MA 261 — SPRING 2018 — 1/08/2018

PROFESSOR: SA BARRETO

OFFICE: MA 640

EMAIL: SABARRE@PURDUE.EDU

OFFICE HOURS MONDAYS 2:00 - 3:00
WEDNESDAYS: 3:00 - 4:00
BY APPOINTMENT

COURSE PAGE: WWW.MATH.PURDUE.EDU/MA261

LECTURE NOTES WILL BE SCANNED AND POSTED ON MY HOME PAGE
WWW.MATH.PURDUE.EDU/~SABARRE

EXAMS: FEB 21, 8:00 - 9:00 PM ELLIOTT
APR 03, 6:30 - 7:30 PM ELLIOTT

QUIZZES DURING (ALMOST) EVERY RECITATION
SEE COURSE CALENDAR ON COURSE WEB PAGE.

GRADES: EXAM 1. 100 POINTS

EXAM 2 : 100 POINTS

FINAL EXAM : 200 POINTS

QUIZZES : 50 POINTS

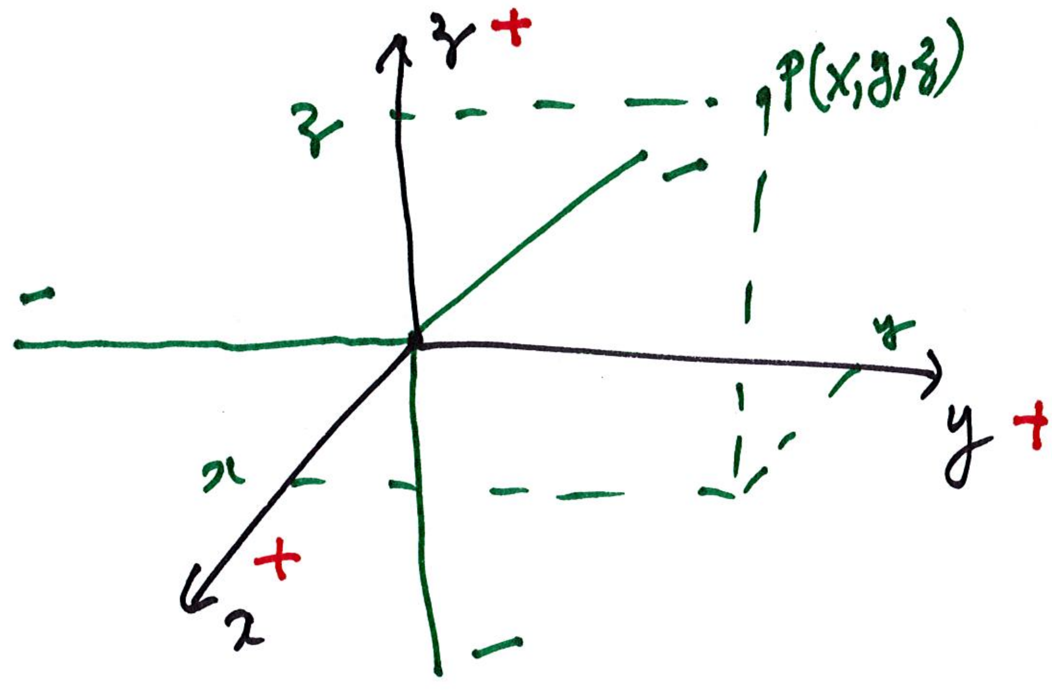
HOMEWORK : 100 POINTS

$\left.\begin{array}{l}\text{Lowest 3 QUIZ GRADES}\\ \text{LOWEST 3 HW GRADES}\end{array}\right\}$ WILL BE DROPPED

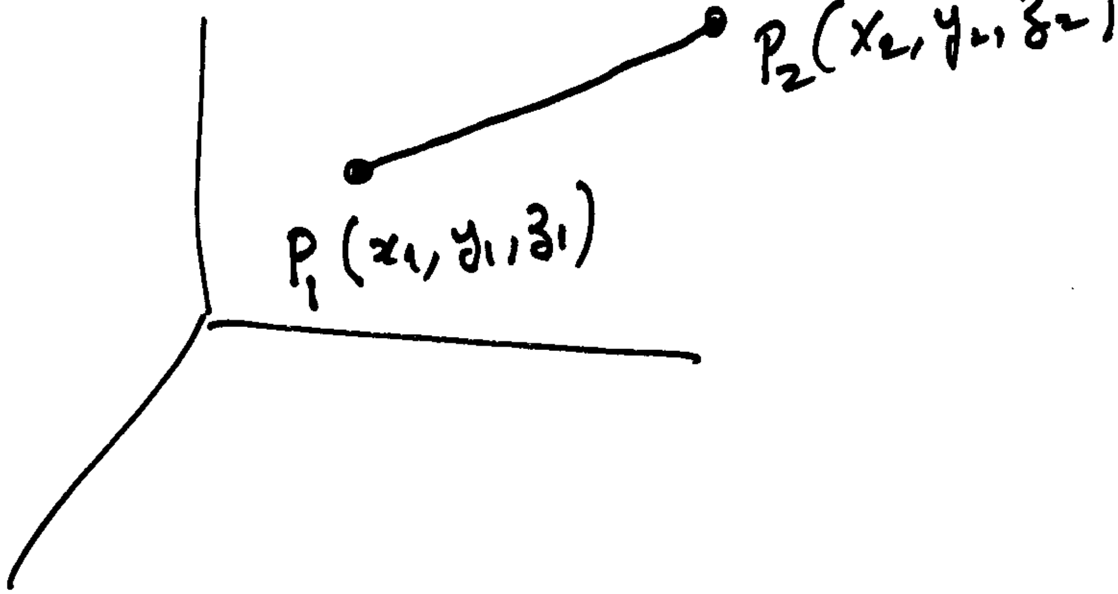
Lesson 1 Review of Vectors and Coordinates

Sections 12.1 to 12.4.

Coordinates in 3-D



Distance Between two points



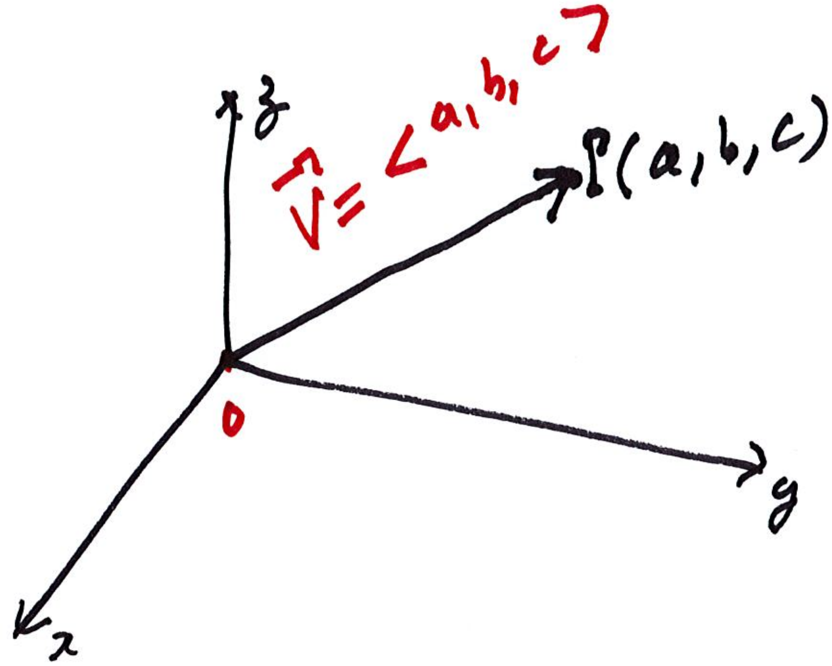
$$|P_1 P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

<u>Ex:</u> $P_1(1,2,3)$; $P_2(8,10,12)$

$$|P_1 P_2| = \sqrt{(8-1)^2 + (10-2)^2 + (12-3)^2} =$$

$$= \sqrt{49 + 64 + 81} \;.$$

Vectors



$\vec{V}$ = Oriented segment from 0 to
P(a, b, c)

Operations with Vectors

Addition: $\vec{V_1} = \langle a_1, b_1, c_1 \rangle$, $\vec{V_2} = \langle a_2, b_2, c_2 \rangle$

$$\vec{V_1} + \vec{V_2} = \langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle$$

Subtraction: $\vec{V_1} - \vec{V_2} = \langle a_1 - a_2, b_1 - b_2, c_1 - c_2 \rangle$

Examples $\vec{v}_1 = \langle 1, 2, 5 \rangle$, $\vec{v}_2 = \langle 2, 8, 9 \rangle$

$$\vec{v}_1 + \vec{v}_2 = \langle 3, 10, 14 \rangle$$

$$\vec{v}_1 - \vec{v}_2 = \langle -1, -6, -4 \rangle.$$

Product by numbers (scalars)

If r is a real number

and $\vec{v} = \langle a, b, c \rangle$

$$r\vec{v} = \langle ra, rb, rc \rangle$$

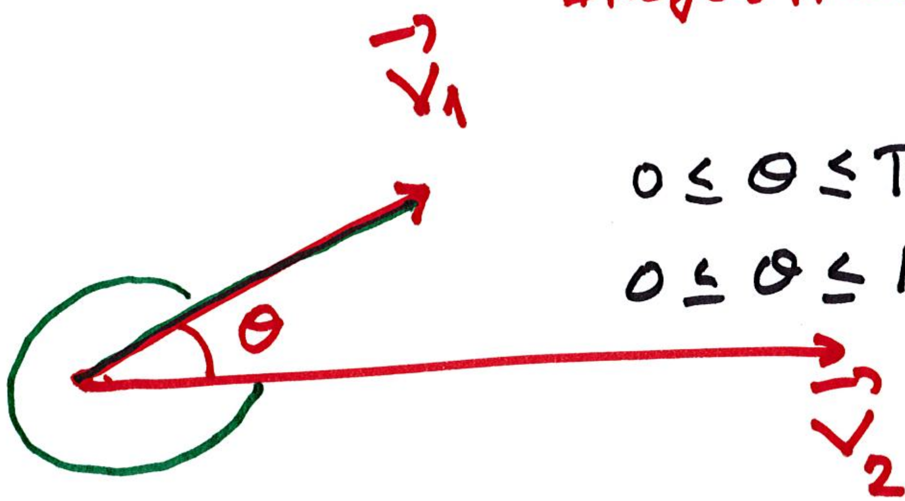
Ex: $\vec{v} = \langle 1, 5, 7 \rangle$

$$\sqrt{2}\,\vec{v} = \langle \sqrt{2}, 5\sqrt{2}, 7\sqrt{2} \rangle.$$

Products of Vectors

1. The dot product
2. The cross product
3. The triple product.

The Dot Product. (Angles and Projections)



$0 \leq \theta \leq \pi$

$0 \leq \theta \leq 180^\circ$

$\vec{V}_1 \cdot \vec{V}_2 = |\vec{V}_1| \cdot |\vec{V}_2| \cdot \cos\theta$

$|\vec{V}_1|$ = Length of $\vec{V}_1$, $|\vec{V}_2|$ = Length of $\vec{V}_2$.

If $\vec{V_1} = \langle a_1, b_1, c_1 \rangle$

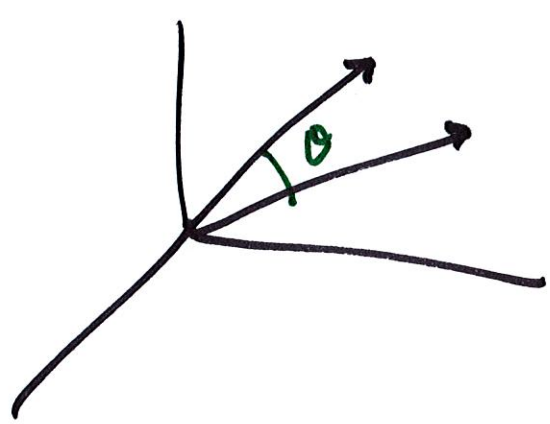
$\vec{V_2} = \langle a_2, b_2, c_2 \rangle$

$$\boxed{\vec{V_1} \cdot \vec{V_2} = a_1 a_2 + b_1 b_2 + c_1 c_2.}$$

$$|\vec{V_1}| = \sqrt{a_1^2 + b_1^2 + c_1^2}.$$

$$|\vec{V_2}| = \sqrt{a_2^2 + b_2^2 + c_2^2}.$$

<u>Example:</u> Find the cosine of the angle between $\vec{V_1} = \langle 1, 2, 3 \rangle$ and $\vec{V_2} = \langle 4, 5, 6 \rangle$



$$\vec{v_1} \cdot \vec{v_2} = |\vec{v_1}| \cdot |\vec{v_2}| \cdot \cos\theta$$

on the other hand.

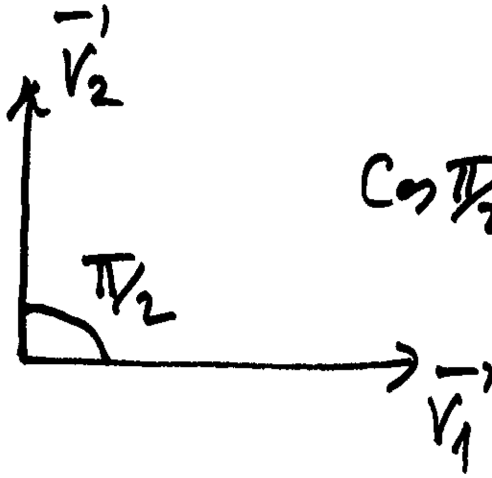
$$\vec{v_1} \cdot \vec{v_2} = 4 + 10 + 18 = 32.$$

$$|\vec{v_1}| = \sqrt{1+4+9} = \sqrt{14}$$

$$|\vec{v_2}| = \sqrt{16+25+36} = \sqrt{77}.$$

$$\sqrt{14} \cdot \sqrt{77} \cdot \cos\theta = 32$$

$$\boxed{\cos\theta = \frac{32}{\sqrt{14} \cdot \sqrt{77}}}$$

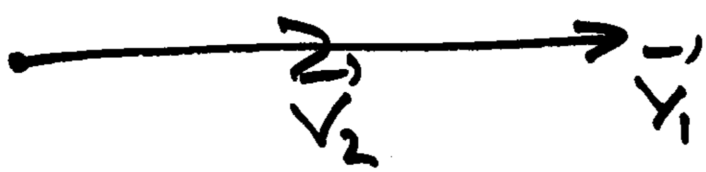


$\cos \pi/2 = 0$

$\vec{V_1}$ and $\vec{V_2}$ are perpendicular (orthogonal) if and only if

$$\vec{V_1} \cdot \vec{V_2} = 0$$

$\vec{V_1}$ and $\vec{V_2}$ are parallel if they lie on the same line

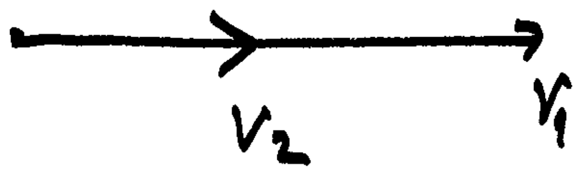


or

Two vectors are parallel

if $\theta = 0$

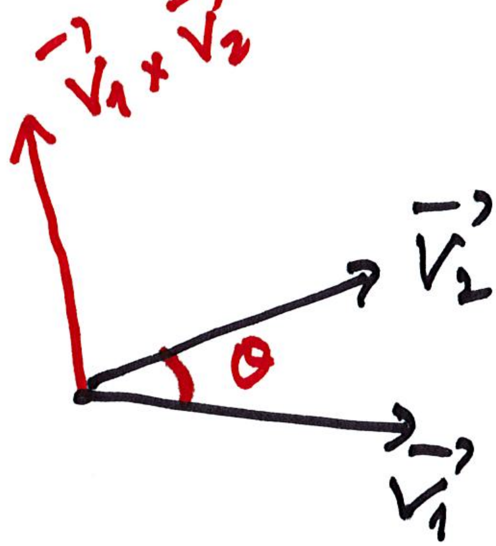
or $\theta = \pi$.



$\underline{\theta = 0}$ $\quad \vec{v}_1 \cdot \vec{v}_2 = |\vec{v}_1| \cdot |\vec{v}_2| \cdot \underbrace{\cos 0}_{\substack{\parallel \\ 1}} = |\vec{v}_1| \cdot |\vec{v}_2|$

$\underline{\theta = \pi}$ $\quad \vec{v}_1 \cdot \vec{v}_2 = - |\vec{v}_1| \cdot |\vec{v}_2|.$

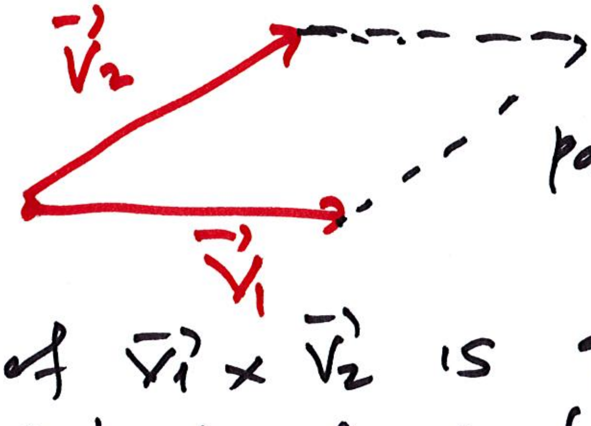
The Cross Product



1. $\vec{V_1} \times \vec{V_2}$ is perpendicular to both $\vec{V_1}$ and $\vec{V_2}$.

2. $|\vec{V_1} \times \vec{V_2}| = |\vec{V_1}| \cdot |\vec{V_2}| \cdot \sin\theta$

// Area of this parallelogram



3. Direction of $\vec{V_1} \times \vec{V_2}$ is given by the right hand rule.

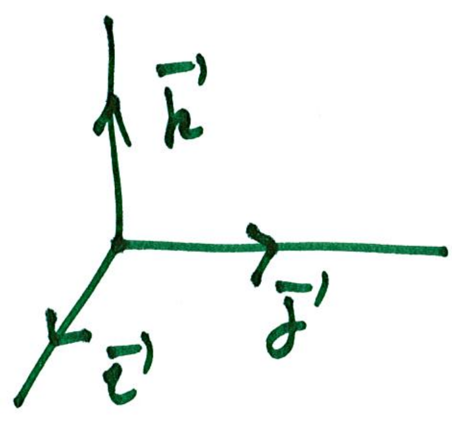
Use coordinates to compute Cross Products.

$$\vec{V}_1 = \langle a_1, b_1, c_1 \rangle = a_1 \vec{i} + b_1 \vec{j} + c_1 \vec{k}$$

$$\vec{V}_2 = \langle a_2, b_2, c_2 \rangle = a_2 \vec{i} + b_2 \vec{j} + c_2 \vec{k}$$

$\vec{i} = \langle 1, 0, 0 \rangle$, $\vec{j} = \langle 0, 1, 0 \rangle$

$\vec{k} = \langle 0, 0, 1 \rangle$



$$\vec{V}_1 = a_1 \vec{i} + b_1 \vec{j} + c_1 \vec{k}$$

$$\vec{V}_2 = a_2 \vec{i} + b_2 \vec{j} + c_2 \vec{k}$$

$$\vec{V_1} \times \vec{V_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - \vec{j} \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$+ \vec{k} \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

Compute: $\langle 1, 2, 3\rangle \times \langle 2, 1, 0\rangle$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 2 & 1 & 0 \end{vmatrix} =$$

$$\vec{i}\begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= \vec{i}\,(0-3) - \vec{j}\,(0-6) + \vec{k}\,(1-4)$$

$$= -3\vec{i} + 6\vec{j} - 3\vec{k}.$$

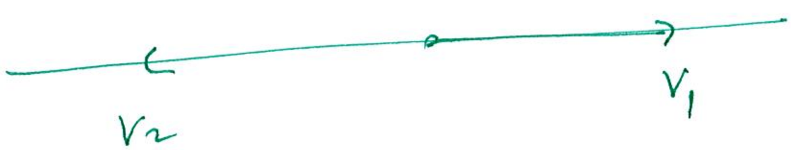
Orthogonal vectors: $\vec{V_1} \cdot \vec{V_2} = 0.$

Parallel vectors $\vec{V_1} \cdot \vec{V_2} = |\vec{V_1}| . |\vec{V_2}|.$

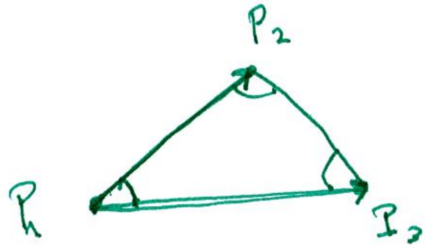
or $\vec{V_1} \cdot \vec{V_2} = - |\vec{V_1}| . |\vec{V_2}|.$

$\vec{V_1}$ and $\vec{V_2}$ lie on the same line

or



Example: Is the triangle with vertices $P_1(1, 2, 1)$, $P_2(-1, 1, 0)$ and $P_3(0, -1, 2)$ a right triangle?



$\vec{V_1} = \overline{P_1 P_2} = \langle -2, -1, -1 \rangle$

$\vec{V_2} = \overline{P_1 P_3} = \langle -1, -3, 1 \rangle$

$\vec{V_3} = \overline{P_2 P_3} = \langle 1, -2, 2 \rangle.$

$\vec{V_1} \cdot \vec{V_2} = 2 + 3 - 1 = 4$; $\vec{V_1} \cdot \vec{V_3} = -2 + 2 - 2 = -2$

$\vec{V_2} \cdot \vec{V_3} = -1 + 6 + 2 = 7.$

The Cross Product

$$\vec{V}_1 = a_1 \vec{i} + b_1 \vec{j} + c_1 \vec{k}$$

$$\vec{V}_2 = a_2 \vec{i} + b_2 \vec{j} + c_2 \vec{k}.$$

$$\vec{V}_1 \times \vec{V}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2. \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - \vec{j} \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + \vec{k} \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$= \vec{i} \left(b_1 c_2 - b_2 c_1 \right) - \vec{j} \left(a_1 c_2 - a_2 c_1 \right)$$

$$+ \vec{k} \left(a_1 b_2 - a_2 b_1 \right)$$

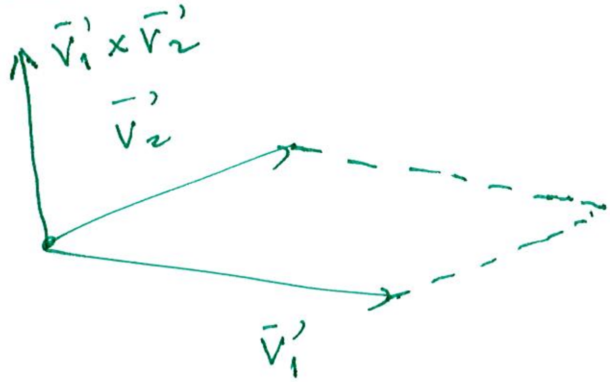
Example: $\vec{v}_1 = \langle 1, 2, 1 \rangle$; $\vec{v}_2 = \langle 0, 2, -1 \rangle$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 0 & 2 & -1 \end{vmatrix}$$

$$= \vec{i}\,(-2-2) - \vec{j}\,(-1-0) + \vec{k}\,(2-0)$$

$$= -4\vec{i} + \vec{j} + 2\vec{k}.$$

Geometric Interpretation:



$\vec{v}_1 \times \vec{v}_2$ is orthogonal to $\vec{v}_1$ and $\vec{v}_2$

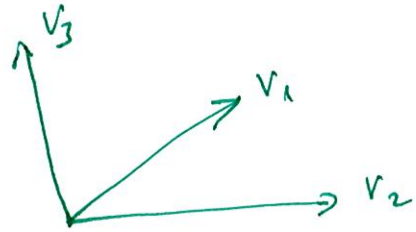
Direction given by the right hand rule.

$|\vec{v}_1 \times \vec{v}_2| =$ Area of the parallelogram

Triple Products:

$$\vec{V_1} \circ (\vec{V_2} \times \vec{V_3}) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$|\vec{V_1} \circ (\vec{V_2} \times \vec{V_3})|$ = Volume of the parallelepiped determined by $\vec{V_1}, \vec{V_2}, \vec{V_3}$



$\vec{V_1}, \vec{V_2}, \vec{V_3}$ are co-planar if

$$\vec{V_1} \circ (\vec{V_2} \times \vec{V_3}) = 0.$$