underline{x = 0}$: $2 - (y-1)^2 = 0 \quad (y-1)^2 = 2$

$y - 1 = \pm\sqrt{2}$

$y = 1 + \sqrt{2}, \ y = 1 - \sqrt{2}.$



<u>When $t=-1$</u> : $x = 2 - t^2 = 1$

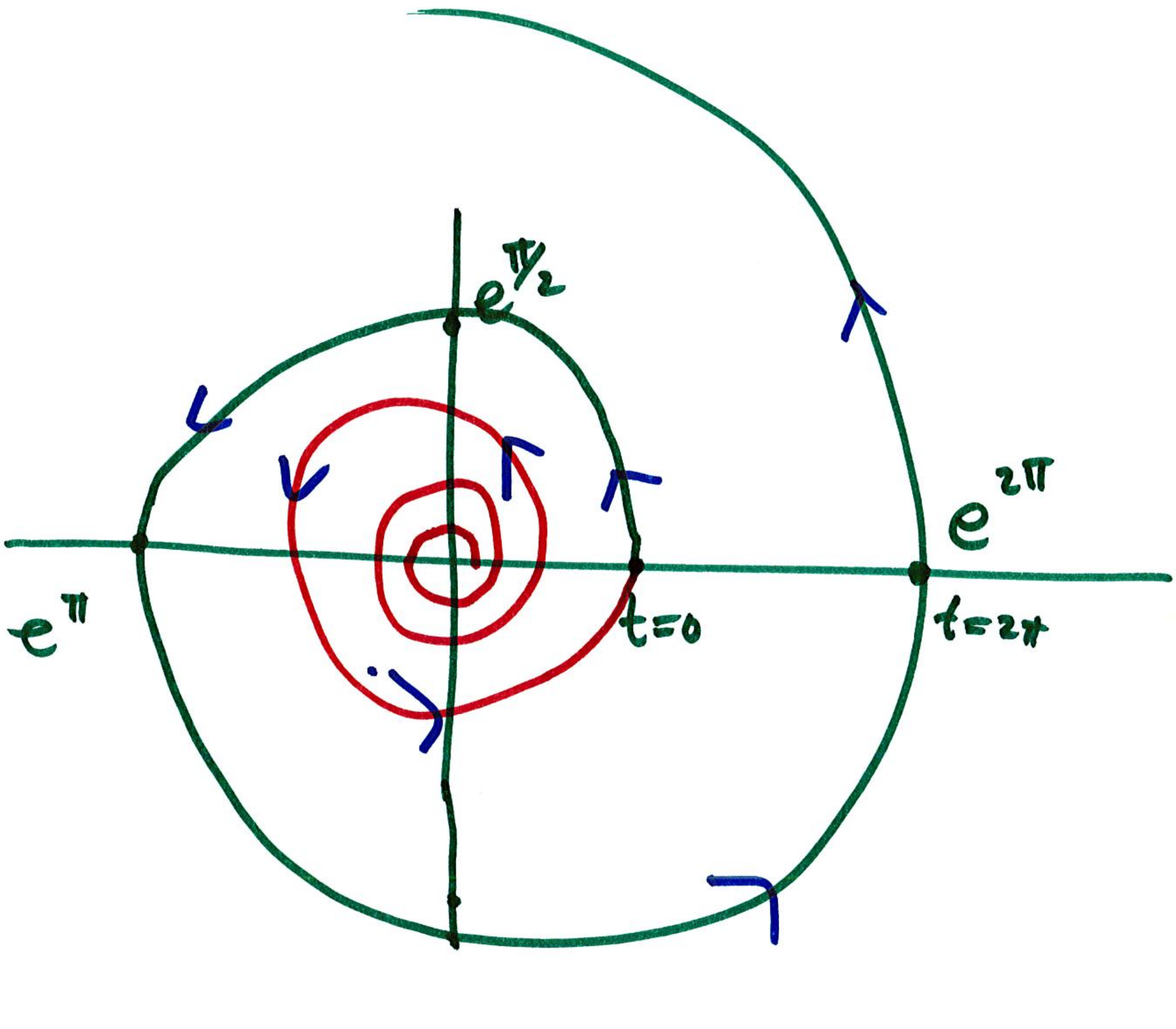
$y = 1 + t = 0$

$t = 0$, $x = 2$, $y = 1$

$t = 1$; $x = 1$, $y = 2$

$$x = e^t \cos t, \quad y = e^t \sin t$$

$$-\infty < t < \infty$$



Spiral.