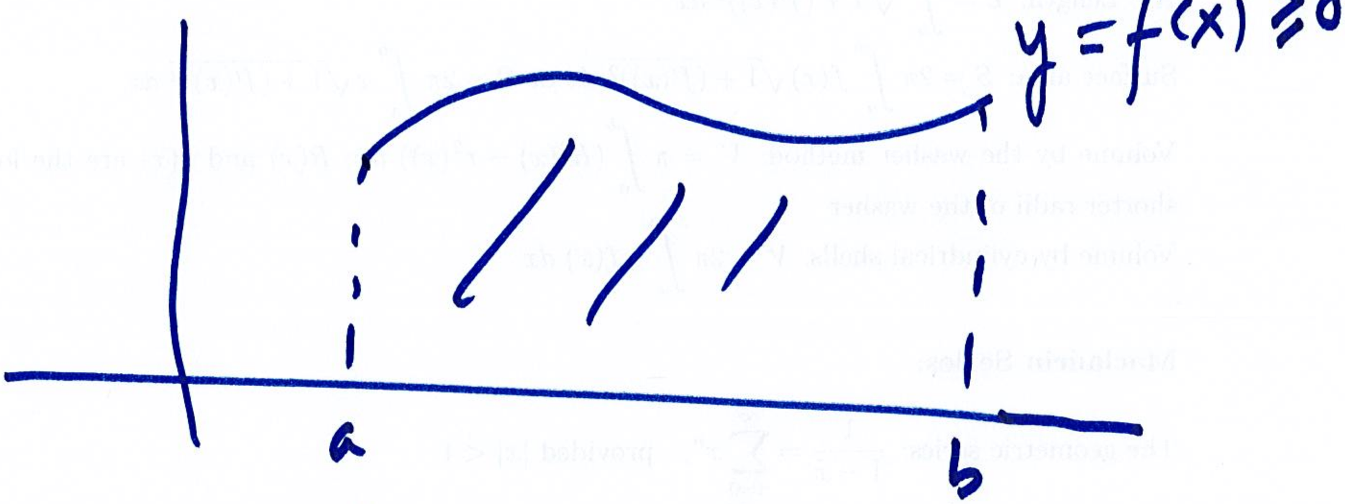
Lesson 5 Section 6.1

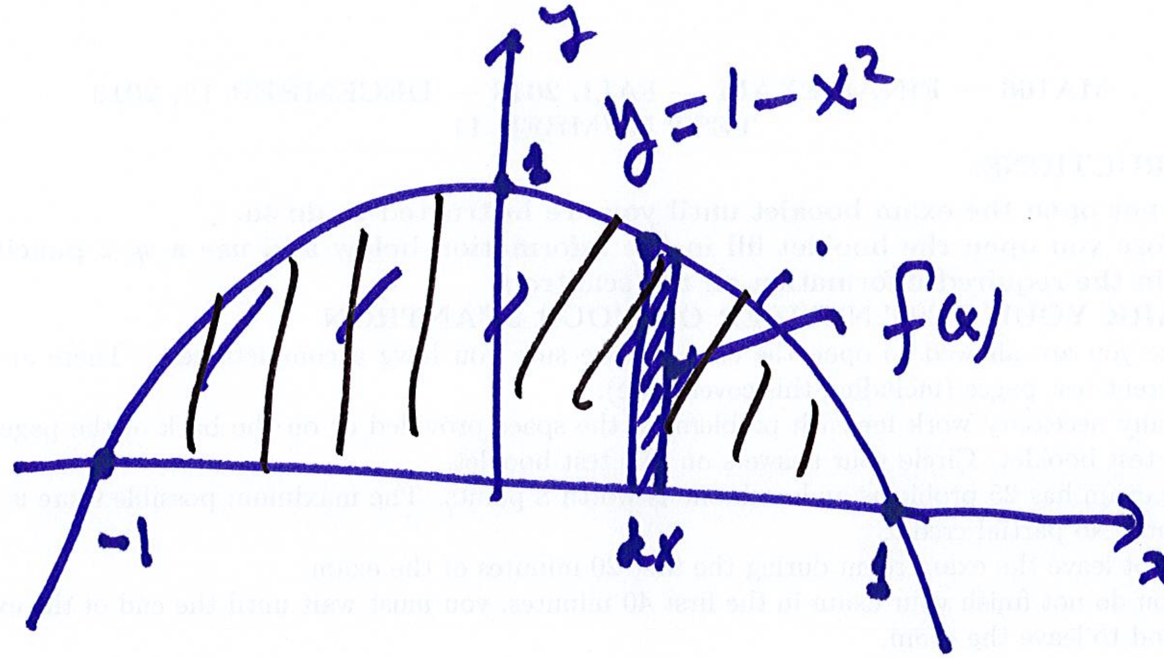
Areas Between Curves

Calc I,



$$\text{Area} = \int_a^b f(x)\,dx.$$

Example: Find the area under the graph of $y = 1 - x^2$ for $-1 \leq x \leq 1$



$$A = \int_{-1}^{1} (1-x^2)\,dx \qquad \underline{\int_{-1}^{1} f(x)\,dx}$$

$$A = x - \frac{x^3}{3}\Bigg|_{-1}^{1} = \left(x - \frac{x^3}{3}\right)\Bigg|_{x=1} - \left(x - \frac{x^3}{3}\right)\Bigg|_{x=-1}$$

$$= \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)$$

$$= \frac{2}{3} - \left(-\frac{2}{3}\right) = \frac{4}{3}.$$

$f(x) = 1 - x^2$ is even.

$$f(-x) = 1 - (-x)^2 = 1 - x^2 = f(x)$$

$$\boxed{A = \int_{-1}^{1} (1 - x^2)\, dx}$$

$$= \int_{-1}^{0} (1 - x^2)\, dx + \int_{0}^{1} (1 - x^2)\, dx$$

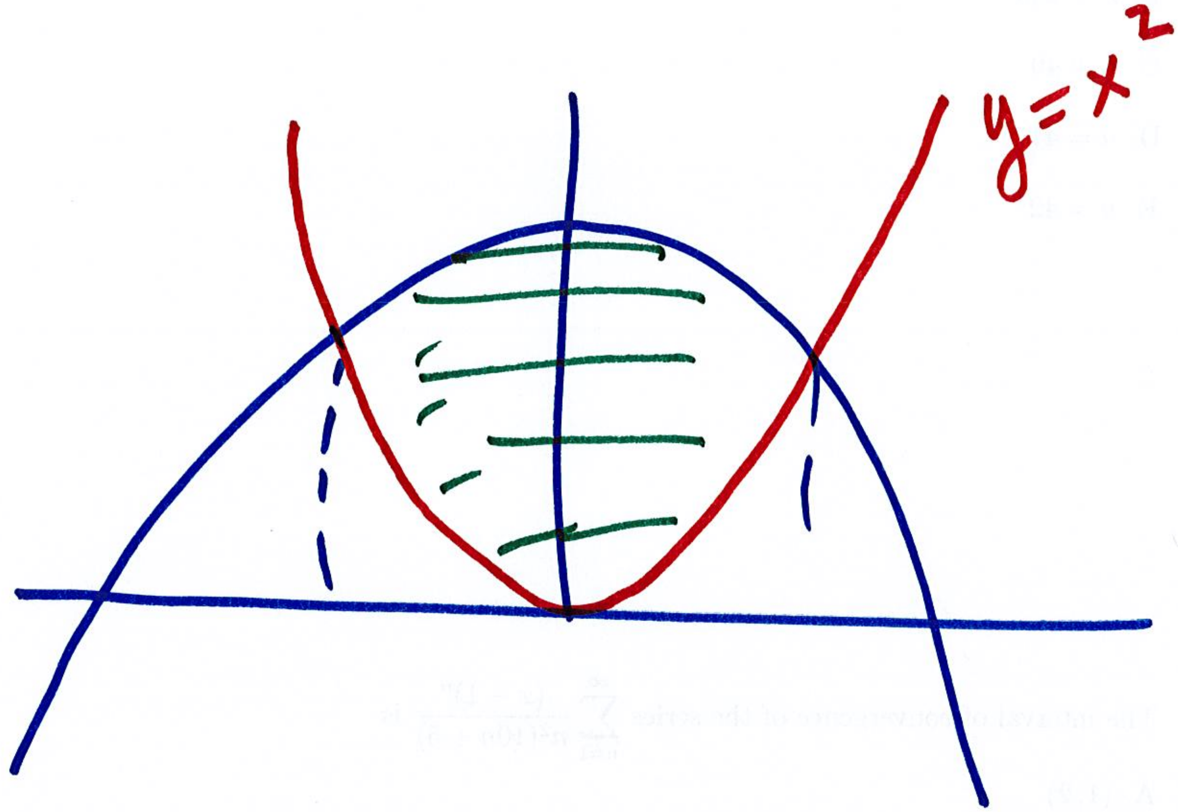
$$\boxed{= 2 \int_{0}^{1} (1 - x^2)\, dx}$$

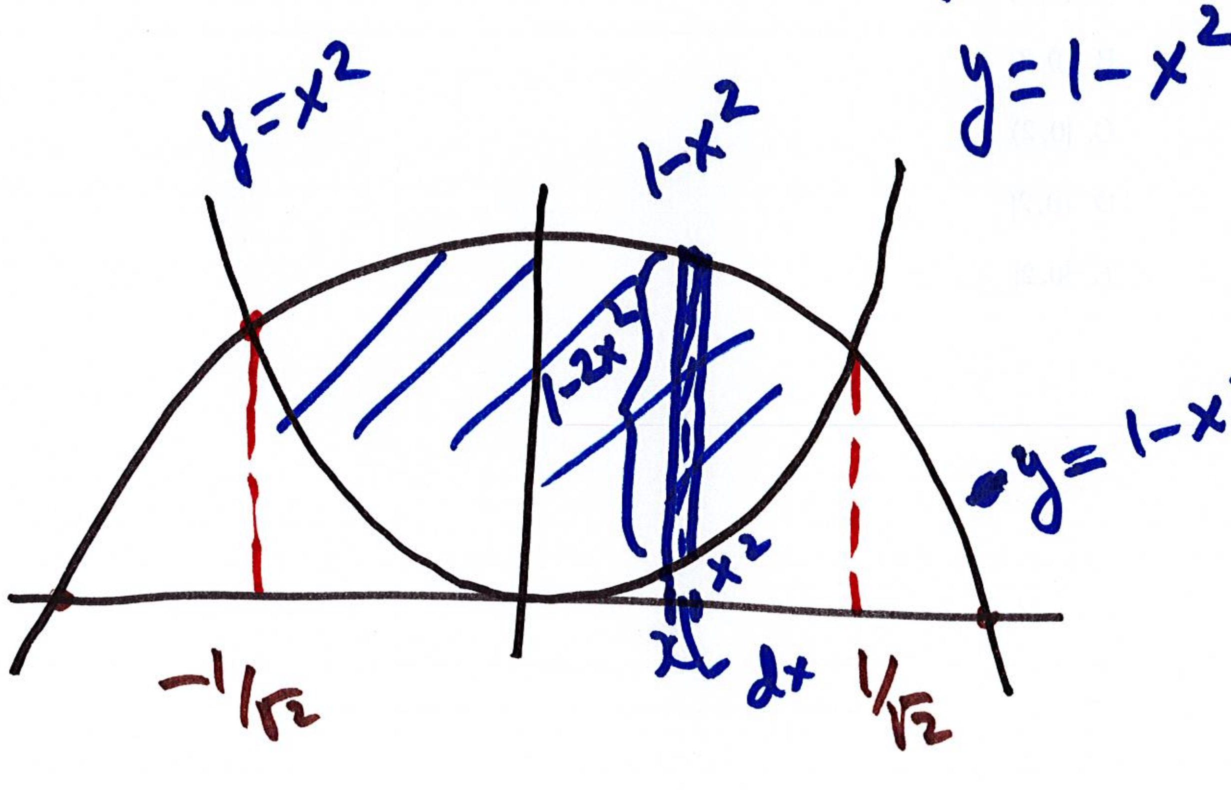
$$= 2 \left(x - \frac{x^3}{3} \right) \Big|_0^1 = 2 \cdot \left(1 - \frac{1}{3}\right)$$

$$= 4/3.$$

Find the area between

$y = 1 - x^2$ and $y = x^2$

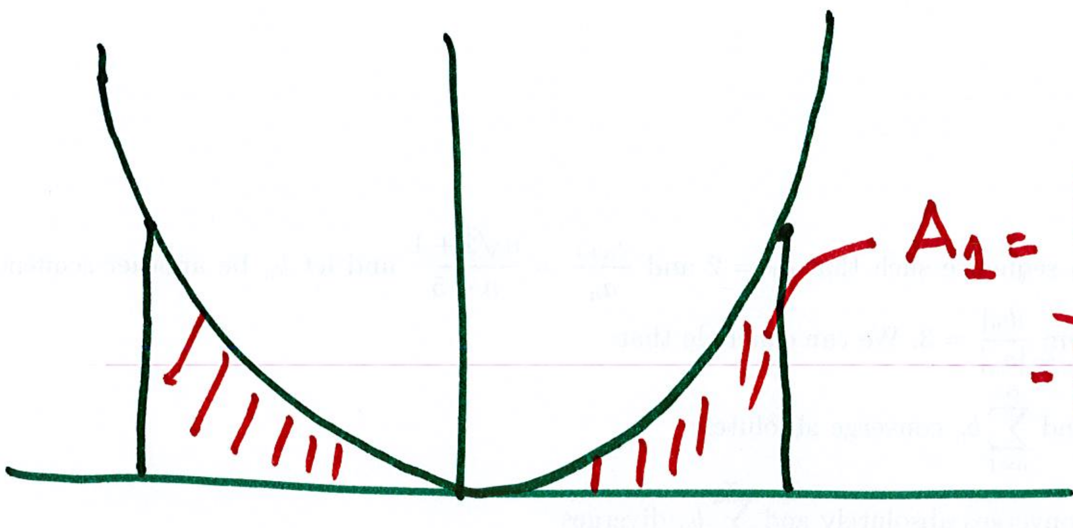




$$\boxed{A = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} (1 - 2x^2)\,dx}$$

$1 - x^2 = x^2$; $1 = 2x^2$

$x^2 = 1/2$; $x = 1/\sqrt{2}$, $x = -1/\sqrt{2}$



$$A_1 = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} x^2\,dx$$

$$A_2 = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} (1 - x^2)\,dx$$

$$A = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} (1 - 2x^2)\, dx$$

$$= 2\int_0^{1/\sqrt{2}} (1 - 2x^2)\, dx$$

$$= 2\left(x - \frac{2}{3}x^3\right)\Big|_0^{1/\sqrt{2}}$$

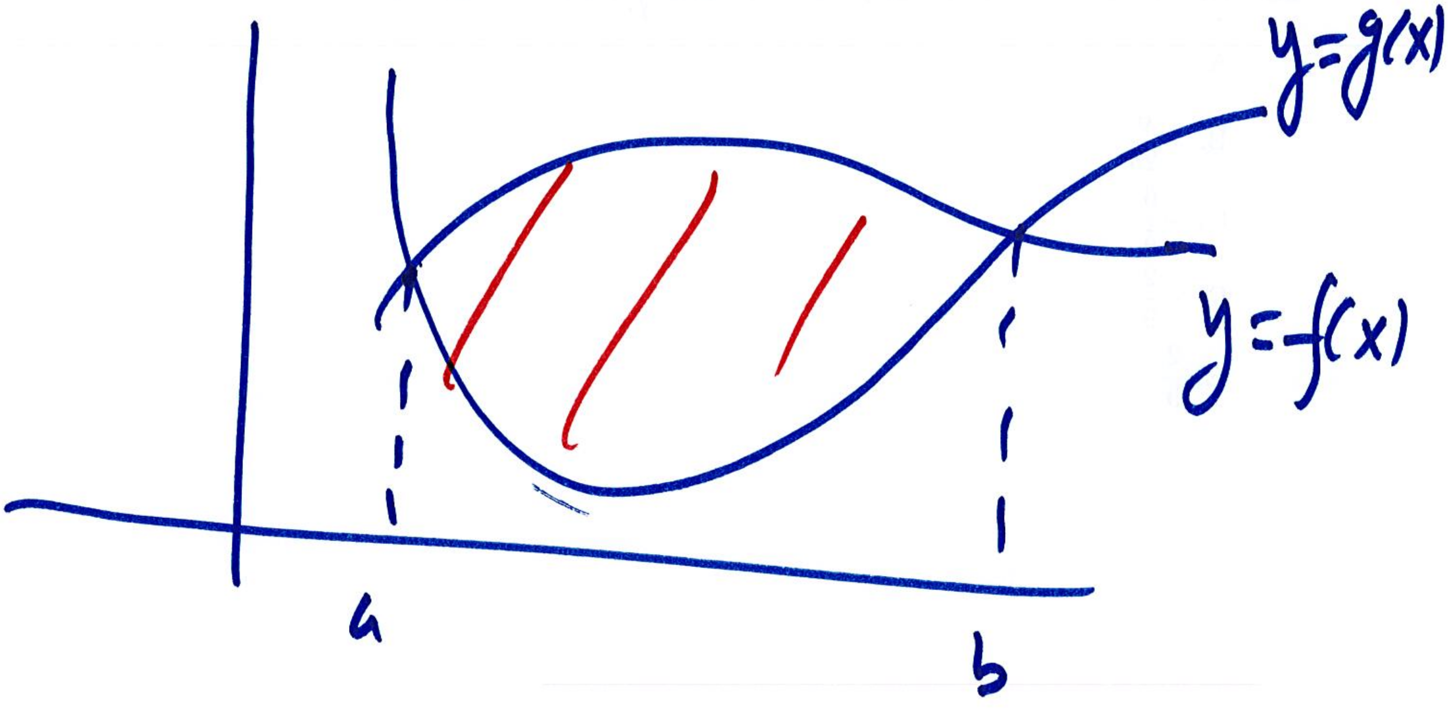
$(\sqrt{2})^3 = 2\sqrt{2}$

$$= 2\left(\frac{\sqrt{2}}{2} - \frac{2}{3}\left(\frac{1}{\sqrt{2}}\right)^3\right)$$

$$= 2\left(\frac{\sqrt{2}}{2} - \frac{2}{3}\,\frac{1}{2\sqrt{2}}\right) = 2\left(\frac{\sqrt{2}}{2} - \frac{1}{3\sqrt{2}}\right)$$

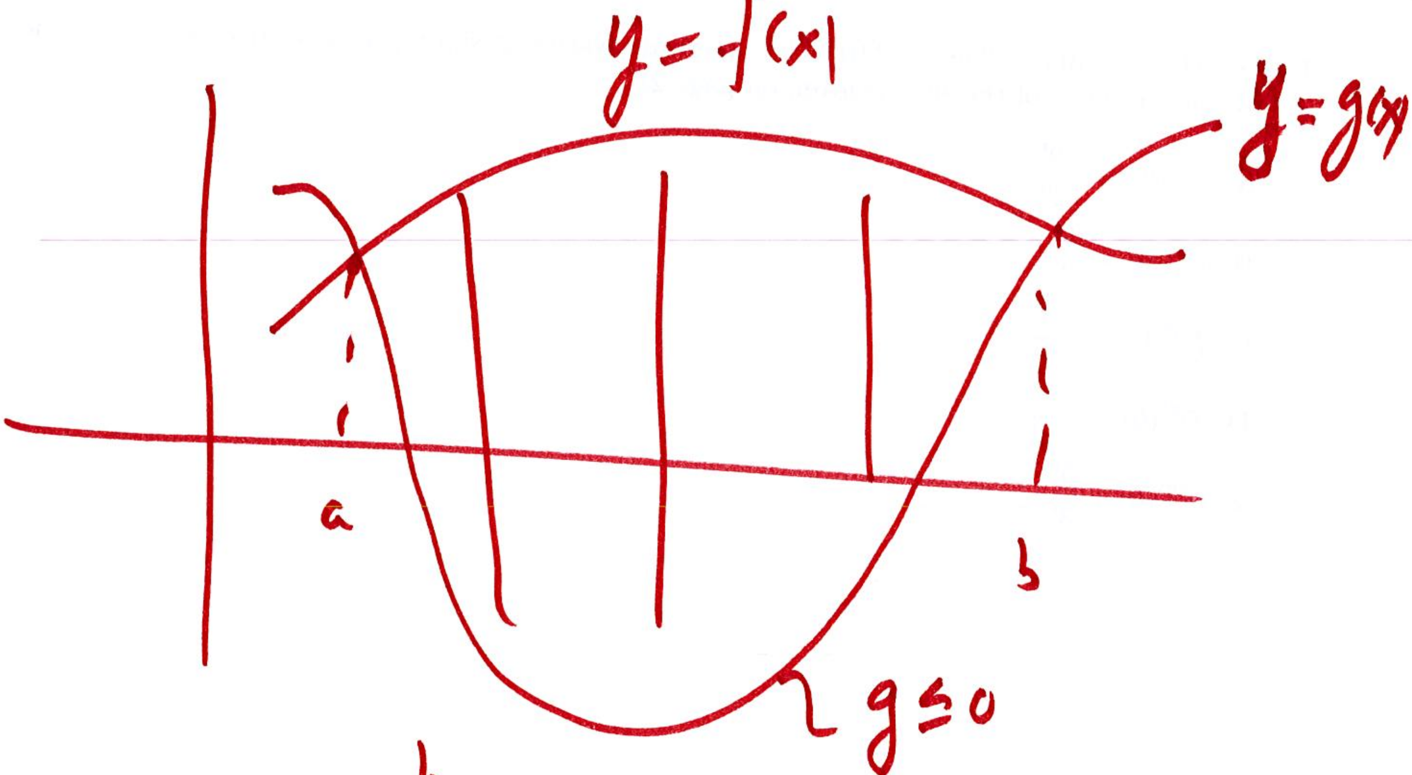
$$= 2\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{6}\right) = \sqrt{2} - \frac{\sqrt{2}}{3}$$

$$= 2\sqrt{2}/3.$$



$f(x) \geq g(x)$ on $[a,b]$

$$A = \int_a^b \left(f(x) - g(x) \right) dx$$

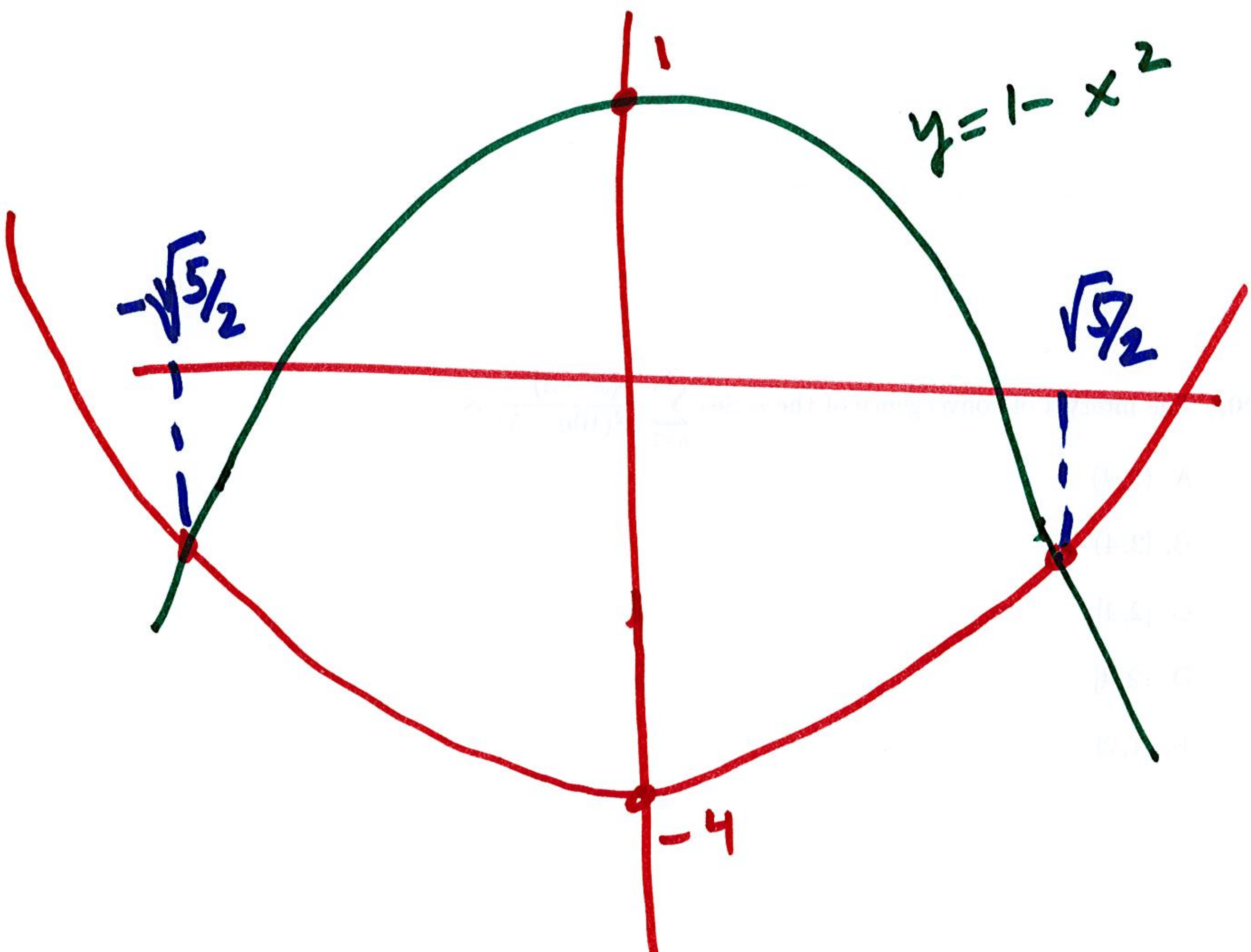


$$A = \int_a^b (f(x) - g(x))\,dx$$

$f(x) \geq g(x)$ on $[a,b]$

<u>Example:</u> $y = 1 - x^2$

$y = x^2 - 4.$



$\cancel{1-x^{2}} = \cancel{x^2} - 4$

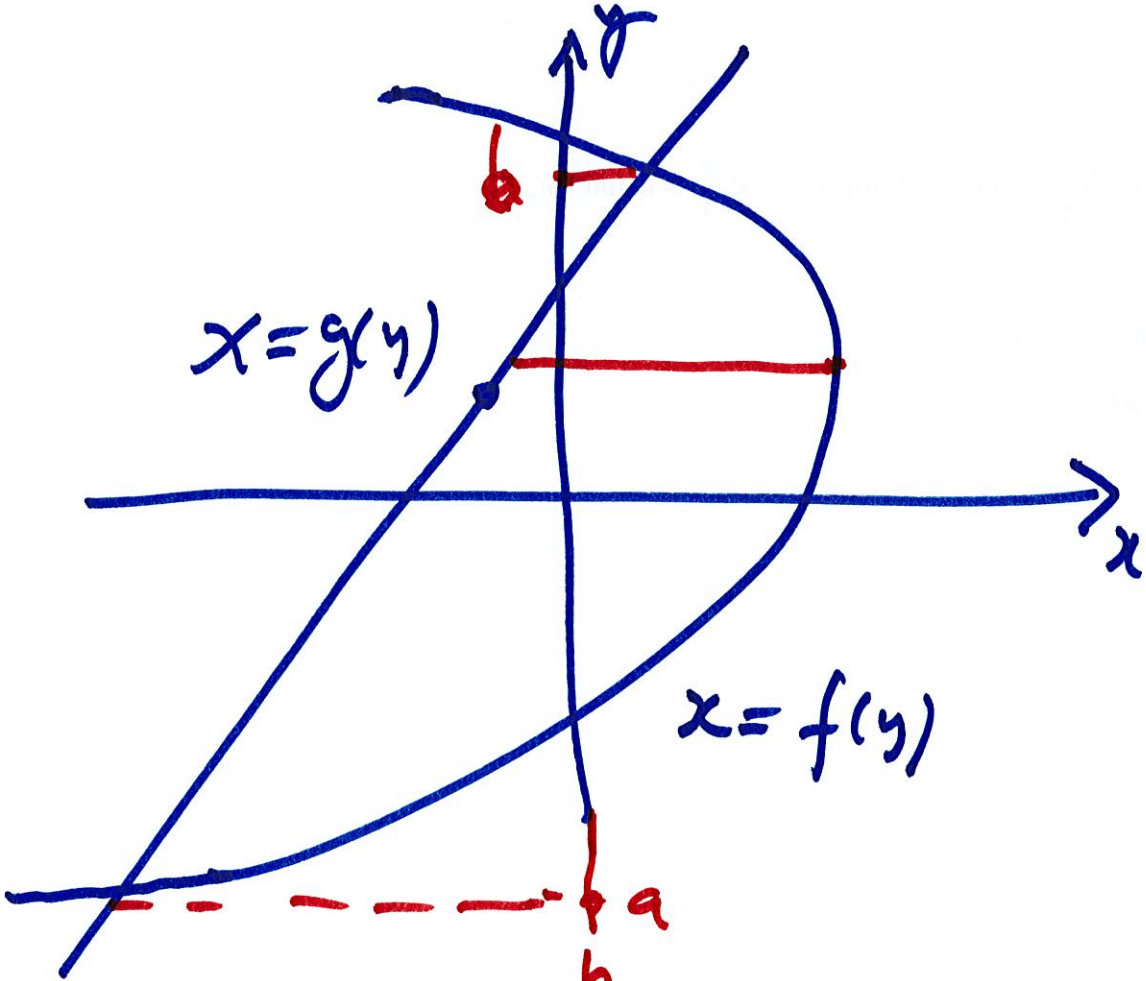
$1 - x^2 = x^2 - 4$

$5 = 2x^2; \quad x^2 = \pm\sqrt{5/2}$

$x^2 = 5/2, \quad x = \pm\sqrt{5/2}.$

$$A = \int_{-\sqrt{5/2}}^{\sqrt{5/2}} \left(1 - x^2 - (x^2 - 4) \right)$$

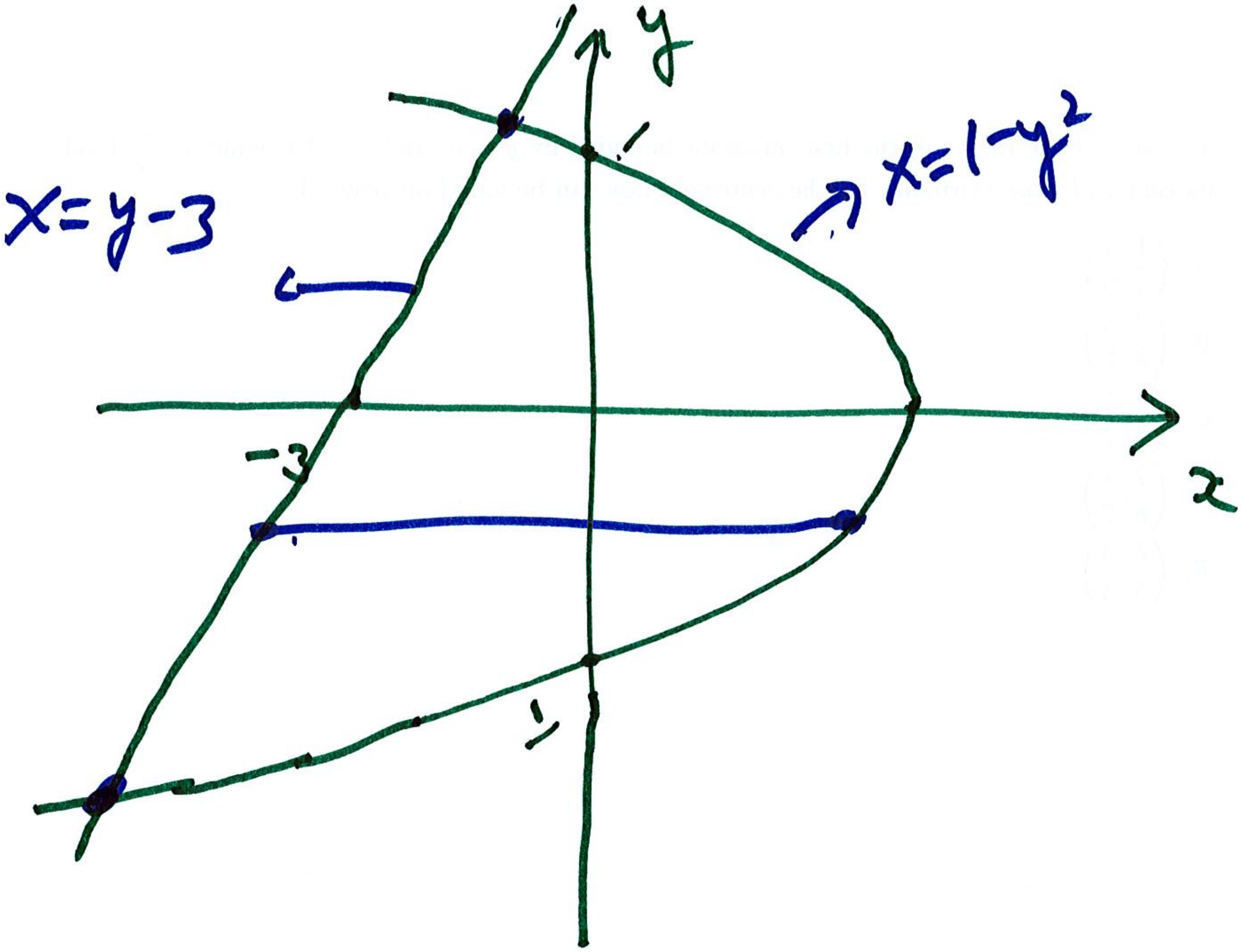
$$= \int_{-\sqrt{5/2}}^{\sqrt{5/2}} (5 - 2x^2)\, dx = 2\int_0^{\sqrt{5/2}} (5 - 2x^2)\, dx$$



$$A = \int_a^b \left(f(y) - g(y) \right) dy$$

<u>Example:</u> $x = 1 - y^2$

$y = x + 3$.



$$x = 1 - y^2 \;; \quad x = y - 3$$

$$1 - y^2 = y - 3 \qquad \cancel{y = 2} \quad \cancel{y = -1}$$

$$y^2 + y - 4 = 0$$

$$y = \frac{-1 \pm \sqrt{1 + 16}}{2} = \cancel{\frac{1 \pm 3}{2}}$$

$$y = \frac{-1+\sqrt{17}}{2}\ ; \quad y = \frac{-1-\sqrt{17}}{2}.$$

$$A = \int_{-\frac{1+\sqrt{17}}{2}}^{\frac{\sqrt{17}-1}{2}} \left(1 - y^2 - (y-3)\right) dy$$

$$= \int_{-\frac{1+\sqrt{17}}{2}}^{\frac{\sqrt{17}-1}{2}} \left(4 - y - y^2\right) dy$$

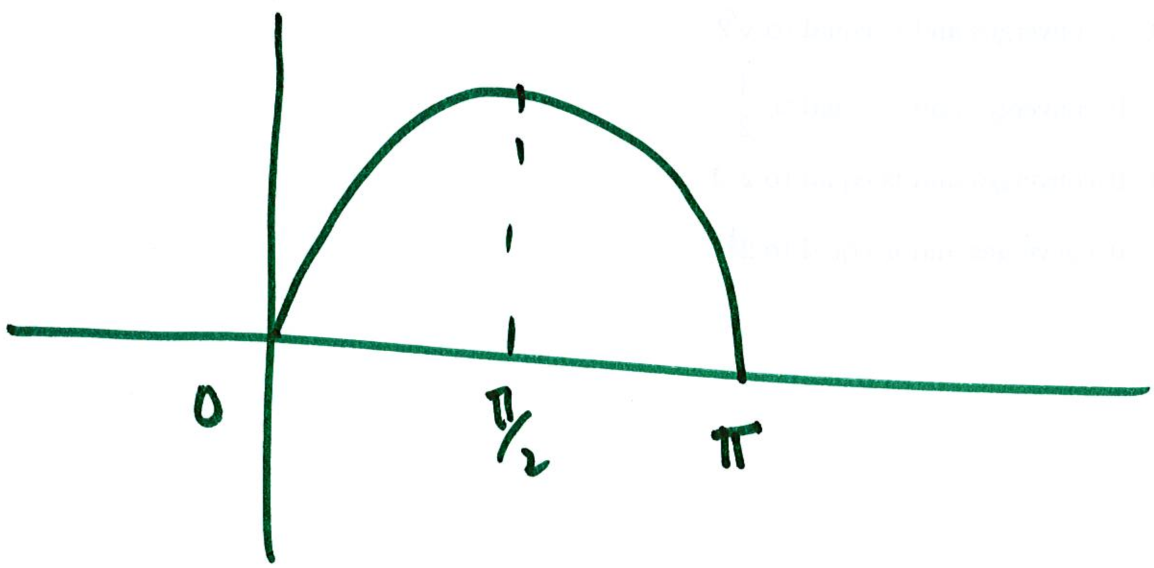
$$= 4y - \frac{y^2}{2} - \frac{y^3}{3} \Bigg|_{-\left(\frac{1+\sqrt{17}}{2}\right)}^{\frac{\sqrt{17}-1}{2}} = \ldots$$

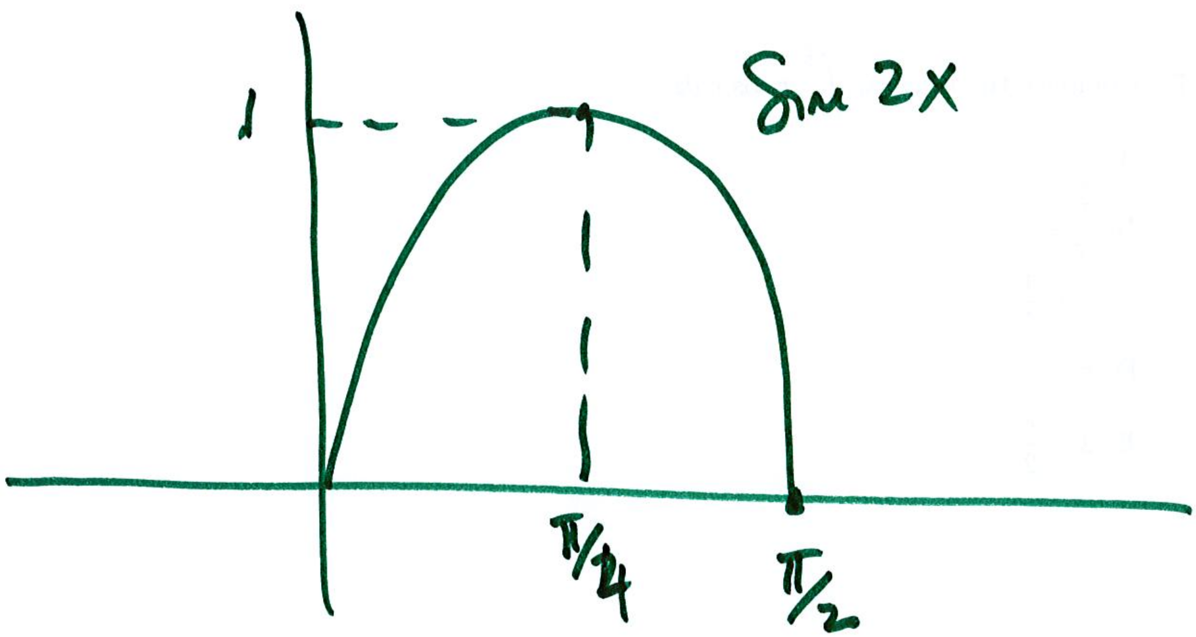
Find the area of the ~~area~~ region bounded by

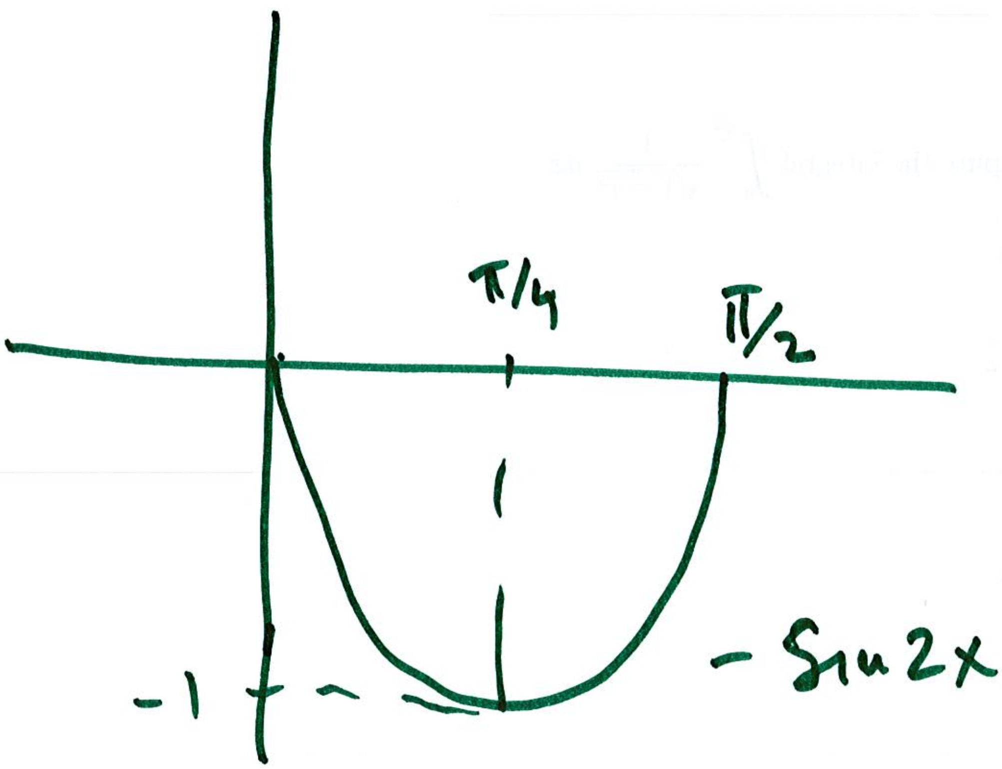
$$y = 2\sin 2x, \quad y = 2 - 2\sin 2x$$

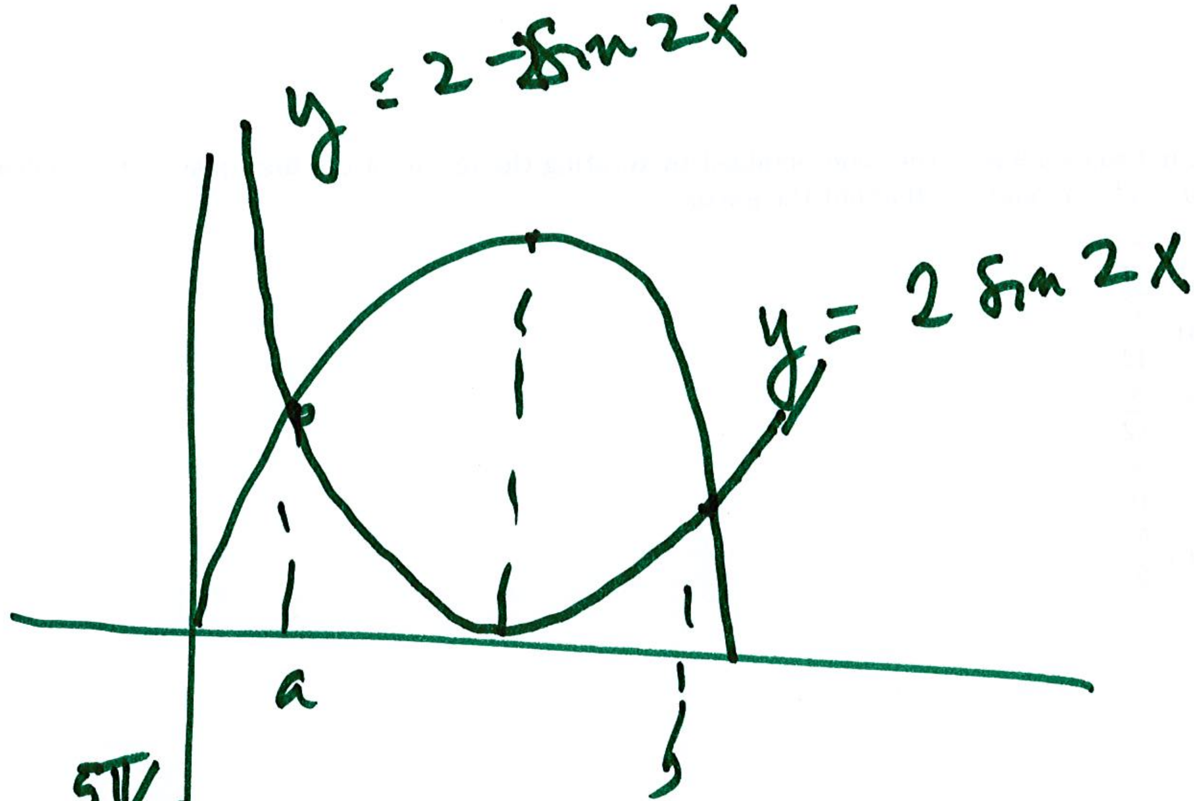
$$0 \le x \le \pi/2.$$

$y = \sin x$




Sin 2x
1
π/4
π/2


π/4
π/2
−1
− Sin 2x



$$\int_{\pi/12}^{5\pi/12} \left[(2 - 2\sin 2x) - 2\sin 2x \right] dx$$

$$2 - 2\sin 2x = 2\sin 2x$$

$$4\sin 2x = 2$$

$$\sin 2x = 1/2$$

$$2x = \pi/6 \; , \quad 2x = 5\pi/6 \, .$$