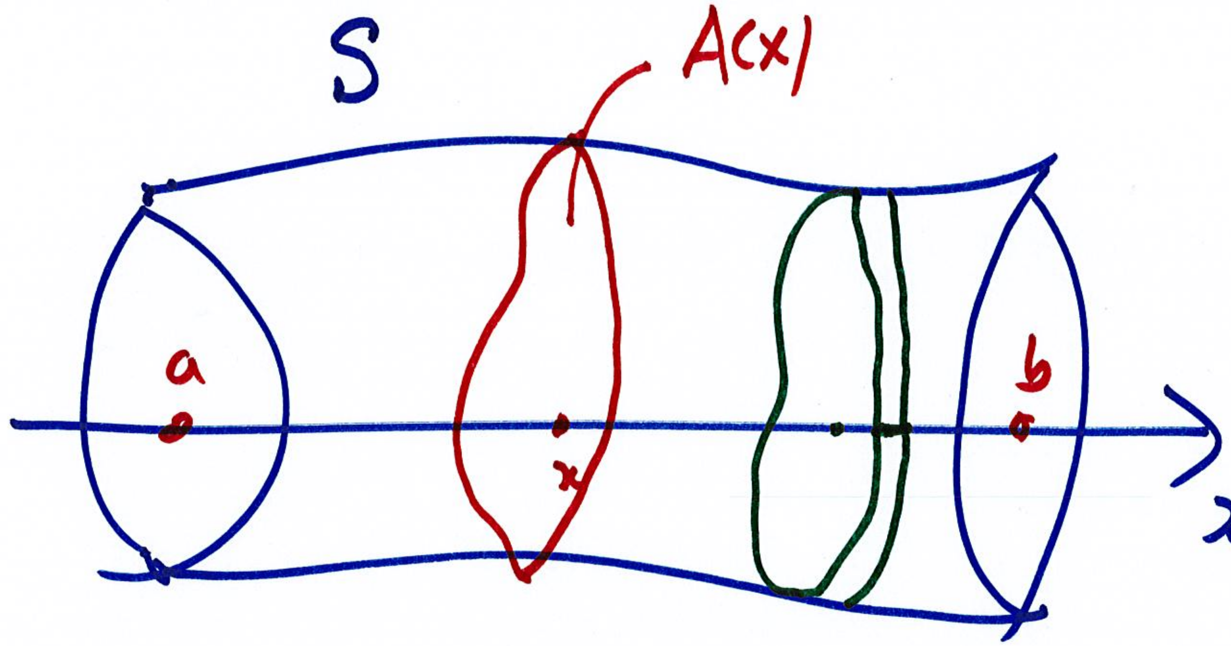
Lesson 6 Section 6.2

Volumes. (Application of Integrals)

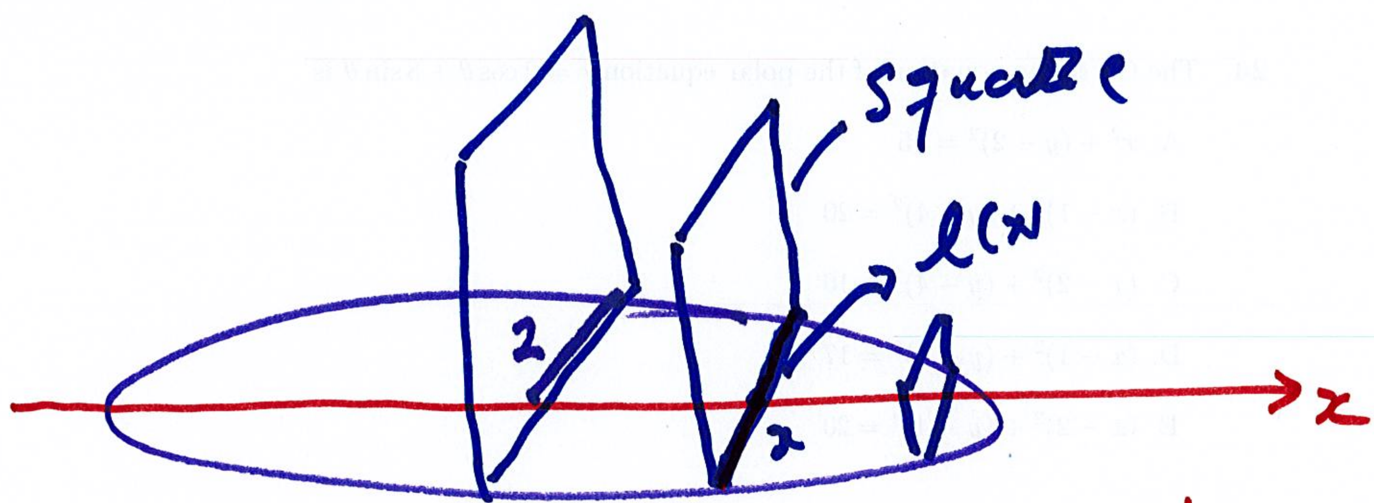


$A(x)$ = area of the slice.

$$dV = A(x)\,dx$$

$$\text{Volume} = \int_a^b A(x)\,dx$$

Example I.



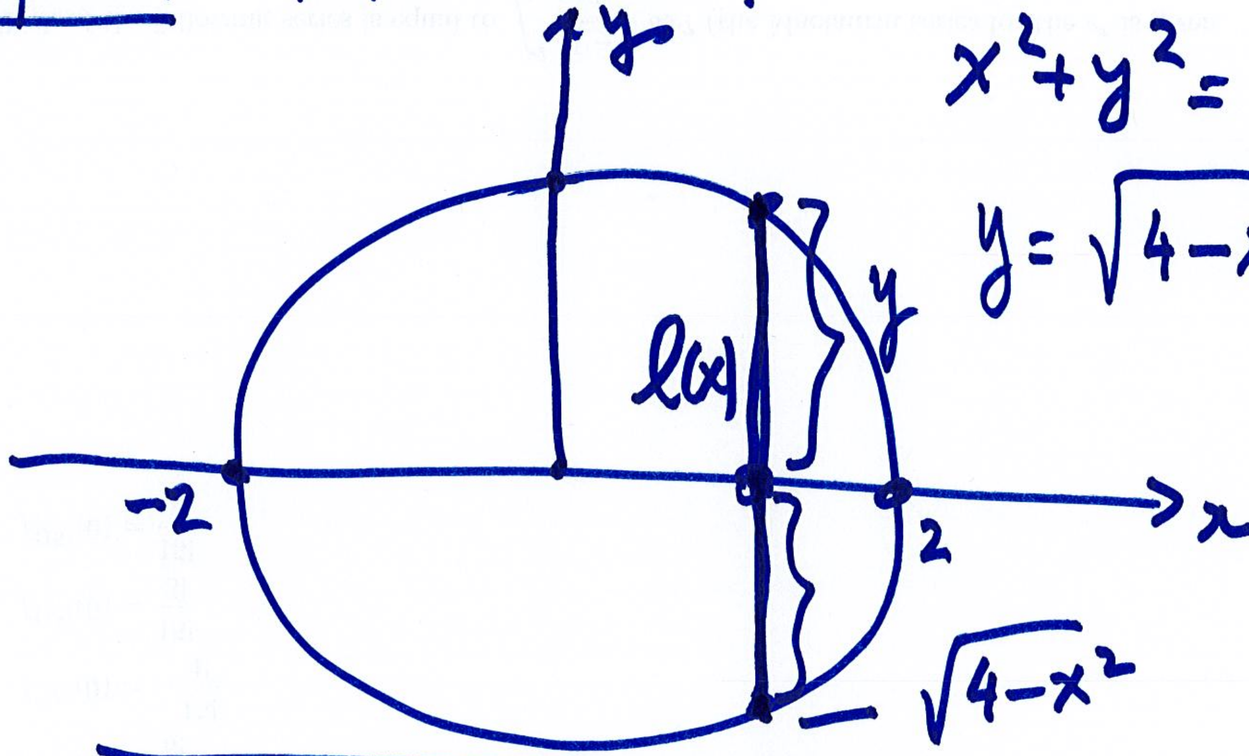
Base of the solid is a circle has radius 2.

Slices are squares.

Apply $V = \int_a^b A(x)\,dx$

Find $A(x)$, a and b.

<u>Step 1:</u> Find $A(x)$.



$x^2 + y^2 = 4$

$y = \sqrt{4-x^2}$

$$\boxed{A(x) = \left(l(x)\right)^2.}$$

$a = -2$, $b = 2$.

$$\ell(x) = 2\sqrt{4-x^2}.$$

$$A(x) = 4(4-x^2).$$

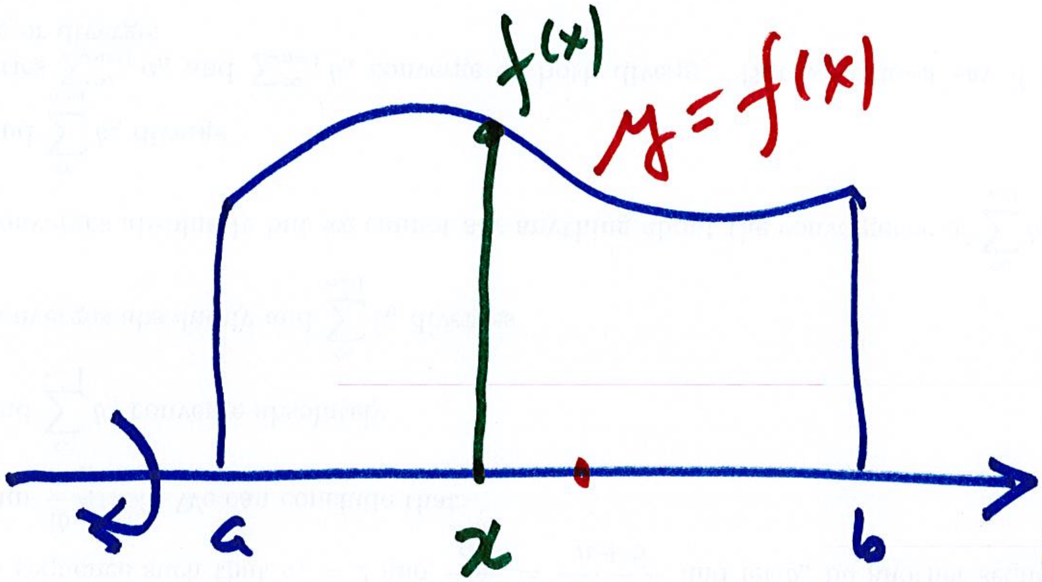
$$V = \int_{-2}^{2} 4(4-x^2)\,dx$$

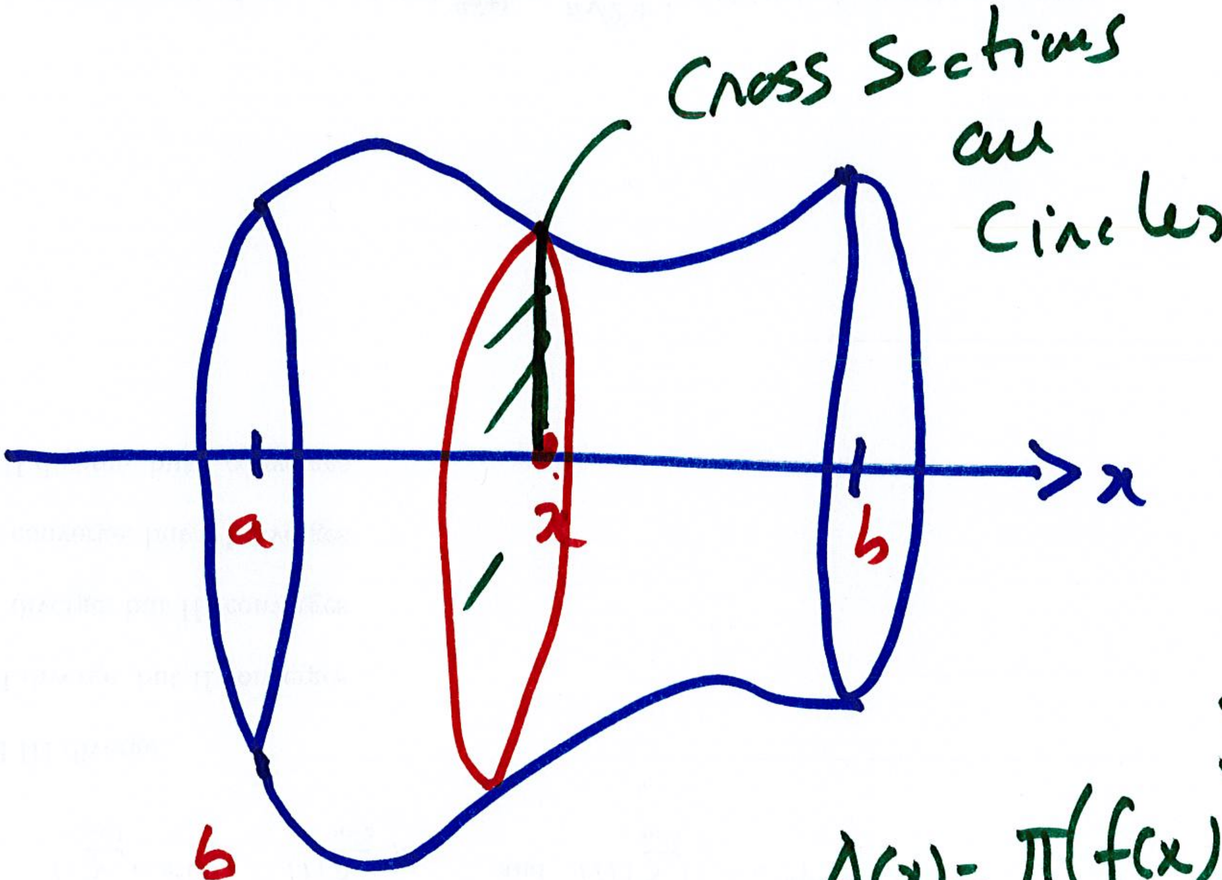
$$= 4 \cdot 2 \int_0^2 (4-x^2)\,dx$$

$$= 8\left(8 - 8/3\right) = 64\left(1 - 1/3\right)$$

$$= \frac{128}{3}.$$

Solids of Revolution



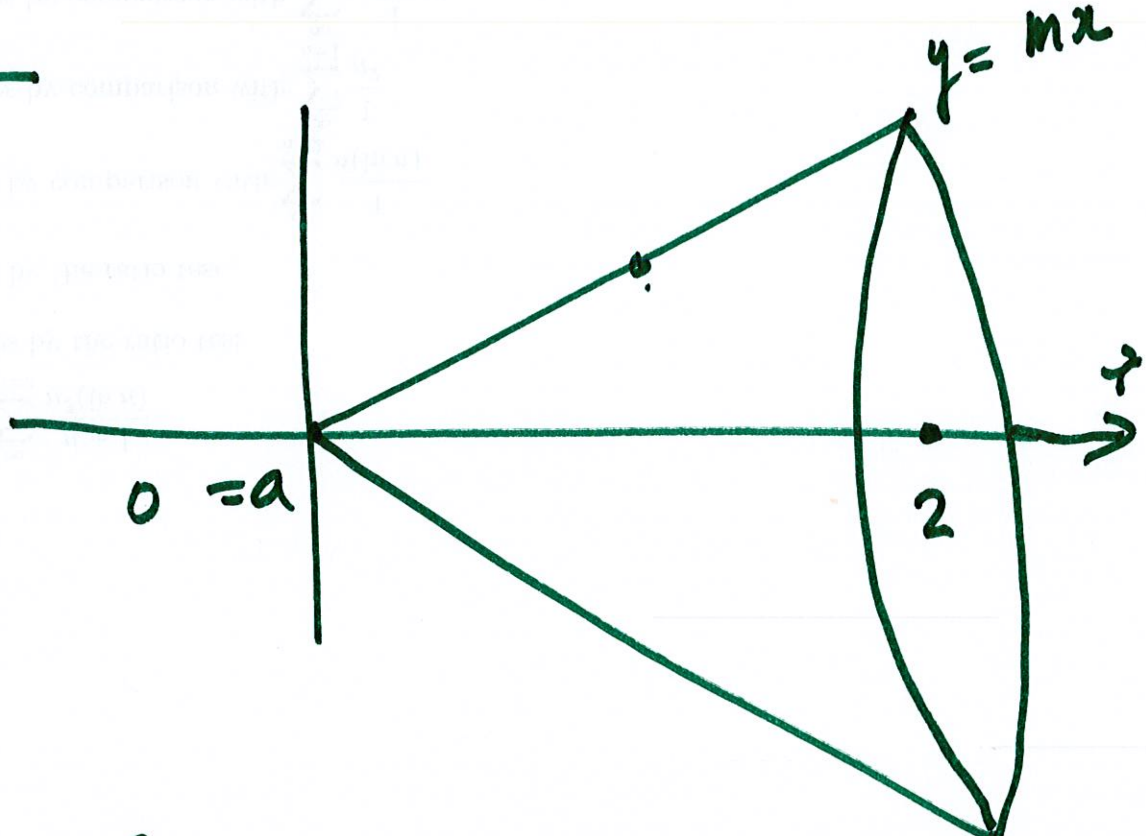


$$A(x) = \pi (f(x))^2$$

$$V = \int_a^b A(x)\, dx$$

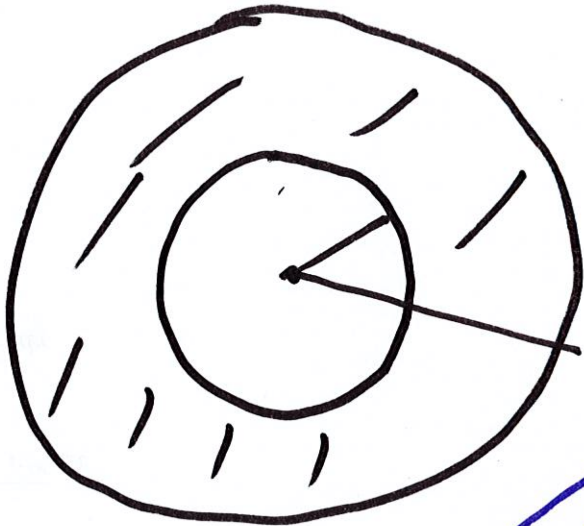
$$V = \pi \int_a^b (f(x))^2 \, dx$$

Example:



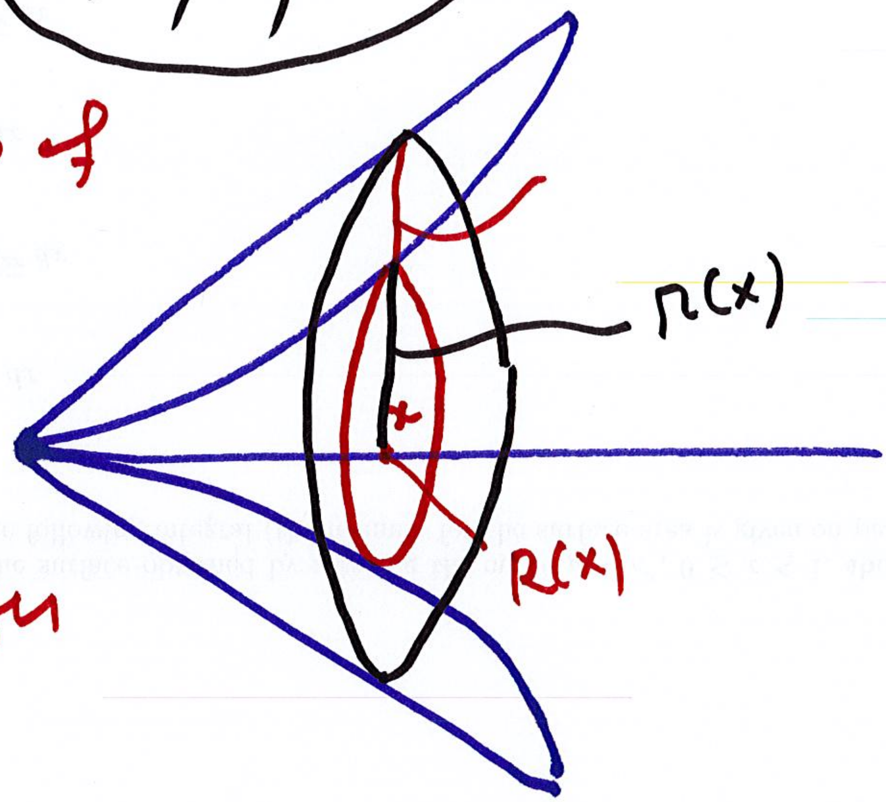
$$V = \int_0^2 \pi (mx)^2 = \pi m^2 \int_0^2 x^2 \, dx$$

$$= \frac{8\pi m^2}{3}.$$



$r(x)$ = radius of the smaller circle

$R(x)$ = radius of the larger one.



$A(x)$ = Area between these circles.

$$= \pi \left(R(x)^2 - r(x)^2 \right)$$

$x^2 = x$

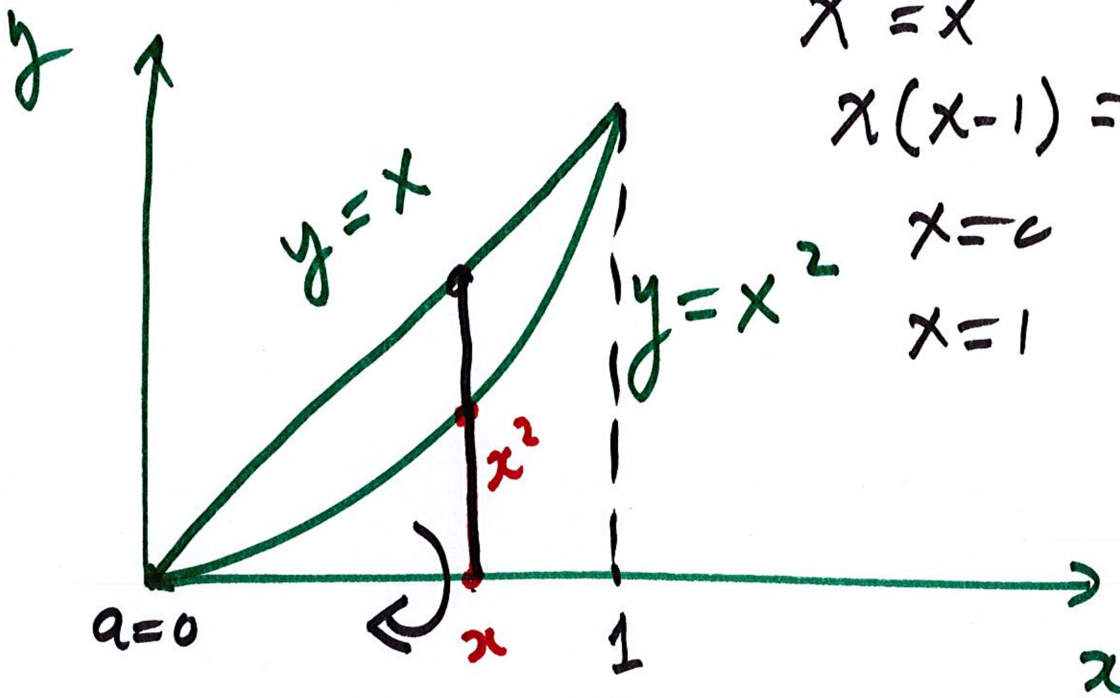
$x(x-1) = 0$

$x = 0$

$x = 1$

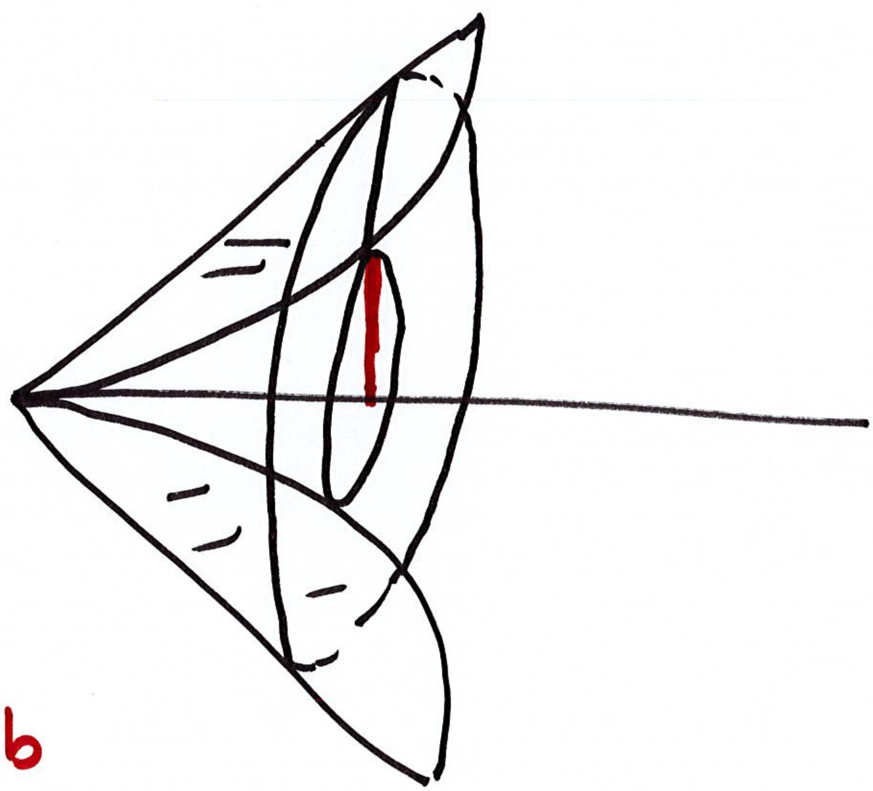
$r(x) = x^2$

$R(x) = x.$



$a = 0$

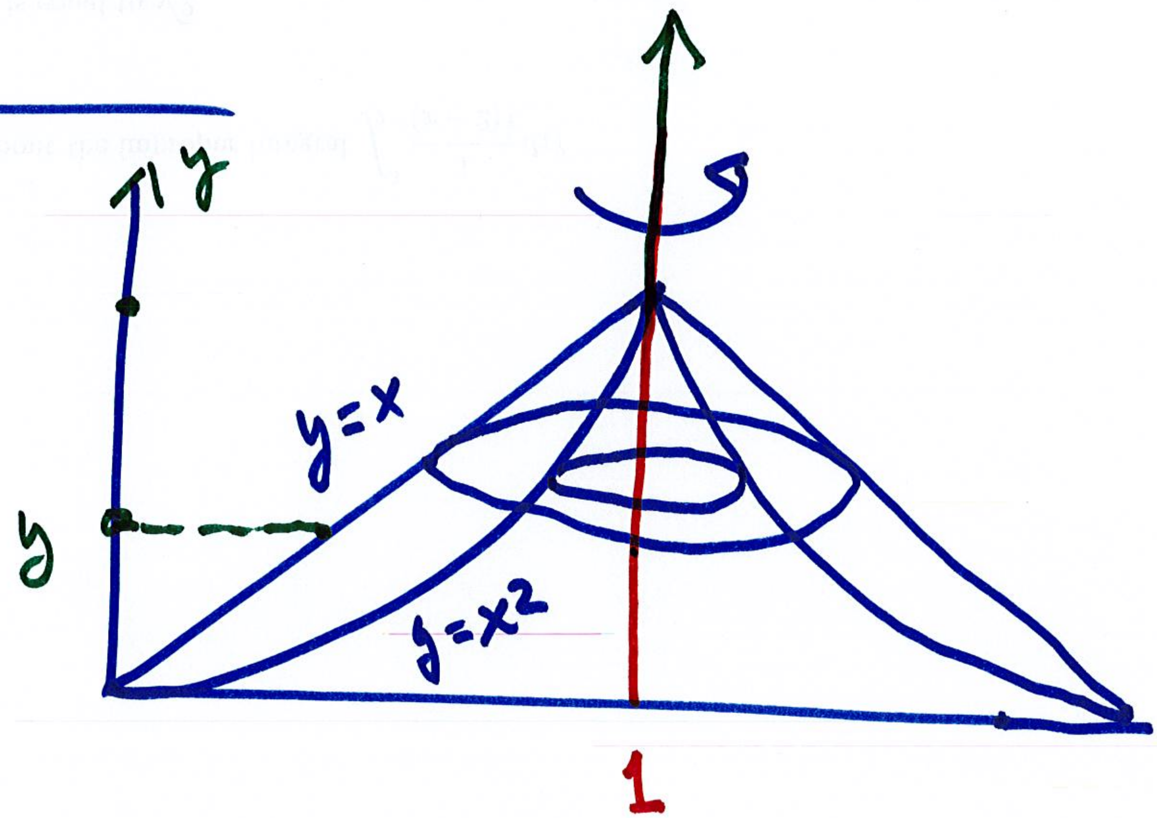
$b = 1$



$$V = \int_a^b A(x)\,dx$$

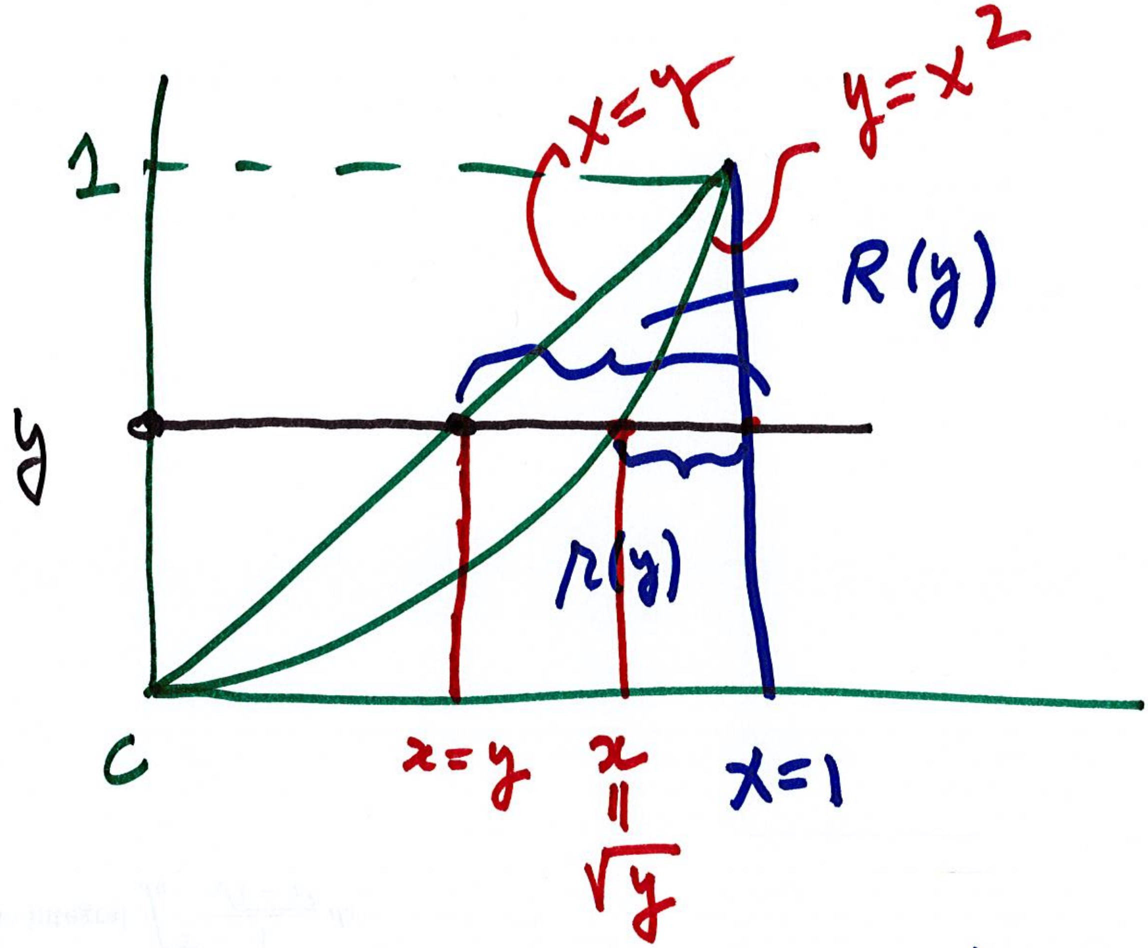
$$A = \pi \int_0^1 (x^2 - x^4)\, dx$$

$$= \pi \left(\frac{1}{3} - \frac{1}{5} \right) = \frac{2\pi}{15}.$$



$y = x^2$

$x = \sqrt{y}$



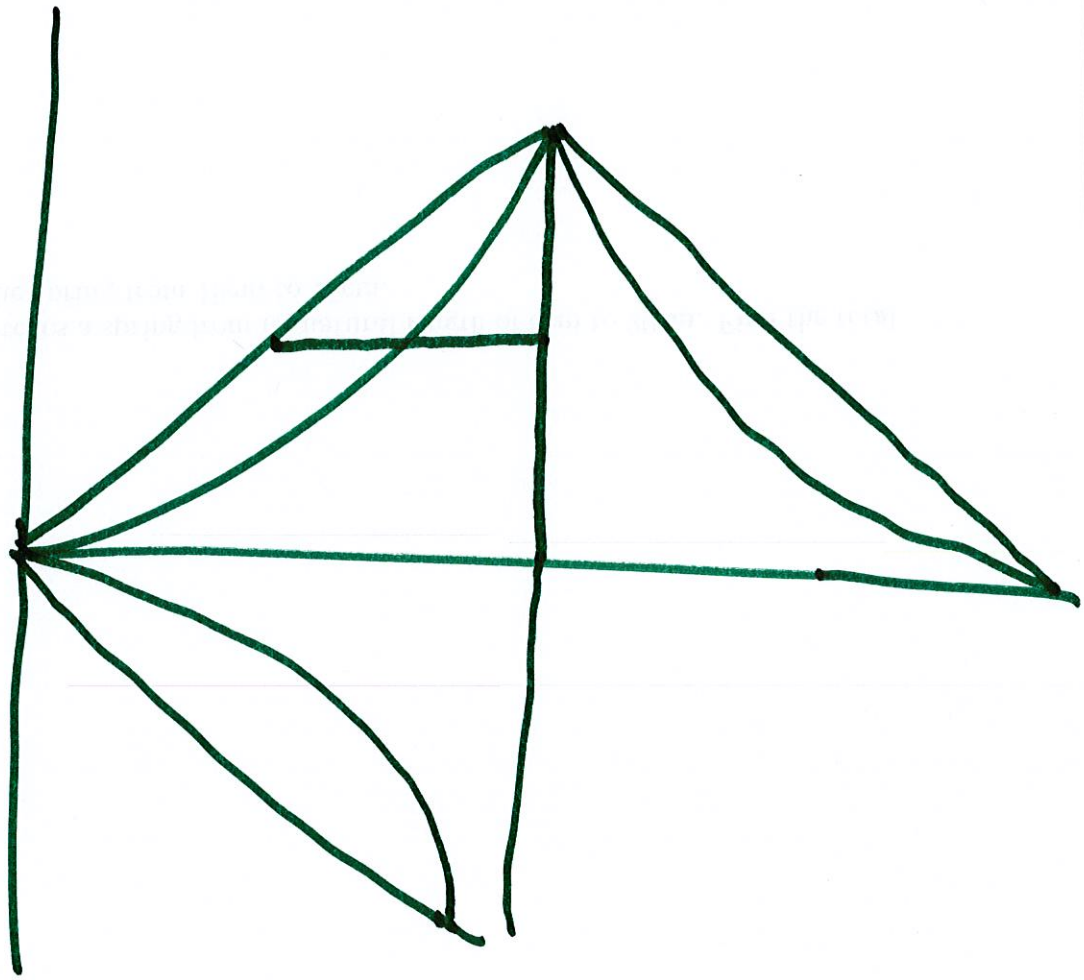
$$A(y) = \pi\left(R(y)^2 - r(y)^2 \right)$$

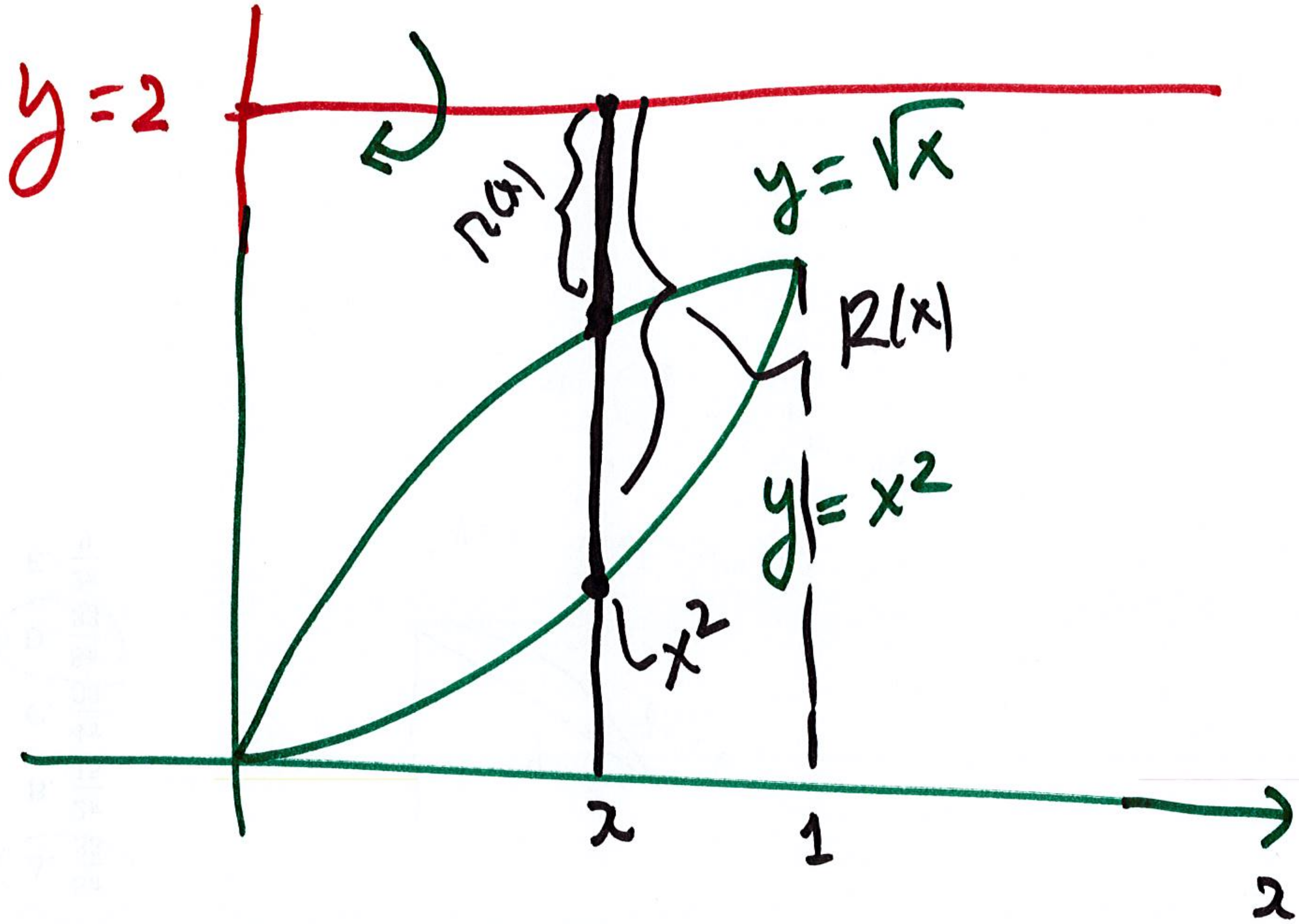
$$R(y) = 1 - y \quad , \quad r(y) = 1 - \sqrt{y}$$

$$A(y) = \pi\left((1-y)^2 - (1-\sqrt{y})^2 \right)$$

$$= \pi\left(\cancel{1} - 2y + y^2 - \cancel{1} + 2\sqrt{y} - y \right)$$

$$V = \pi \int_0^1 \left(y^2 + 2\sqrt{y} - 3y \right) dy = \ldots$$





$$A(x) = \pi\left(R(x)^2 - r(x)^2\right)$$

$$R(x) = 2 - x^2 \quad ; \quad r(x) = 2 - \sqrt{x}$$

$$A(x) = \pi\left(\left(2 - x^2\right)^2 - \left(2 - \sqrt{x}\right)^2\right).$$

$$V = \pi \int_0^1 \left((2-x^2)^2 - (2-\sqrt{x})^2 \right) dx.$$