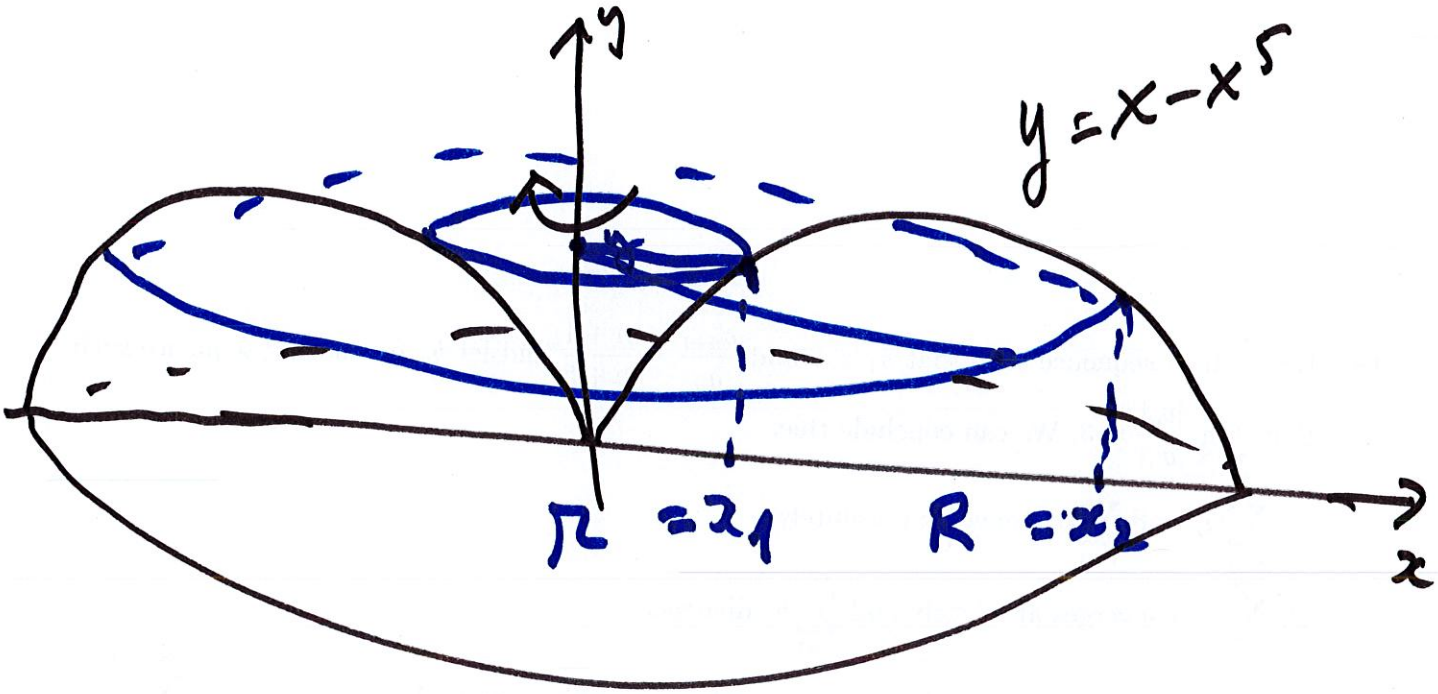
MA 166 - LESSON 7

Volumes by Cylindrical Shells

Example :



$y = x - x^5$, Solve for x.

The washers method does not work here

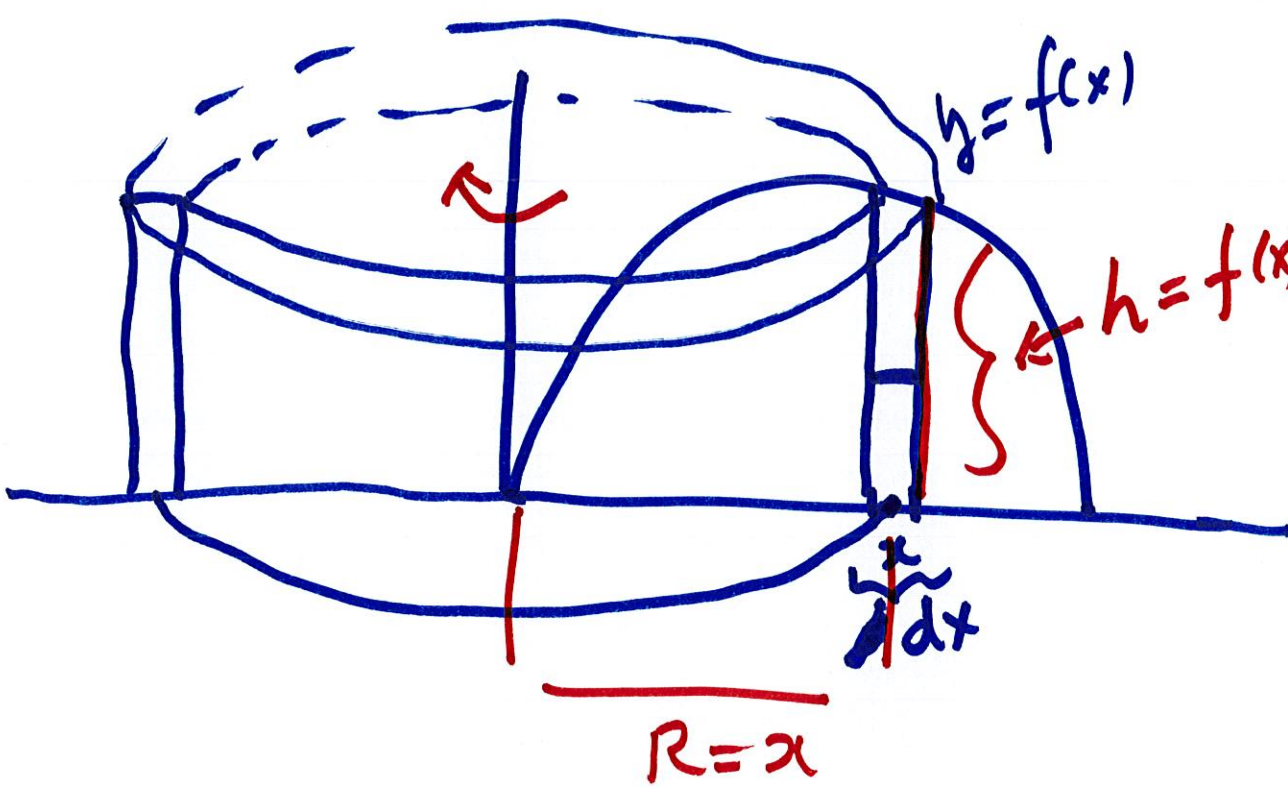
because I cannot solve

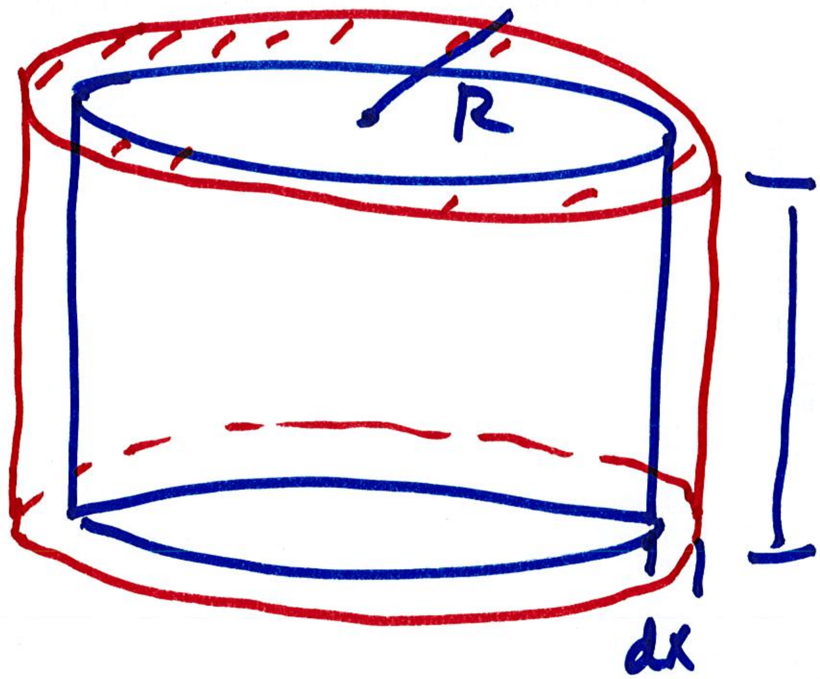
$$y = x - x^2$$

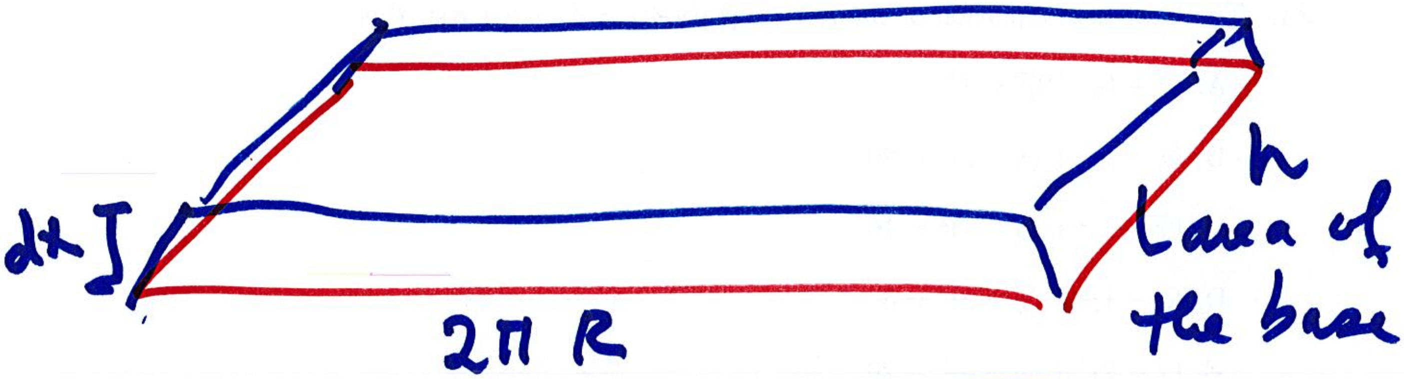
~~in~~ as $x =$ a function

of y.

Alternative for computing Volumes. - Shells Method.







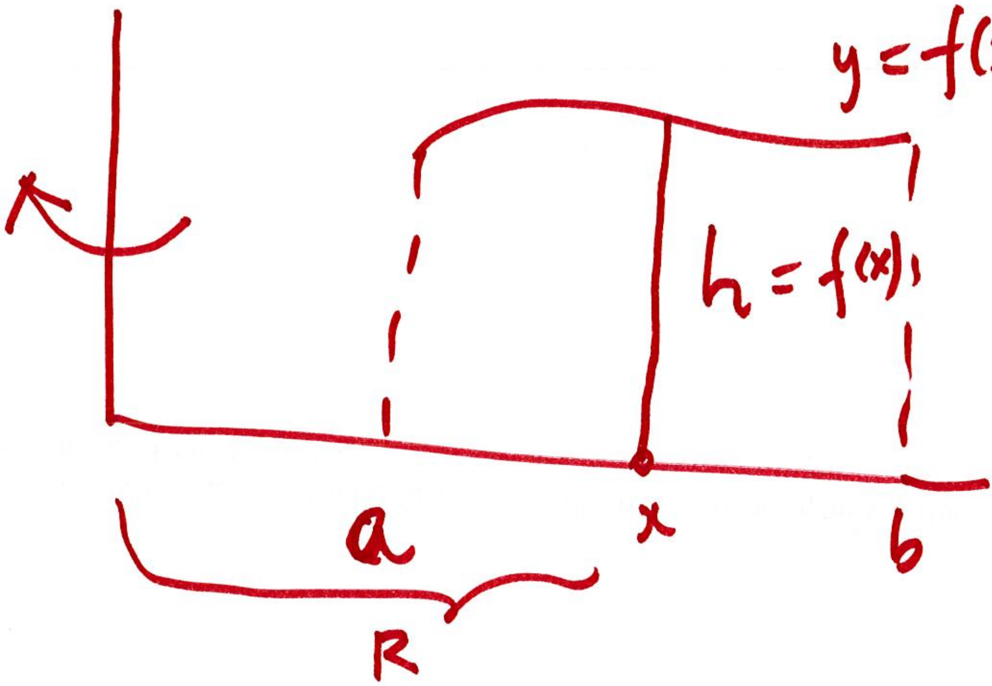
$$\boxed{dV \cong 2\pi \underset{\substack{\uparrow \\ x}}{R}\ \underset{\substack{\uparrow \\ f(x)}}{h}\ dx}$$

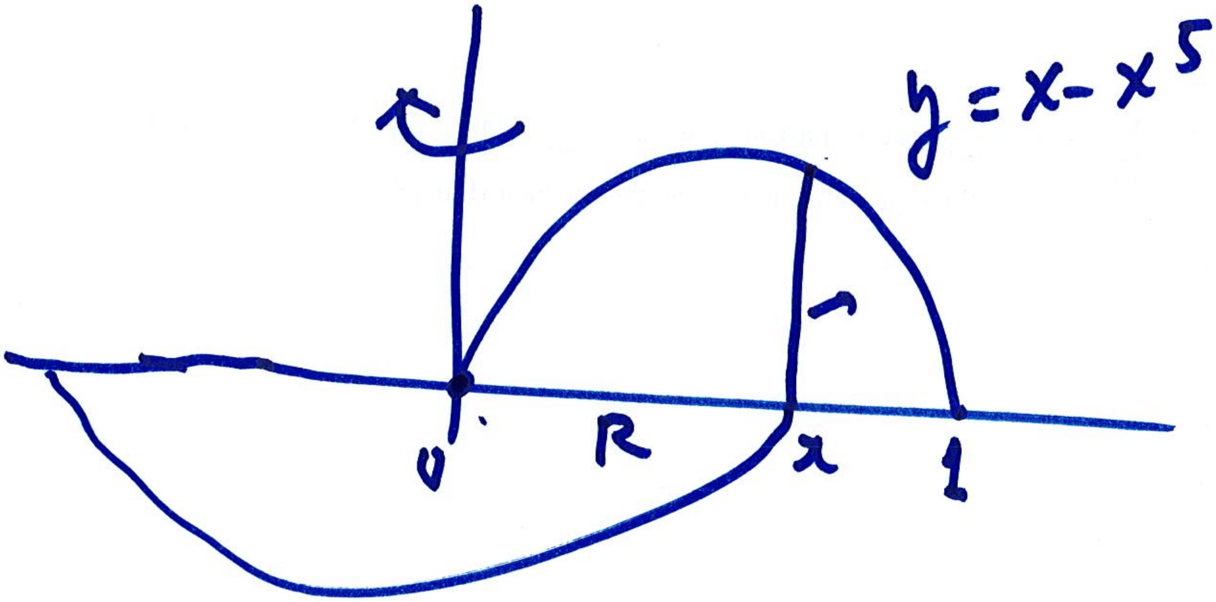
Volume of the shell

$$dV = 2\pi x f(x)\,dx$$

Volume of the Solid

$$V = \int_a^b 2\pi x f(x)\,dx.$$





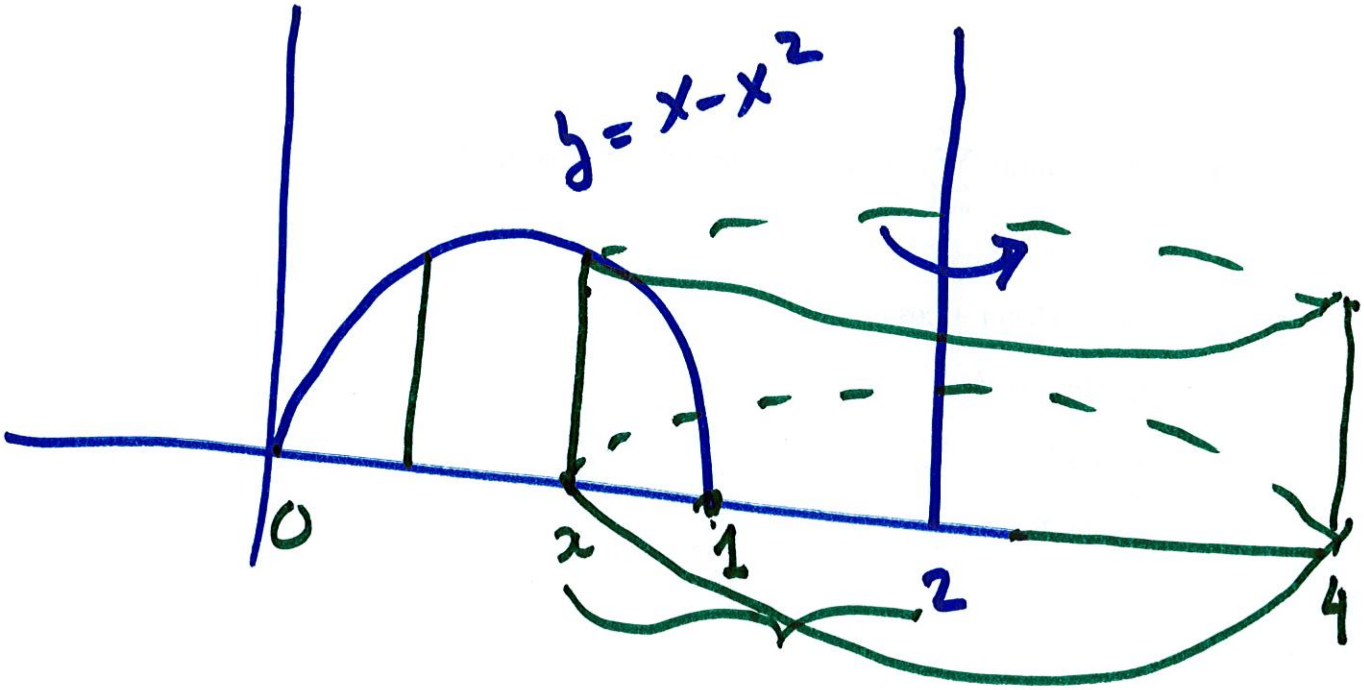
$$R = x \ , \ h = f(x) = x - x^5$$

$$V = 2\pi \int_0^1 x\,(x - x^5)\,dx$$

$$= 2\pi \int_0^1 (x^2 - x^6)\,dx = \frac{x^3}{3} - \frac{x^7}{7}\Big|_0^1$$

$$= 2\pi\left(\frac{1}{3} - \frac{1}{7}\right) = \frac{4 \times 2\pi}{21}$$

$$= \frac{8\pi}{21}.$$



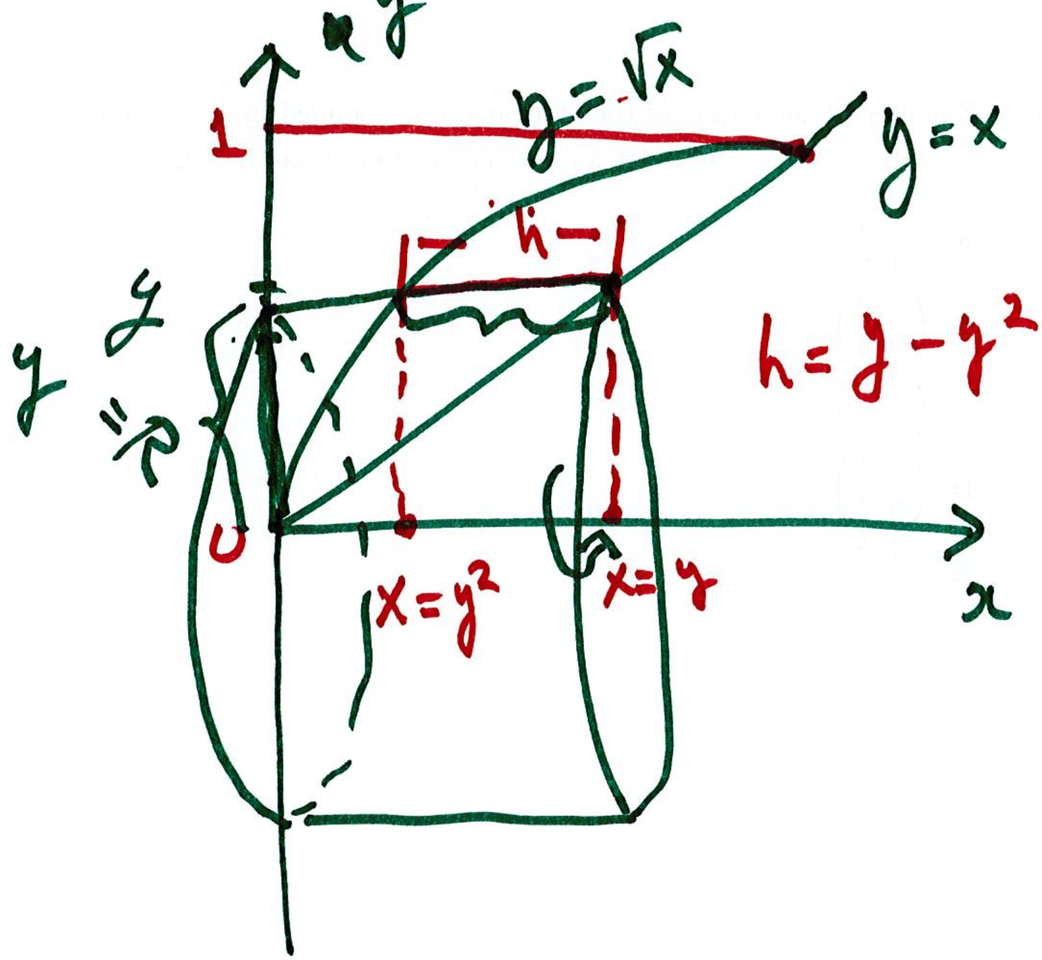
Radius = $2-x$

$h = (x-x^2)$

$$V = 2\pi \int_0^1 (2-x)\cdot(x-x^2)\, dx$$

$$= 2\pi \int_0^1 (2x - 2x^2 - x^2 + x^3)\, dx$$

$$= 2\pi \int_0^1 (2x + x^3 - 3x^2)\, dx$$

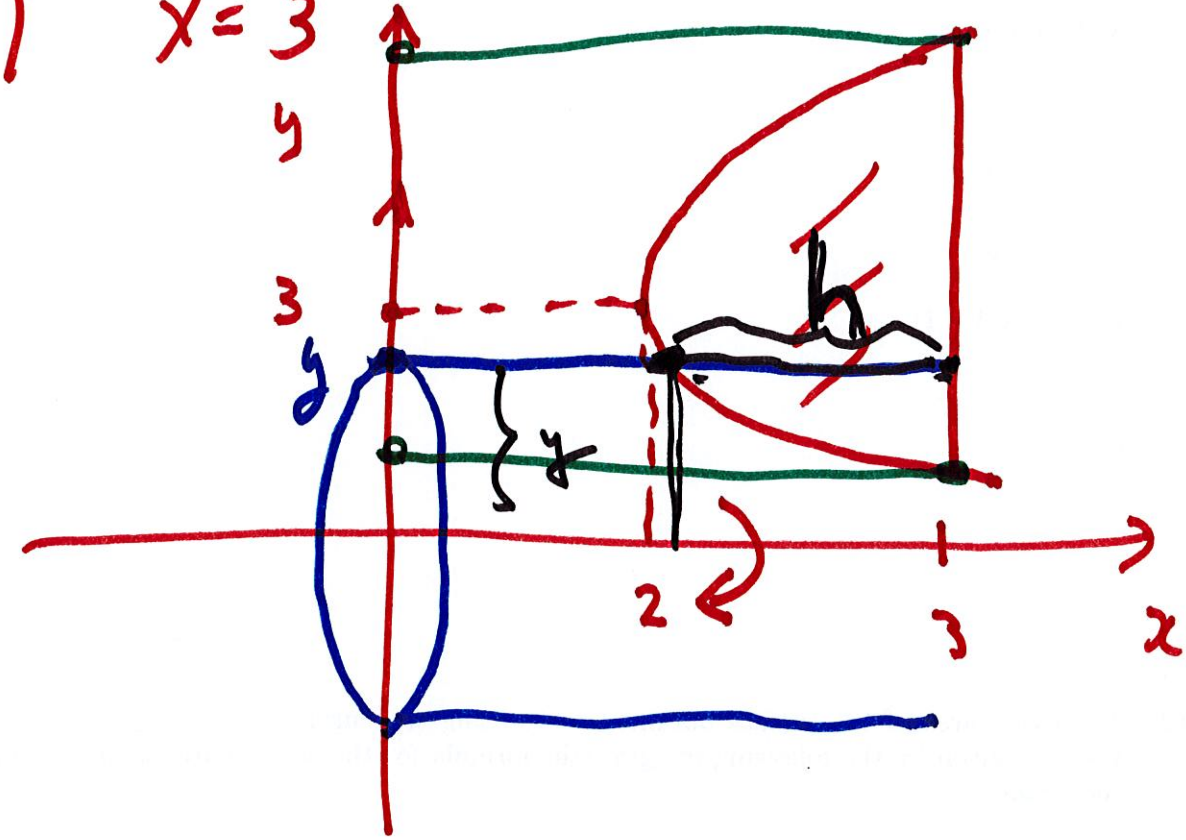


$$dv = 2\pi \underline{\underline{R}} \cdot h \, dy$$

$$dv = 2\pi \, y(y-y^2)\, dy$$

$$V = 2\cdot\pi \int_0^1 y(y-y^2)\, dy.$$

$$\begin{cases} x = \underline{2 + (y-3)^2} \\ x = 3 \end{cases}$$



$$R = y; \quad h = 3 - \left(2 + (y-3)^2\right)$$
$$= 1 - (y-3)^2.$$

$$V = 2\pi \int_2^4 y \left(1 - (y-3)^2\right) dy.$$

$$2 + (y-3)^2 = 3$$

$$(y-3)^2 = 1$$

$y - 3 = 1$ or $y - 3 = -1$

$y = 4$ or $y = 2$.

Region bounded by

$y = x^2$ and

$y = 6x - 2x^2$

$6x - 2x^2 = x^2$

$6x = 3x^2$

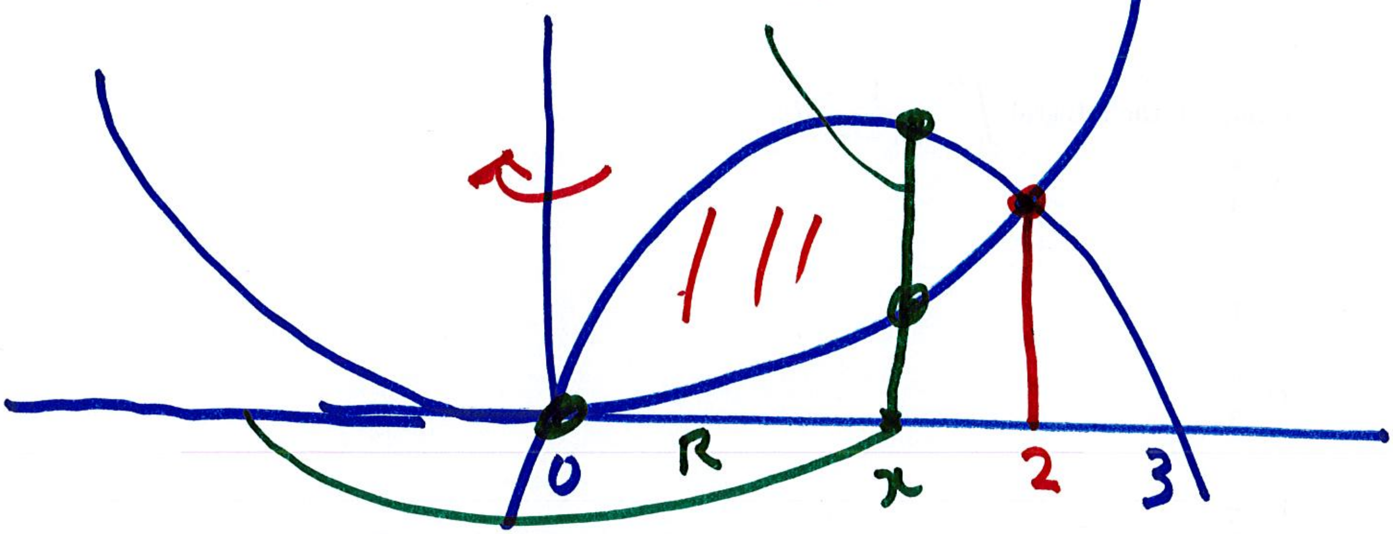
$x = 0$ $\quad 6 = 3x$

$x = 2$

Rotate about the y-axis.

$h = 6x - 2x^2 - x^2$

$= 6x - 3x^2$



$6x - 2x^2 = 0 \qquad x(6 - 2x) = 0$

$x = 0, \quad x = 3$

$$V = 2\pi \int_0^2 x\left(6x - 3x^2\right)dx$$