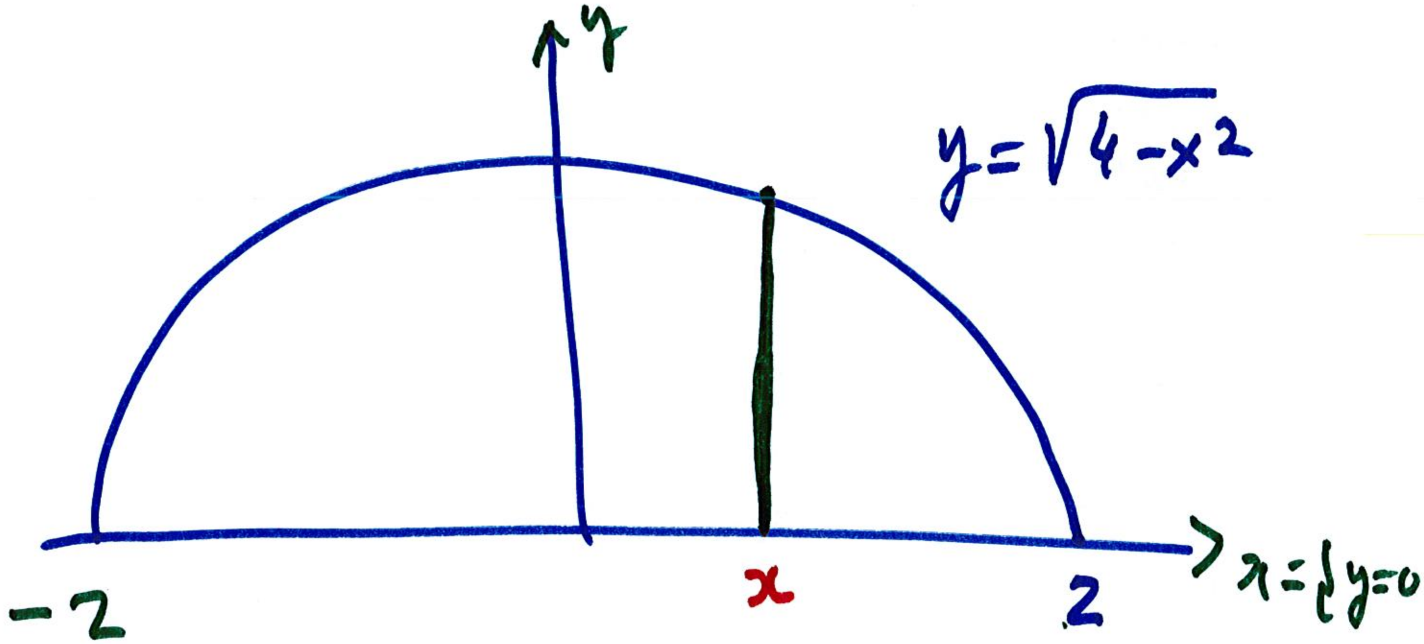
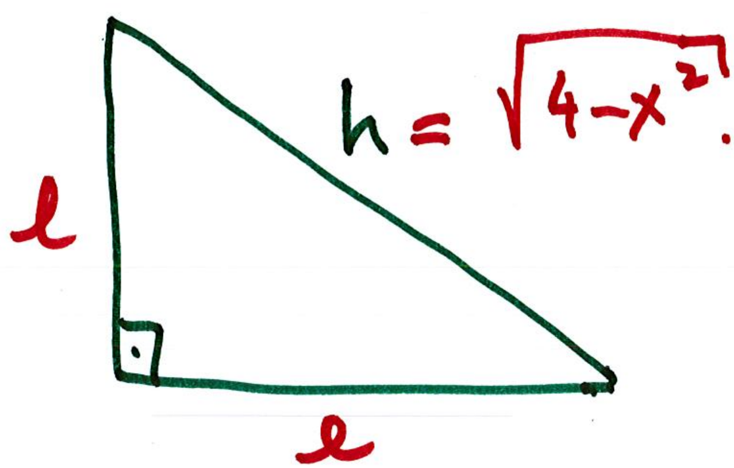
Review For Exam I

#9, Spring 18





$$h^2 = l^2 + l^2 = 2l^2$$

$$h = l\sqrt{2} \qquad \boxed{l = \frac{h}{\sqrt{2}}}$$

Area of the triangle

$$A = \frac{1}{2} l^2 \quad , \quad l = \frac{1}{\sqrt{2}} \sqrt{4-x^2}$$

$$l^2 = \frac{1}{2}(4-x^2)$$

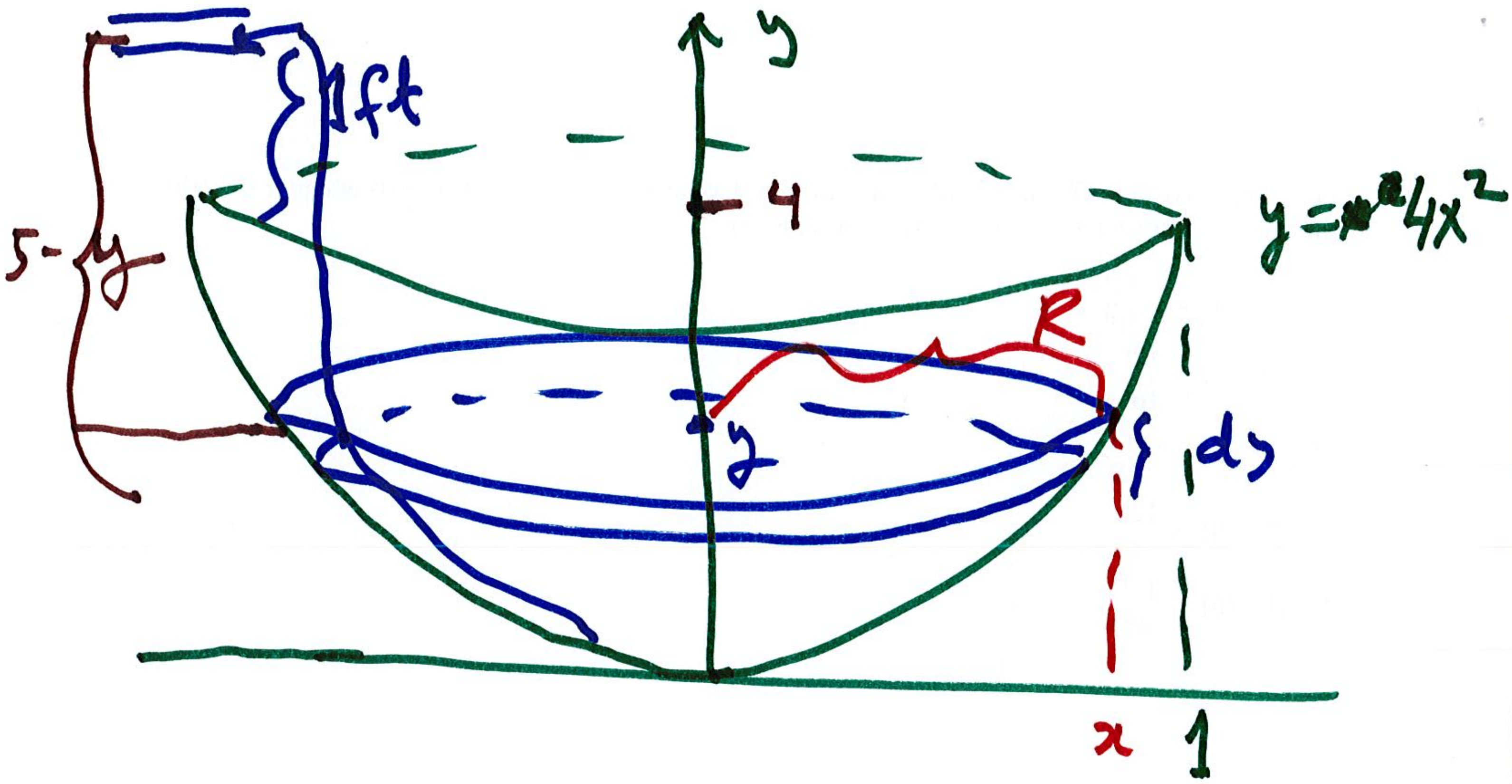
$$A(x) = \frac{1}{4}(4-x^2)$$

$$V = \int_{-2}^{2} A(x)\,dx = \frac{1}{4}\int_{-2}^{2}(4-x^2)\,dx$$

$$= \frac{1}{4}\cdot 2\int_0^2 (4-x^2)\,dx$$

$$= \frac{1}{2}\left(4x - \frac{x^3}{3}\right)\Big|_0^2 = \frac{1}{2}\left(8 - \frac{8}{3}\right)$$

$$= \frac{1}{2}8\left(1-\frac{1}{3}\right) = \frac{8}{3}$$



<u>Step 1</u>: Find the weight of the slab

$$= \underbrace{\rho}_{\text{density}} \cdot \underline{\text{Area of cross section}} \cdot dy$$

$y = 4x^2$

$x^2 = y/4$

$x = \sqrt{y/4}$

Area $= \pi \cdot R^2$

$$\boxed{\text{Area} = \pi \cdot \left(y/4\right).}$$

Weight of the slab:

$$= \pi \rho / 4 \; y \, dy$$

Step 2: Find the work necessary to move the slab.

$$dW = \frac{\pi}{4} \rho \; y \, dy \cdot \text{distance}.$$

$$dW = \pi/4 \, \rho \; y \, (5-y) \, dy.$$

$$\text{Work} = \pi/4 \, \rho \int_0^4 y(5-y) \, dy$$

$$= \pi/4 \, \rho \cdot \int_0^4 (5y - y^2) \, dy =$$

$$= \pi \rho/4 \cdot \left(5y^2/2 - y^3/3 \right)_0^4$$

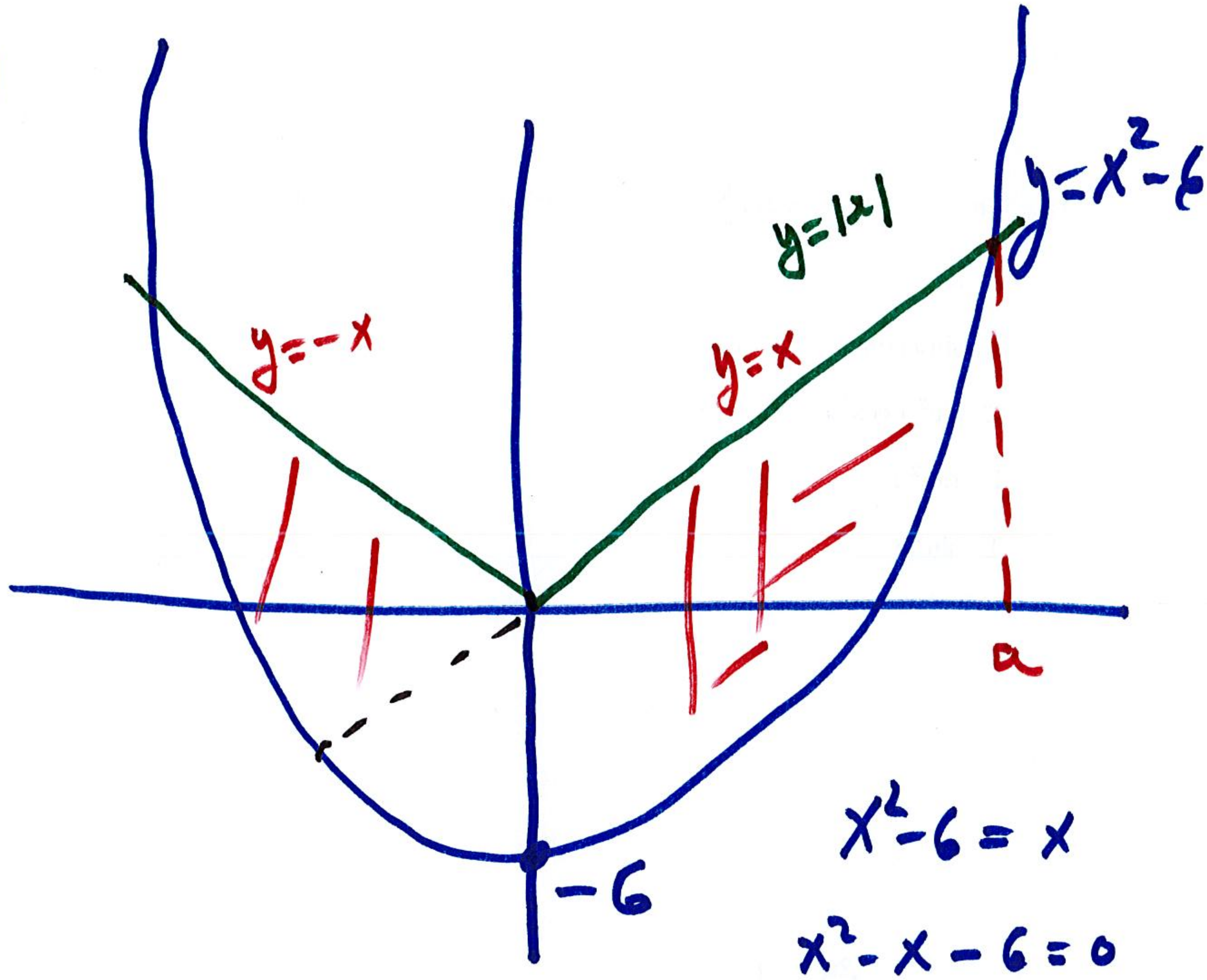
$$= \pi \rho/4 \left(\frac{80}{2} - \frac{64}{3} \right)$$

$$= \pi \rho/4 \left(40 - \frac{64}{3} \right)$$

$$= \frac{\pi}{4} \rho \left(\frac{120 - 64}{3} \right)$$

$$= \frac{\pi}{4} \rho \cdot \frac{56}{3} = \frac{14 \rho \pi}{3}.$$

#6)



$$x^2 - 6 = x$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = -2$$

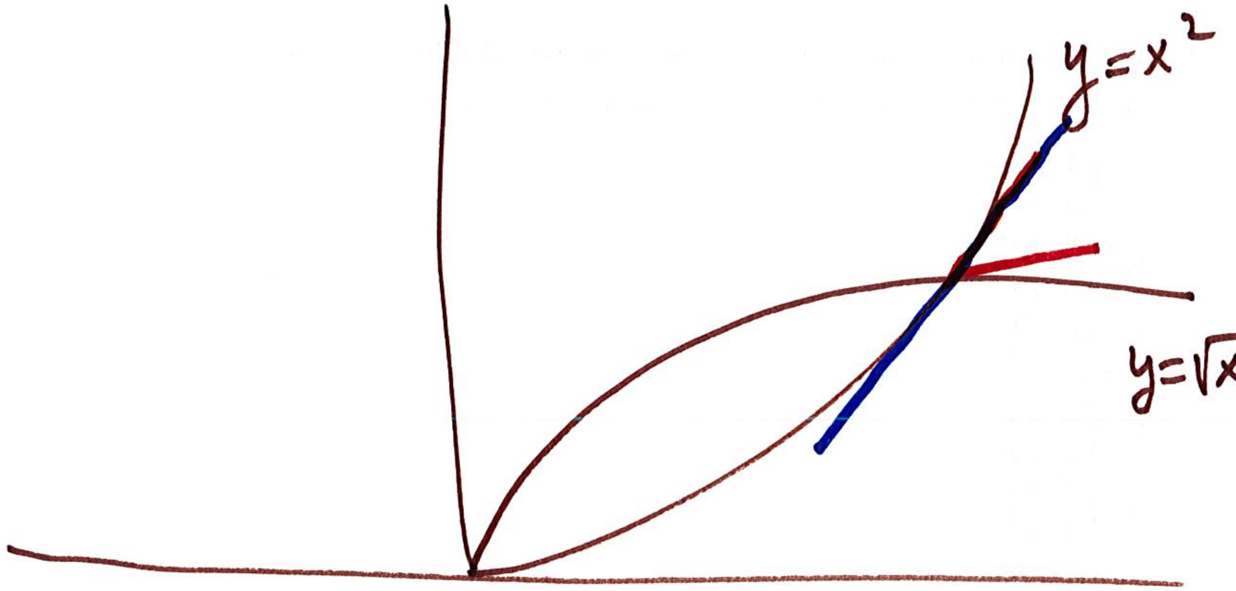
$$\boxed{x = 3}$$

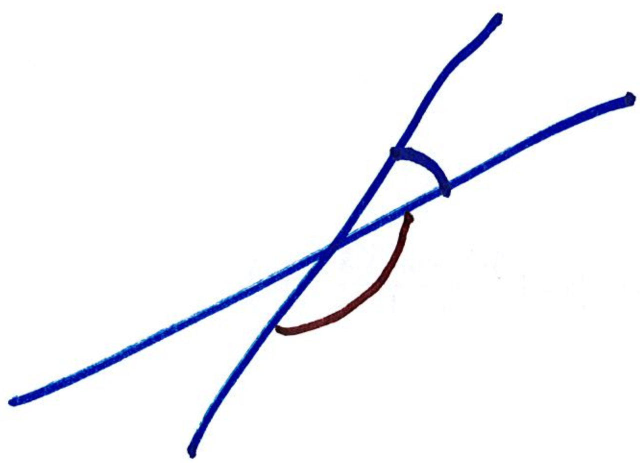
$$A = 2\int_0^a \left[x - (x^2 - 6)\right] dx$$

$$= 2\int_0^3 (x - x^2 + 6)\, dx$$

$$= 2\left(\frac{x^2}{2} - \frac{x^3}{3} + 6x\right)\Big|_0^3 = 2\left(\frac{9}{2} - 9 + 18\right)$$

$$= 2\left(18 - \frac{9}{2}\right) = \underline{27}$$





Solution: $y = x^2$, $y' = 2x$,

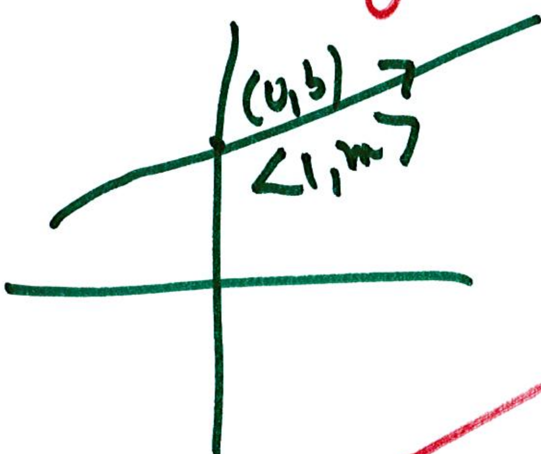
At $x=1$, $y' = 2$.

$$y - 1 = 2(x-1)$$

$$\boxed{y = 2x - 1}$$

Equation of a line using vectors

$$y = mx + b$$



$$(x, y) = (x, mx+b)$$

$$= (x, mx) + (0, b)$$

$$= x(1, m) + \underline{(0, b)}$$

$$y = \sqrt{x}, \quad y' = \frac{1}{2} x^{-1/2}$$

$$= x^{1/2}$$

$$y' = 1/2$$

$$y - 1 = \frac{1}{2}(x-1)$$

$$\boxed{y = x/2 + 1/2}$$

Equation of the line:

$$y = x/2 + 1/2 \qquad (x, y) = (x, x/2 + 1/2)$$

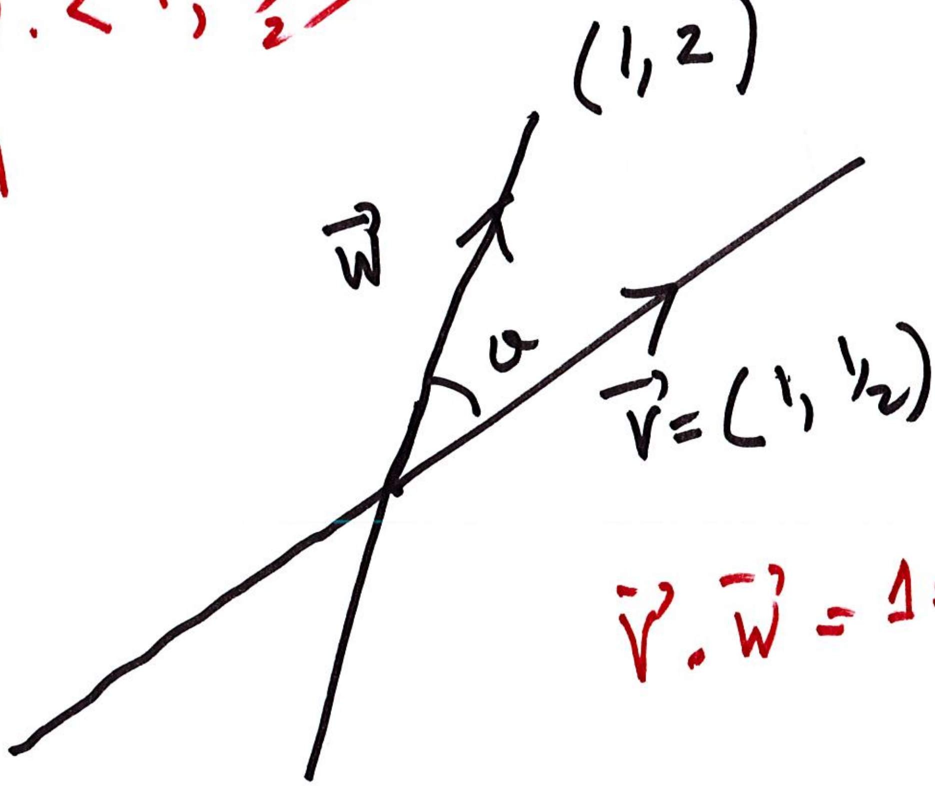
$$= (x, x/2) + (0, 1/2)$$

$$= x(1, 1/2) + (0, 1/2).$$

$$y = 2x - 1 \qquad (x, 2x-1) = x(1, 2) + (0, -1)$$

$\langle 1, 2\rangle \cdot \langle 1, \frac{1}{2}\rangle$

$= 1 + 1$



$\vec{v} \cdot \vec{w} = 1 + 1 = 2$

$$\vec{v} \cdot \vec{w} = |\vec{v}| \cdot |\vec{w}| \cdot \cos\theta$$

$$\cos\theta = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|}$$

$$\cos\theta = \frac{2}{\sqrt{1+4} \cdot \sqrt{1+1/4}} = \frac{2}{\sqrt{5} \cdot \frac{\sqrt{5}}{2}}$$

$$= \frac{4}{5}.$$